International Journal of Mathematical Archive-8(2), 2017, 124-128
Available online through www.ijma.info ISSN 2229 – 5046

SOME RESULTS ON SQUARE FUZZY GRAPHS

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(Received On: 20-01-17; Revised & Accepted On: 14-02-17)

ABSTRACT

Special fuzzy graph can be obtained from given fuzzy graph, Square fuzzy graph. In this paper we find the relation between the fuzzy graph and its square fuzzy graph. We find what happens in the square fuzzy graphs if two fuzzy graphs are isomorphic. We also find whether the square fuzzy graph is regular if the given fuzzy graph is regular.

Keywords: *Square fuzzy graphs, regular fuzzy graphs, strong fuzzy graphs, complete fuzzy graphs, order and size of fuzzy graphs.*

AMS Mathematics subject classification (2010):

INTRODUCTION

Azriel Rosenfeld introduces fuzzy graphs in 1975. It has been growing fast and has numerous application in various fields. NagoorGani and Ratha introduced regular fuzzy graphs, total degree and totally regular fuzzy graph. We have defined the square fuzzy graph of G(σ, μ) with underlying crisp graph $G^*(V, E)$ is denoted by $G^2(\sigma^2, \mu^2)$ has same vertex set as in G (i.e. $\sigma = \sigma^2$) and the edge set is $\mu^2(u, v) = \mu(u, v)$ if $(u, v) \in E$ and $\mu^2(u, v) = \sigma(u) \wedge \sigma(v)$ if $(u, v) \notin E$, if u and v are joined by a path of length is less than or equal to two in $G^*(V, E)$. We have proved that square fuzzy graph $G^2(\sigma^2, \mu^2)$ is complete if G (σ, μ) is a strong fuzzy graph and $G^*(V, E)$ is a cycle of length ≰ 5. We proved that two fuzzy graphs are isomorphic then their square fuzzy graphs are also isomorphic. We have proved that G (σ, μ) is weak isomorphic to the square fuzzy graph $G^2(\sigma^2, \mu^2)$. We have illustrated an example that square fuzzy graph of a regular fuzzy graph need not be regular.

2. PRELIMINARIES

A fuzzy graph G is a pair of function (σ,μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of G: (σ,μ) is denoted by G*(V, E) where $E \subseteq V \times V$, G: (σ, μ) is called **connected fuzzy graph** if for all u, v ∈V there exists at least one nonzero path between u and v. G is called **strong fuzzy graph** if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all (u, v) ∈ E and **complete fuzzy graph** of $\mu(u, v) = \sigma(u) \wedge \sigma(v), \forall u, v \in V$. The **degree of a vertex** is of G: (σ, μ) is defined as $d_G(u) = \sum_{uv \in E} \mu(u\ v)$. The **order of a fuzzy graph** G: (σ, μ) is defined as $O(G) = \sum_{u \in V} \sigma(u)$. The **size of fuzzy graph**G(σ, μ) is defined as $q(G) = \sum_{uv \in E} \mu(u, v)$ .A **homomorphism** of fuzzy graphs G: (σ, μ) and G′: (σ′, μ′) with underlying crisp graphs $G^*(V, E)$ and $G^{*\prime}(V', E')$ respectively is abijectivemap h: V →V′ which satisfies.$\sigma(x) \le \sigma'(h(x)), \forall x \in V$ and$\mu(x, y) \le \mu'(h(x), h(y)), \forall x, y \in V$.A **weak isomorphism** from G to G′ is a map h: V →V′ which bijective homomorphism that satisfies $\sigma(x) = \sigma'(h(x)), \forall x \in V$.

3. SQUARE FUZZY GRAPHS

Definition 3.1: Let G: (σ, μ) be a fuzzy graph with underlying crisp graph $G^*(V, E)$. Then the **Square fuzzy graph** of G is denoted by $G^2(\sigma^2, \mu^2)$ and is defined as

$\sigma^2(\mu) = \sigma(u), \forall u \in V$

and $\mu^2(u, v) = \begin{cases} \mu(u, v), \text{ if } (u, v) \in E \\ \sigma(u) \wedge \sigma(v), \text{ if } (u, v) \notin E. \end{cases}$

if u and v are joined by a path of length is less than or equal to two in $G^*(V, E)$.

