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BIPOLAR VALUED MULTI FUZZY SUBSEMIGROUPS OF A SEMIGROUP

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ABSTRACT

In this paper, we study some of the properties of bipolar valued multi fuzzy subsemigroup and prove some results on these.

Key Words: *Bipolar valued fuzzy subset, bipolar valued multi fuzzy subset, bipolar valued multi fuzzy subsemigroup, product, pseudo bipolar valued multi fuzzy coset.*

INTRODUCTION

In 1965, Zadeh [12] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [5]. Lee [7] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [–1, 1]. In a bipolar valued fuzzy subset, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [–1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [7, 8]. We introduce the concept of bipolar valued multi fuzzy subsemigroup and established some results.

1. PRELIMINARIES

1.1 Definition: A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form $A = \{ < x, A^+(x), A^-(x) >/ x\in X\}$, where $A^+ : X\rightarrow [0, 1]$ and $A^- : X\rightarrow [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

1.2 Example: A = {< a, 0.4, –0.7 >, < b, 0.8, –0.5 >, < c, 0.7, –0.4 >} is a bipolar valued fuzzy subset of X = {a, b, c}.

1.3 Definition: A bipolar valued multi fuzzy set (BVMFS) A in X is defined as an object of the form $A = \{< x, A_i^+(x), A_i^-(x) >/ x\in X\}$, where $A_i^+: X\rightarrow [0, 1]$ and $A_i^- : X\rightarrow [-1, 0]$. The positive membership degrees $A_i^+(x)$ denote the satisfaction degree of an element x to the property corresponding to a bipolar valued multi fuzzy set A and the negative membership degrees $A_i^-(x)$ denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued multi fuzzy set A.

1.4 Example: A = {< a, 0.3, 0, 2, 0.3, –0.3, –0.7, –0.4 >, < b, 0.2, 0.5, 0.6, –0.7, –0.2, –0.6 >, < c, 0.5, 0.4, 0.7, –0.4, –0.2, –0.1 >} is a bipolar valued multi fuzzy subset with order three of X = {a, b, c}.

1.5 Definition: Let S be a semigroup. A bipolar valued multi fuzzy subset A of S is said to be a bipolar valued multi fuzzy subsemigroup of S if the following conditions are satisfied

(i) $A_i^+(xy) \geq \min \{A_i^+ (x), A_i^+ (y)\}$

(ii) $A_i^- (xy) \leq \max \{A_i^- (x), A_i^- (y)\}$ for all x and y in S.

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1.6 Example: Let S = {1, −1, i, −i} be a semigroup with respect to the ordinary multiplication. Then A = {< 1, 0.6, 0.6, 0.5, −0.6, −0.7, −0.3 >, < −1, 0.5, 0.5, 0.4, −0.5, −0.6, −0.2 >, < i, 0.3, 0.2, 0.3, −0.4, −0.5, −0.1 >, < −i, 0.3, 0.2, 0.3, −0.4, −0.5, −0.1 >} is a bipolar valued multi fuzzy subsemigroup of S.

1.7 Definition: Let $A = \langle A_i^+, A_i^- \rangle$ and $B = \langle B_i^+, B_i^- \rangle$ be any two bipolar valued multi fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as $A\times B = \{ \langle (x, y), (A_i\times B_i)^+(x, y), (A_i\times B_i)^-(x, y) \rangle$ / for all x in G and y in H } where $(A_i\times B_i)^+(x, y) = \min \{A_i^+(x), B_i^+(y) \}$ and $(A_i\times B_i)^-(x, y) = \max \{A_i^-(x), B_i^-(y) \}$ for all x in G and y in H.

1.8 Definition: Let $A = \langle A_i^+, A_i^- \rangle$ be a bipolar valued multi fuzzy subset in a set S, the strongest bipolar valued multi fuzzy relation on S, that is a bipolar valued multi fuzzy relation on A is $V = \{\langle (x, y), V_i^+(x, y), V_i^-(x, y) \rangle$ / x and y in S} given by $V_i^+(x, y) = \min \{A_i^+(x), A_i^+(y)\}$ and $V_i^-(x, y) = \max \{ A_i^-(x), A_i^-(y) \}$ for all x and y in S.

