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UNIQUENESS OF ENTIRE FUNCTIONS CONCERNING q-SHIFT DIFFERENE POLYNOMIALS

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ABSTRACT

In this paper, we investigate the uniqueness problem of q-shift difference polynomials sharing a small functions. With the notion of weakly weighted sharing and relaxed weighted sharing we extend some well known previous results.

1. INTRODUCTION, DEFINITIONS AND RESULTS

By a meromorphic function we shall always mean a meromorphic function in the complex plane. Let k be a positive integer or infinity and $a \in C \cup \{\infty\}$. Set $E(a,f) = \{z{:}\, f(z) - a = 0\}$, where a zero point with multiplicity k is counted k times in the set. If these zeros points are only counted once, then we denote the set by $\bar{E}(a,f)$. Let f and g be two nonconstant meromorphic functions. If $E(a,f) = E(a,g)$, then we say that f and g share the value a CM; if $\bar{E}(a,\ f) = \bar{E}(a,\ g)$, then we say that f and g share the value a IM. We denote by $E_{k)}(a,f)$ the set of all a-points of f with multiplicities not exceeding k, where an a-point is counted according to its multiplicity. Also we denote by $\bar{E}_{k)}(a,\ f)$ the set of distinct a-points of f with multiplicities not greater than k. It is assumed that the reader is familiar with the notations of Nevanlinna theory such as $T(r,f), m(r,f), N(r,f), \bar{N}(r,f), S(r,f)$ and so on, that can be found, for instance, in [4], [12]. We denote by $N_{k)}(r,\frac{1}{(f-a)})$ the counting function for zeros of $f-a$ with multiplicity less or equal to k, and by $\bar{N}_{k)}(r,\frac{1}{(f-a)})$ the corresponding one for which multiplicity is not counted. Let $N_{(k}(r,\frac{1}{(f-a)})$ be the counting function for zeros of $f-a$ with multiplicity atleast k and $\bar{N}_{(k}(r,\frac{1}{(f-a)})$ the corresponding one for which multiplicity is not counted.

Set

$$N_k\left(r,\frac{1}{(f-a)}\right) = \bar{N}\left(r,\frac{1}{f-a}\right) + \bar{N}_{(2}\left(r,\frac{1}{f-a}\right) + \cdots + \bar{N}_{(k}\left(r,\frac{1}{f-a}\right).$$

Let $N_E(r,a;\ f,g)\big(\bar{N}_E(r,a;\ f,g)\big)$be the counting function (reduced counting function) of all common zeros of $f-a$ and $g-a$ with the same multiplicities and $N_0(r,a;\ f,g)$ $(\bar{N}_0(r,a;\ f,g))$ the counting function (reduced counting function) of all common zeros of $f-a$ and $g-a$ ignoring multiplicities. If

$$\bar{N}\left(r,\frac{1}{f-a}\right) + \bar{N}\left(r,\frac{1}{g-a}\right) - 2N_E(r,a;\ f,g) = S(r,f) + S(r,g),$$

then we say that f and g share a "CM". On the other hand, if

$$\bar{N}\left(r,\frac{1}{f-a}\right) + \bar{N}\left(r,\frac{1}{g-a}\right) - 2\bar{N}_0(r,a;\ f,g) = S(r,f) + S(r,g),$$

then we say that $f$and g share a "IM".

We now explain in the following definition the notion of weakly weighted sharing which was introduced by Lin and Lin [8].

Definition 1 ([8]): Let f and g share a "IM" and k be a positive integer or ∞. $\bar{N}^E_{k)}(r,a;f,g)$ denotes the reduced counting function of those a −points of f whose multiplicities are equal to the corresponging a-points of g, and both of their multiplicities are not greater than k.

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$\overline{N}^0_{(k}(r, a; f, g)$ denotes the reduced counting function of those a-points of f which are a-points of g, and both of their multiplicities are not less than k.