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Example. 3.2:

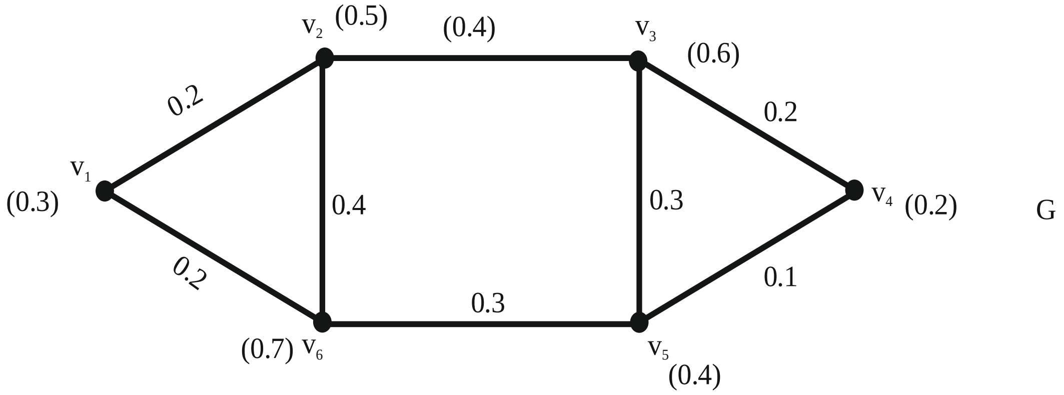

Figure-1: Fuzzy Graph G(σ, μ)

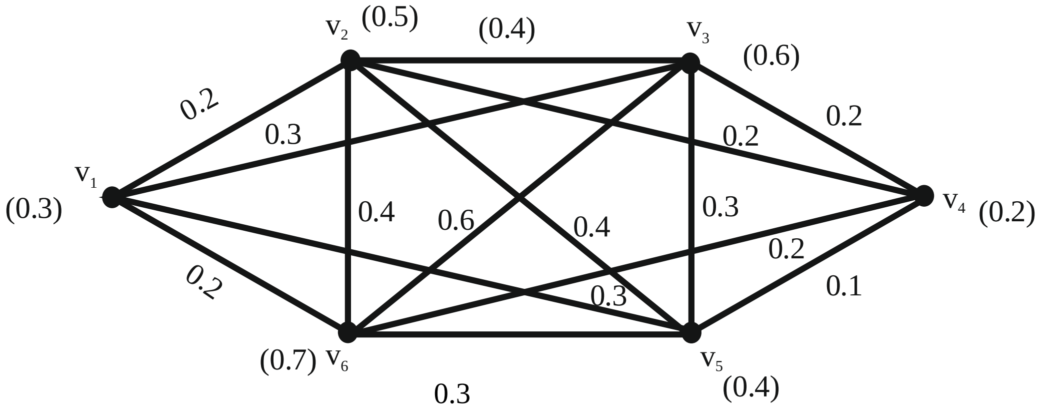

Figure-2: Square Fuzzy Graph $G^2(\sigma^2, \mu^2)$

Theorem 3.3: Let G (σ, μ) be a strong fuzzy graph and G^* (V, E) is cycle of length $n \leq 5$. Then the square fuzzy graph $G^2(\sigma^2, \mu^2)$ is a complete fuzzy graph.

Proof: Given G (σ, μ) be a strong fuzzy graph and G^*(V, E) is a cycle of length $n \leq 5$.

Since G is a strong fuzzy graph if $(u, v) \in E$. Then $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ in G^2 also.

Since G^* is a cycle of length ≤ 5 if u and v are non adjacent than there is a path of length 2.

$\therefore \mu(u, v) = \sigma(u) \wedge \sigma(v)$ in G^2,if $(u, v) \notin E$.

$\therefore$ Any two points are joined by an effective edge.

$\therefore$ $G^2(\sigma^2, \mu^2)$ is a complete fuzzy graph.

Theorem 3.4: If two fuzzy graphs are isomorphic then their square fuzzy graphs are also isomorphic.

Proof: Let G (σ, μ) and G′ (σ′, μ′) are two isomorphic fuzzy graphs.

There exist an isomorphic $h: V \rightarrow V'$ such that the edges and points in G and G´ are preserved.

Also the distance between any two points in G^* (V, E) and G'^* (V′, E′) are preserved.

If u and v having the distance less than are equal to two in G^* (V, E) then h(u) and h(w) having the distance less than are equal to two in G'^* (V′, E′)

If u and v are made adjacent in $G^2(\sigma^2, \mu^2)$ and h(u) and h(v) are made adjacent in $G'^2(\sigma'^2, \mu'^2)$.

$\therefore$ $G^2(\sigma^2, \mu^2)$ and $G'^2(\sigma'^2, \mu'^2)$ are isomorphic since h itself is a bijective map from V^2 to V'^2, which preserves the edges and points of G^2 and G'^2.

Theorem 3.5: Fuzzy graph G (σ, μ) is weak isomorphic to the square fuzzy graph G^2 (σ^2, μ^2).

Proof: Let G (σ, μ) be a fuzzy graph and G^2 (σ^2, μ^2) be the square fuzzy graph of G.
Since G and G^2 has same point set

$$\therefore \sigma(x) = \sigma^2(x), \quad \forall x \in V. \qquad (1)$$

If $(u, v) \in E$, the edge set of G then $\mu(u, v)$ is in $G^2 \Rightarrow \mu^2(u, v) = \mu(u, v)$ if $(u, v) \in E$.