1.9 Definition: Let $A = \langle A_i^+, A_i^- \rangle$ be a bipolar valued multi fuzzy subsemigroup of a semigroup S and a in S. Then the **pseudo bipolar valued multi fuzzy coset** $(aA)^p = \langle (aA_i^+)^p, (aA_i^-)^p \rangle$ is defined by $(aA_i^+)^p(x) = p(a)\, A_i^+(x)$ and $(aA_i^-)^p(x) = p(a)A_i^-(x)$, for every x in S and for some p in P.

2. PROPERTIES

2.1 Theorem: Let $A = \langle A_i^+, A_i^- \rangle$ be a bipolar valued multi fuzzy subsemigroup of a semigroup S.
(i) If $A_i^+(xy) = 0$, then either $A_i^+(x) = 0$ or $A_i^+(y) = 0$ for x and y in S.
(ii) If $A_i^-(xy) = 0$, then either $A_i^-(x) = 0$ or $A_i^-(y) = 0$ for x and y in S.

Proof: Let x and y in S.

(i) By the definition $A_i^+(xy) \geq \min \{A_i^+(x), A_i^+(y)\}$ which implies that $0 \geq \min \{A_i^+(x), A_i^+(y)\}$. Therefore either $A_i^+(x) = 0$ or $A_i^+(y) = 0$.
(ii) By the definition $A_i^-(xy) \leq \max \{ A_i^-(x), A_i^-(y)\}$ which implies that $0 \leq \max \{A_i^-(x), A_i^-(y)\}$. Therefore either $A_i^-(x) = 0$ or $A_i^-(y) = 0$.

2.2 Theorem: If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then

$$H = \{x\in S \mid A_i^+(x) = 1, A_i^-(x) = -1\}$$ is either empty or a subsemigroup of S.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $A_i^+(xy) \geq \min\{A_i^+(x), A_i^+ (y)\} = \min\{1, 1\} = 1$. Therefore $A_i^+ (xy) = 1$. And $A_i^- (xy) \leq \max\{A_i^- (x), A_i^- (y)\} = \max \{-1, -1\}= -1$. Therefore $A_i^- (xy) = -1$. That is $xy\in H$. Hence H is a subsemigroup of S. Hence H is either empty or a subsemigroup of S.

2.3 Theorem: If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of S, then $H = \{x\in S \mid A_i^+(x) = H(A_i^+)$ and $A_i^- (x) = H(A_i^-)\}$ is a subsemigroup of S.

Proof: Here $H = \{x\in S \mid A_i^+(x) = H(A_i^+)$ and $A_i^-(x) = H(A_i^-)\}$. Let x, y in H. Then $A_i^+(xy) \geq \min \{A_i^+(x), A_i^+(y) \} = \min \{H(A_i^+), H(A_i^+)\} = H(A_i^+)$. Hence $A_i^+(xy) = H(A_i^+)$. Also $A_i^-(xy) \leq \max \{A_i^-(x), A_i^-(y) \} = \max \{ H(A_i^-), H(A_i^-)\} = H(A_i^-)$. Hence $A_i^- (xy) = H(A_i^-)$. Therefore $xy\in H$. Hence H is a subsemigroup of S.

2.4 Theorem: If $A = \langle A_i^+, A_i^- \rangle$ and $B = \langle B_i^+, B_i^- \rangle$ are two bipolar valued multi fuzzy subsemigroups of a semigroup S, then their intersection $A\cap B$ is a bipolar valued multi fuzzy subsemigroup of S.