Definition 2 ([8]): For $a \in C \cup \{\infty\}$, if k is a positive integer or ∞ and

$$\overline{N}_{k)}\left(r, \frac{1}{(f-a)}\right) - \overline{N}^E_{k)}(r, a; f, g) = S(r, f),$$
$$\overline{N}_{k)}\left(r, \frac{1}{(g-a)}\right) - \overline{N}^E_{k)}(r, a; f, g) = S(r, g),$$
$$\overline{N}_{(k+1}\left(r, \frac{1}{(f-a)}\right) - \overline{N}^0_{(k+1}(r, a; f, g) = S(r, f),$$
$$\overline{N}_{(k+1}\left(r, \frac{1}{(g-a)}\right) - \overline{N}^0_{(k+1}(r, a; f, g) = S(r, g),$$

or of $k = 0$ and $\overline{N}\left(r, \frac{1}{f-a}\right) - \overline{N}_0(r, a;\ f, g) = S(r, f),\quad \overline{N}\left(r, \frac{1}{g-a}\right) - \overline{N}_0(r, a;\ f, g) = S(r, g),$
then we say f and g weakly share a with weight k. Here we write f, g share *"(a,k)"* to mean that f, g weakly share a with weight k.

Now it is clear from Definition 2 that weakly weighted sharing is a scaling between IM and CM.

Recently, A. Banerjee and S. Mukherjee [1] introduced another sharing notion which is also a scaling between IM and CM but weaker than weakly weighted sharing.

Definition 3 ([1]): We denote by $\overline{N}(r, a; f| = p; g| = q)$ the reduced counting function of common a-points of f and g with multiplicities p and q, respectively.

Definition 4 ([1]): Let f, g share a "IM". Also let k be a positive integer or ∞ and $a \in C \cup \{\infty\}$. If

$$\sum_{p,q \le K} \overline{N}(r, a; f| = p; g| = q) = S(r),$$

then we say f and g share a with weight k in a relaxed manner. Here we write f and g share $(a, k)^*$ to mean that f and g share a with weight k in a relaxed manner.

W.K Hayman proposed the following well-known conjecture in [5].

Hayman's conjecture: If an entire function f satisfies $f^n f' \neq 1$ for all positive integers $n \in N$, then f is a constant.

It has been verified by Hayman himself in [6] for the case $n > 1$ and Clunie in [3] for the case $n \geq 1$, respectively.

It is well-known that if f and g share four distinct values CM, then f is Mobius transformation of g. In 2011, Liu and Cao [10], have obtained results on the uniqueness and value distribution of q-shift difference polynomials. Some of them are stated below.

Theorem A. [10, Theorem 1.1]: Let $f(z)$ be a transcendental meromorphic (resp. entire) function with zero order, and let m, n be positive integers and a, q be non-zero complex constants. If $n \geq 6 (resp. n \geq 2)$, then $f^n(z)(f^m(z) - a)f(qz + c) - \alpha(z)$ has infinitely many zeros, where $\alpha(z)$ is a non-zero small function with respect to f. In particular, if $f(z)$ is a transcendental entire function and $\alpha(z)$ is a non-zero rational function, then m and n can be any positive integers.

Theorem B [10, Theorem 1.5]: Let $f(z)$ and $g(z)$ be a transcendental entire functions with zero order. If $n \geq m + 5$, and $f^n(z)(f^m(z) - a)f(qz + c)$and $g^n(z)(g^m(z) - a)g(qz + c)$ share a non-zero polynomial $p(z)$ CM, then $f(z) \equiv g(z)$.

In 2015, on the basis of Theorems A and B, Q. Zhao and J. Zhang [14] study the k-th derivative of q-shift difference polynomials and proved the following results.

Theorem C: Let $f(z)$ be a transcendental meromorphic function with zero order, and let n, k be positive integers. If $n > k + 5$, then $(f^n(z)f(qz + c))^{(k)} - 1$ has infinitely many zeros.

Theorem D: Let $f(z)$ be a transcendental entire function with zero order, and let n, k be positive integers, then $(f^n(z)f(qz + c))^{(k)} - 1$ has infinitely many zeros.

Theorem E: Let $f(z)$ be a transcendental entire functions with zero order, and let n, k be positive integers. If $n > 2k+5$, and $(f^n(z)f(qz+c))^{(k)}$ and $g^n(z)g(qz+c))^{(k)}$ share z CM, then $f = tg$ for a constant t with $t^{n+1} = 1$.

Theorem F: Let $f(z)$ be a transcendental entire functions with zero order, and let n, k be positive integers. If $n > 2k+5$, and $(f^n(z)f(qz+c))^{(k)}$ and $g^n(z)g(qz+c))^{(k)}$ share 1 CM, then $f = tg$ for a constant t with $t^{n+1} = 1$.

When sharing a single value IM, and obtain the following theorems.

Theorem G: Let $f(z)$ and $g(z)$ be transcendental entire functions with zero order, and let n, k be positive integer. If $n > 5k+11$, and $(f^n(z)f(qz+c))^{(k)}$ and $g^n(z)g(qz+c))^{(k)}$ share z IM, then $f = tg$ for a constant t with $t^{n+1} = 1$.

Theorem H: Let $f(z)$ and $g(z)$ be transcendental entire functions with zero order, and let n, k be positive integer. If $n > 5k+11$, and $(f^n(z)f(qz+c))^{(k)}$ and $g^n(z)g(qz+c))^{(k)}$ share 1 IM, then $f = tg$ for a constant t with $t^{n+1} = 1$.
In this paper by introducing the small function $\alpha(z)$, we prove the following results.

Theorem 1: Let $f(z)$ and $g(z)$ be a transcendental entire functions of finite order, and $\alpha(z)$ be a small function with respect to both $f(z)$ and $g(z)$. Suppose that c is a non-zero complex constant and $n \geq 4k+m+6$ is an integer. If $(f^n(z)(f^m(z)-1)f(qz+c))^{(k)}$ and $(g^n(z)(g^m(z)-1)g(qz+c))^{(k)}$ share "$(\alpha(z),2)$", then $f(z) \equiv g(z)$.

Theorem 2: Let $f(z)$ and $g(z)$ be a transcendental entire functions of finite order, and $\alpha(z)$ be a small function with respect to both $f(z)$ and $g(z)$. Suppose that c is a non-zero complex constant and $n > 6k+3m+8$ is an integer. If $(f^n(z)(f^m(z)-1)f(qz+c))^{(k)}$ and $(g^n(z)(g^m(z)-1)g(qz+c))^{(k)}$ share $(\alpha(z),2)^*$, then $f(z) \equiv g(z)$.

Without the notions of weakly weighted sharing and relaxed weighted sharing we prove the following theorem.

Theorem 3: Let $f(z)$ and $g(z)$ be a transcendental entire functions of finite order, and $\alpha(z)$ be a small function with respect to both $f(z)$ and $g(z)$. Suppose that c is a non-zero complex constant and $n > 10k+5m+12$ is an integer. If $\overline{E}_{2)}\left(\alpha(z), f^n(z)(f^m(z)-1)f(qz+c)\right) = \overline{E}_{2)}(\alpha(z), g^n(z)(g^m(z)-1)g(qz+c))$ then $f(z) \equiv g(z)$.

2. LEMMAS

In this section, we present some lemmas which play an important role in the proof of the main results. We will denote by H the following function;

$$H = \left(\frac{F''}{F'} - \frac{2F'}{F-1}\right) - \left(\frac{G''}{G'} - \frac{2G'}{G-1}\right)$$

Lemma 1 ([1]): H be defined as above. If F and G share "(1,2)" and $H \not\equiv 0$, then

$$T(r,F) \leq N_2\left(r,\frac{1}{F}\right) + N_2\left(r,\frac{1}{G}\right) + N_2(r,F) + N_2(r,G) - \sum_{p=3}^{\infty} \overline{N}_{(p}\left(r,\frac{G}{G'}\right) + S(r,F) + S(r,G),$$

and the same inequality holds for $T(r,G)$.