Proof: Let $A = \{< x, A_i^+(x), A_i^-(x) > / x\in S\}$, $B = \{< x, B_i^+(x), B_i^-(x) > / x\in S \}$. Let $C = A\cap B$ and $C = \{< x, C_i^+(x), C_i^-(x) > / x\in S\}$. Now $C_i^+(xy) = \min \{A_i^+(xy), B_i^+(xy)\}\geq \min \{\min \{A_i^+(x), A_i^+(y)\}, \min \{B_i^+(x), B_i^+(y) \} \}\geq \min \{\min \{A_i^+(x), B_i^+(x)\}, \min \{A_i^+(y), B_i^+(y)\}\}= \min \{C_i^+(x), C_i^+(y)\}$. Therefore $C_i^+(xy) \geq \min \{C_i^+(x), C_i^+(y)\}$. Also $C_i^-(xy) = \max \{A_i^-(xy), B_i^-(xy)\}\leq \max\{ \max \{A_i^-(x), A_i^-(y)\}, \max \{B_i^-(x), B_i^-(y)\}\} \leq \max \{ \max \{ A_i^-(x), B_i^-(x)\}, \max \{A_i^-(y), B_i^-(y)\}\} = \max \{C_i^-(x), C_i^-(y)\}$. Therefore $C_i^-(xy) \leq \max\{C_i^-(x), C_i^-(y)\}$. Hence $A\cap B$ is a bipolarvalued multi fuzzy subsemigroup of S.

2.5 Theorem: The intersection of a family of bipolar valued multi fuzzy subsemigroups of a semigroup S is a bipolar valued multi fuzzy subsemigroup of S.

Proof: The Theorem is true by Theorem 2.4.

2.6 Theorem: If $A= \langle A_i^+, A_i^-\rangle$ and $B = \langle B_i^+, B_i^- \rangle$ are any two bipolar valued multi fuzzy subsemigroups of the semigroups S_1 and S_2 respectively, then $A\times B = \langle (A_i\times B_i)^+, (A_i\times B_i)^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of $S_1\times S_2$.

Proof: Let A and B be two bipolar-valued multi fuzzy subsemigroups of the semigroups S_1 and S_2 respectively. Let x_1 and x_2 be in S_1, y_1 and y_2 be in S_2. Then (x_1, y_1) and (x_2, y_2) are in $S_1 \times S_2$. Now, $(A_i \times B_i)^+[(x_1, y_1)(x_2, y_2)] = (A_i \times B_i)^+(x_1x_2, y_1y_2) = \min \{A_i^+(x_1x_2), B_i^+(y_1y_2)\} \geq \min \{\min \{A_i^+(x_1), A_i^+(x_2)\}, \min \{B_i^+(y_1), B_i^+(y_2)\}\} = \min \{\min \{A_i^+(x_1), B_i^+(y_1)\}, \min\{A_i^+(x_2), B_i^+(y_2)\}\} = \min \{(A_i \times B_i)^+(x_1, y_1), (A_i \times B_i)^+(x_2, y_2)\}$. Therefore $(A_i \times B_i)^+ [(x_1, y_1)(x_2, y_2)] \geq \min\{ (A_i \times B_i)^+(x_1, y_1), (A_i \times B_i)^+(x_2, y_2)\}$. Also $(A_i \times B_i)^-[(x_1, y_1)(x_2, y_2)] = (A_i \times B_i)^-(x_1x_2, y_1y_2) = \max \{A_i^-(x_1x_2), B_i^-(y_1y_2)\} \leq \max \{\max \{A_i^-(x_1), A_i^-(x_2)\}, \max \{B_i^-(y_1), B_i^-(y_2)\}\} = \max \{\max \{A_i^-(x_1), B_i^-(y_1)\}, \max \{A_i^-(x_2), B_i^-(y_2)\}\} = \max \{(A_i \times B_i)^- (x_1, y_1), (A_i \times B_i)^- (x_2, y_2)\}$. Therefore $(A_i \times B_i)^-[(x_1, y_1)(x_2, y_2)] \leq \max \{(A_i \times B_i)^-(x_1, y_1), (A_i \times B_i)^-(x_2, y_2)\}$. Hence A×B is a bipolar valued multi fuzzy subsemigroup of $S_1 \times S_2$.

2.7 Theorem: Let $A = \langle A_i^+, A_i^- \rangle$ be a bipolar valued multi fuzzy subset of a semigroup (S, .) and $V = \langle V_i^+, V_i^- \rangle$ be the strongest bipolar valued multi fuzzy relation of S. If A is a bipolar valued multi fuzzy subsemigroup of S, then V is a bipolar valued multi fuzzy subsemigroup of S×S.

Proof: Suppose that A is a bipolar valued multi fuzzy subsemigroup of S. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in S×S. We have $V_i^+ (xy) = V_i^+ [(x_1, x_2)(y_1, y_2)] = V_i^+ (x_1y_1, x_2y_2) = \min \{A_i^+ (x_1y_1), A_i^+ (x_2y_2) \} \geq \min \{\min \{ A_i^+ (x_1), A_i^+ (y_1)\}, \min \{A_i^+ (x_2), A_i^+ (y_2)\}\} = \min \{\min \{A_i^+ (x_1), A_i^+ (x_2)\}, \min \{A_i^+ (y_1), A_i^+ (y_2)\}\} = \min \{V_i^+(x_1, x_2), V_i^+(y_1, y_2)\} = \min \{V_i^+(x), V_i^+(y)\}$. Therefore $V_i^+(xy) \geq \min \{V_i^+(x), V_i^+(y)\}$for all x and y in S×S. Also we have $V_i^-(xy)= V_i^-[(x_1, x_2)(y_1, y_2)] = V_i^-(x_1y_1, x_2y_2) = \max\{A_i^-(x_1y_1), A_i^-(x_2y_2)\} \leq \max \{\max \{A_i^-(x_1), A_i^-(y_1)\}, \max \{A_i^-(x_2), A_i^-(y_2)\}\} = \max \{\max \{A_i^-(x_1), A_i^-(x_2) \}, \max \{ A_i^-(y_1), A_i^-(y_2)\}\} = \max \{V_i^-(x_1, x_2), V_i^-(y_1, y_2)\} = \max \{V_i^-(x), V_i^-(y)\}$. Therefore $V_i^-(xy) \leq \max \{V_i^-(x), V_i^-(y) \}$ for all x and y in S×S. Hence V is a bipolar valued multi fuzzy subsemigroup of S×S.

2.8 Theorem: Let $A = \langle A_i^+, A_i^- \rangle$ be a bipolar valued multi fuzzy subsemigroup of a semigroup S. Then the pseudo bipolar valued multi fuzzy coset $(aA)^p = \langle (a\ A_i^+)^p, (a\ A_i^-)^p \rangle$ is a bipolar valued multi fuzzy subsemigroup of the semigroup S, for every a in S and p in P.

Proof: Let A be a bipolar valued multi fuzzy subsemigroup of the semigroup S. For every x and y in S, we have $(aA_i^+)^p(xy) = p(a)A_i^+(xy) \geq p(a) \min \{A_i^+(x), A_i^+(y) \}= \min \{p(a)A_i^+(x), p(a)A_i^+(y)\}= \min \{(aA_i^+)^p(x), (aA_i^+)^p(y)\}$. Therefore $(aA_i^+)^p(xy) \geq \min \{(aA_i^+)^p(x), (aA_i^+)^p(y)\}$ for x and y in S. And $(aA_i^-)^p(xy) = p(a)A_i^-(xy) \leq p(a) \max \{A_i^-(x), A_i^-(y)\}= \max\{p(a)A_i^-(x), p(a)A_i^-(y)\}= \max\{(aA_i^-)^p(x), (aA_i^-)^p(y)\}$. Therefore $(aA_i^-)^p(xy) \leq \max \{(aA_i^-)^p(x), (aA_i^-)^p(y)\}$ for x and y in S. Hence $(aA)^p$ is a bipolar valued multi fuzzy subsemigroup of the semigroup S.

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