Lemma 2 ([1]): Let H be defined as above. If F and G share $(1,2)^*$ and $H \not\equiv 0$, then

$$T(r,F) \leq N_2\left(r,\frac{1}{F}\right) + N_2\left(r,\frac{1}{G}\right) + N_2(r,F) + N_2(r,G) + \overline{N}\left(r,\frac{1}{F}\right) + \overline{N}(r,F) - m\left(r,\frac{1}{G-1}\right) + S(r,F) + S(r,G),$$

and the same inequality holds for $T(r,G)$.

Lemma 3 ([13]): Let H be defined as above. If $H \equiv 0$ and

$$\limsup_{r\to\infty} \frac{\overline{N}\left(r,\frac{1}{F}\right) + \overline{N}(r,F) + \overline{N}\left(r,\frac{1}{G}\right) + \overline{N}(r,G)}{T(r)} < 1, r \in I,$$

where $T(r) = \max\{T(r,F), T(r,G)\}$ and I is a set with infinite linear measure then $F \equiv G$ or $FG \equiv 1$.

Lemma 4 ([2]): Let $f(z)$ be a meromorphic function in the complex plane of finite order $\sigma(f)$, and let η be a fixed non-zero complex number. Then for each $\epsilon > 0$, one had

$$T(r, f(z+\eta)) = T(r, f(z)) + O\left(r^{\sigma(f)-1+\epsilon}\right) + O(\log r)$$

Lemma 5 ([11]): Let $f(z)$ be an entire function of finite order $\sigma(f)$, c is a fixed non-zero complex number, and

$$P(z) = a_n f^n(z) + a_{n-1} f^{n-1}(z) + \cdots + a_1 f(z) + a_0$$

where $a_j\,(j = 0,1,\ldots,n)$ are constants. If $F(z) = P(z)f(z+c)$, then

$$T(r,F) = (n+1)T(r,f) + O\left(r^{\sigma(f)-1+\epsilon}\right) + O(\log r).$$

Lemma 6 ([9]): Let F and G be two nonconstant entire functions, and $p \geq 2$ an integer. If $\bar{E}_{p)}(1,F) = \bar{E}_{p)}(1,G)$ and $H \not\equiv 0$, then

$$T(r,F) = N_2\left(r,\frac{1}{F}\right) + N_2\left(r,\frac{1}{G}\right) + 2\bar{N}\left(r,\frac{1}{F}\right) + \bar{N}\left(r,\frac{1}{G}\right) + S(r,F) + S(r,G).$$

Lemma 7 ([7]): Let $f(z)$ be a nonconstant meromorphic function, and let s, k be two positive integers. Then

$$N_s\left(r,\frac{1}{f^{(k)}}\right) \leq T\left(r,f^{(k)}\right) - T(r,f) + N_{s+k}\left(r,\frac{1}{f}\right) + S(r,f),$$

$$N_s\left(r,\frac{1}{f^{(k)}}\right) \leq k\bar{N}(r,f) + N_{s+k}\left(r,\frac{1}{f}\right) + S(r,f).$$

Clearly, $\bar{N}\left(r,\frac{1}{f^{(k)}}\right) = N_1\left(r,\frac{1}{f^{(k)}}\right)$.

3. PROOF OF THEOREM 1

Let $F(z) = \frac{[f^n(f^m-1)f(qz+c)]^{(k)}}{\alpha(z)}$, $G(z) = \frac{[g^n(g^m-1)g(qz+c)]^{(k)}}{\alpha(z)}$, Then $F(z)$ and $G(z)$ share "(1,2)" except the zeros or poles of $\alpha(z)$. By Lemma 5, we have

$$T\left(r,F(z)\right) = T\left(r,f^n(f^m-1)f(qz+c)\right) + k\bar{N}(r,f) + S(r,f). \tag{1}$$

$$T\left(r,G(z)\right) = T\left(r,g^n(g^m-1)g(qz+c)\right) + k\bar{N}(r,g) + S(r,g). \tag{2}$$

Also from Lemma 7, we obtain

$$\begin{aligned} N_2\left(r,\tfrac{1}{F}\right) &\leq N_{k+2}\left(r,\tfrac{1}{f^n(f^m-1)f(qz+c)}\right) + S(r,f) \\ &\leq (k+2)N\left(r,\tfrac{1}{f}\right) + N\left(r,\tfrac{1}{f^m-1}\right) + N\left(r,\tfrac{1}{f(qz+c)}\right) + k\bar{N}(r,f) + S(r,f) \\ &\leq (2k+m+3)T(r,f) + S(r,f) \end{aligned} \tag{3}$$

and

$$N_2\left(r,\tfrac{1}{G}\right) \leq (2k+m+3)T(r,g) + S(r,g) \tag{4}$$

Suppose $H \not\equiv 0$, then by Lemma 1 and Lemma 4, we have

$$T(r,F) + T(r,G) \leq 2N_2\left(r,\frac{1}{F}\right) + 2N_2\left(r,\frac{1}{G}\right) + S(r,f) + S(r,g)$$

$$(n+m+1)[T(r,f) + T(r,g)] \leq (4k+2m+6)[T(r,f) + T(r,g)] + S(r,f) + S(r,g)$$

$$(n-4k-m-5)[T(r,f) + T(r,g)] \leq O\left(r^{\sigma(f)-1+\epsilon}\right) + O\left(r^{\sigma(g)-1+\epsilon}\right) + S(r,f) + S(r,g) \tag{5}$$

which contradicts with $n > 4k+m+6$. Thus we have $H \equiv 0$. Note that

$$\begin{aligned} \bar{N}\left(r,\tfrac{1}{F}\right) + \bar{N}\left(r,\tfrac{1}{G}\right) &\leq (2k+m+2)T(r,f) + (2k+m+2)T(r,g) + S(r,f) + S(r,g) \\ &\leq T(r). \end{aligned}$$

Where $T(r) = \max\{T(r,F), T(r,G)\}$. By Lemma 3, we deduce that either $F \equiv G$ or $FG \equiv 1$. Next we will consider the following two cases, respectively.

Case-1: $F \equiv G$, thus $f^n(f^m-1)f(qz+c) \equiv g^n(g^m-1)g(qz+c)$. Let $\varphi(z) = \frac{f(z)}{g(z)}$. If $\varphi^{n+m}(z)(qz+c) \not\equiv 1$, we have

$$g^m(z) = \frac{\varphi^n(z)\varphi(qz+c)-1}{\varphi^{n+m}(z)\varphi(qz+c)-1} \tag{6}$$

Then $\varphi(z)$ is a transcendental meromorphic function of finite order since $g(z)$ is transcendental. By Lemma 4, we have

$$T(r,\varphi(qz+z)) = T(r,\varphi(z)) + S(r,\varphi). \tag{7}$$

If $\varphi^{n+m}(z)\varphi(z+c) = k(\neq 1)$, where k is a constant, the Lemma 4 and (7) imply that

$$(n+m)T(r,\varphi(z)) = T(r,\varphi(z+c)) + O(1) = T(r,\varphi(z)) + O\left(r^{\sigma(\varphi(z))-1+\epsilon}\right) + O(logr)$$

which contradicts with $n \geq 4k+m+6$. Thus $\varphi^{n+m}(z)\varphi(qz+c)$ is not a constant.

Suppose that there exists a point z_0 such that $\varphi^{n+m}(z_0)\varphi(qz_0+c) = 1$. Then $\varphi^{n}(z_0)\varphi(qz_0+c) = 1$ since $g(z)$ is an entire functions. Hence $\varphi^m(z_0) = 1$ and

$$\overline{N}\left(r,\frac{1}{\varphi^{n+m}(z)\varphi(z+c)-1}\right) \leq \overline{N}\left(r,\frac{1}{\varphi^{m}(z)-1}\right) \leq mT(r,\varphi(z)) + O(1).$$

We apply the second Nevanlinna fundamental theorem to $\varphi^{n+m}(z)\varphi(qz+c)$:

$$\begin{aligned} T(r,\varphi^{n+m}(z)\varphi(qz+c)) &\leq \overline{N}(r,\varphi^{n+m}(z)\varphi(z+c)) + \overline{N}\left(r,\frac{1}{\varphi^{n+m}(z)\varphi(z+c)}\right) \\ &+\overline{N}\left(r,\frac{1}{\varphi^{n+m}(z)\varphi(qz+c)-1}\right) + S(r,\varphi). \\ &\leq (m+5)T(r,\varphi(z)) + S(r,\varphi). \end{aligned}$$

By Lemma 5 we deduce

$$(n-m-4)T(r,\varphi(z)) \leq O\left(r^{\sigma(\varphi(z))-1+\epsilon}\right) + S(r,\varphi), \tag{8}$$

which contradicts with $n \geq 4k+m+6$. So $\varphi^{n+m}(z)\varphi(qz+c) \equiv 1$. Thus $\varphi(z) \equiv 1$, that is $f(z) \equiv g(z)$.

Case-2: $F(z)G(z) \equiv 1$, that is

$$f^n(f^m-1)f(qz+c)g^n(g^m-1)g(qz+c) \equiv \alpha^2(z). \tag{9}$$

Since f and g are transcendental entire functions, we can deduce from (9) that $N\left(r,\frac{1}{f}\right) = S(r,f), N(r,f) = S(r,f)$ and $N\left(r,\frac{1}{f-1}\right) = S(r,f)$. Then $\delta(0,f) + \delta(\infty,f) + \delta(1,f) = 3$, which contradicts the deficiency relation. This completes the proof of Theorem 1.

4. PROOF OF THEOREM 2

Let $F(z) = \frac{[f^n(f^m-1)f(qz+c)]^{(k)}}{\alpha(z)}$, $G(z) = \frac{[g^n(g^m-1)g(qz+c)]^{(k)}}{\alpha(z)}$,

Then $F(z)$ and $G(z)$ share $(1,2)^*$ except the zeros or poles of $\alpha(z)$. Obviously

$$\begin{aligned} &2N_2\left(r,\frac{1}{F}\right) + 2N_2\left(r,\frac{1}{G}\right) + \overline{N}\left(r,\frac{1}{F}\right) + \overline{N}\left(r,\frac{1}{G}\right) + S(r,F) + S(r,G) \\ &\leq (6k+3m+8)T(r,f) + (6k+3m+8)T(r,g) + S(r,f) + S(r,g). \end{aligned} \tag{10}$$

According to (10) and Lemma 2, we can prove Theorem 2 in a similar way as in Section 3.

5. PROOF OF THEOREM 3

Let $F(z) = \frac{[f^n(f^m-1)f(qz+c)]^{(k)}}{\alpha(z)}$, $G(z) = \frac{[g^n(g^m-1)g(qz+c)]^{(k)}}{\alpha(z)}$,

Then $\overline{E}_{2)}(\alpha(z),[f^n(f^m-1)f(qz+c)]^{(k)}) = \overline{E}_{2)}(\alpha(z),[g^n(g^m-1)g(qz+c)]^{(k)})$ except the zeros or poles of $\alpha(z)$. Obviously

$$\begin{aligned} &2N_2\left(r,\frac{1}{F}\right) + 2N_2\left(r,\frac{1}{G}\right) + 3\overline{N}\left(r,\frac{1}{F}\right) + 3\overline{N}\left(r,\frac{1}{G}\right) + S(r,F) + S(r,G) \\ &\leq (10k+5m+12)T(r,f) + (10k+5m+12)T(r,g) + S(r,f) + S(r,g). \end{aligned} \tag{11}$$

Using (11) and Lemma 6, we can prove Theorem 3 in a similar way as in Section 3.

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