If $(u, v) \notin E$, the edge set of G then $\mu(u, v) = \sigma(u) \wedge \sigma(v)$

$$\therefore \Rightarrow \mu^2(u, v) = \sigma(u) \wedge \sigma(v), \text{ if } (u, v) \in E.$$
$$\therefore \mu(u, v) \leq \mu^2(u, v), \quad \forall (u, v) \in E^2 \qquad (2)$$

$\therefore$ By (1) and (2), G (σ, μ) and G^2 (σ^2, μ^2) are weak isomorphic.

Theorem 3.6: Let G (σ, μ) be a fuzzy graph and G^2 (σ^2, μ^2) be the its square fuzzy graph. Then

(i) $O(G) = O(G^2)$ [i.e. Order of G = Order of G^2]
(ii) Size of G $\leq$ Size of G^2

Proof: From the definition, number of points are same in G (σ, μ) and G^2 (σ^2, μ^2) with their respective weights.

$$\therefore \text{Order of } G = \sum\sigma(u)$$

Order of $G^2 = \sum\sigma^2(u) = \sum\sigma(u)$

$$\therefore O(G) = O(G^2)$$

(ii) If (u, v) be an edge in G (σ, μ) then (u, v) is also be an edge in G^2 (σ^2, μ^2)

If u and v are at a distance 2 in G^* (V, E) then they are made adjacent in G^2 (σ^2, μ^2)

$\therefore$ The number of edges are more in G^2 (σ^2, μ^2)

$\therefore$ Size of G $\leq$ Size of G^2.

We have illustrated an example that a square fuzzy graph of a regular fuzzy graph need not be regular.

Example 3.7:

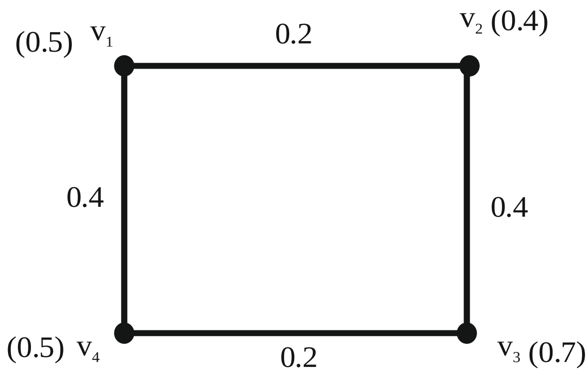

Figure-3: G: (σ, μ) regular

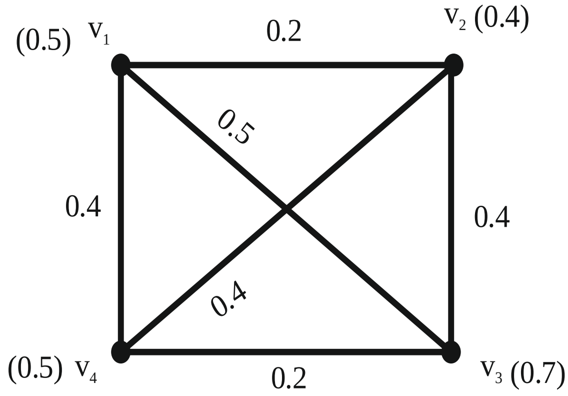

Figure-4: G^2: (σ^2, μ^2) not regular

$d(v_1) = 0.2 + 0.4 + 0.5 = 1.1$
$d(v_2) = 0.2 + 0.4 + 0.4 = 1.0$
$d(v_3) = 0.2 + 0.4 + 0.5 = 1.1$
$d(v_4) = 0.2 + 0.4 + 0.4 = 1.0$
$d(v_1) \neq d(v_2)$

$\therefore$ G_2 (σ^2, μ^2) is not regular.

The following example shows that if G is a bipartite fuzzy graph then G^2 is need not be a bipartite fuzzy graph.

Example 3.8

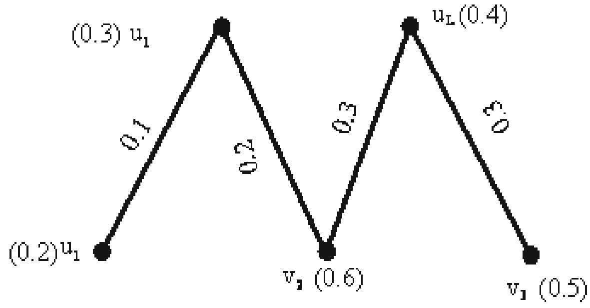

Figure-5: Bipartite fuzzy graph, G (σ, μ)

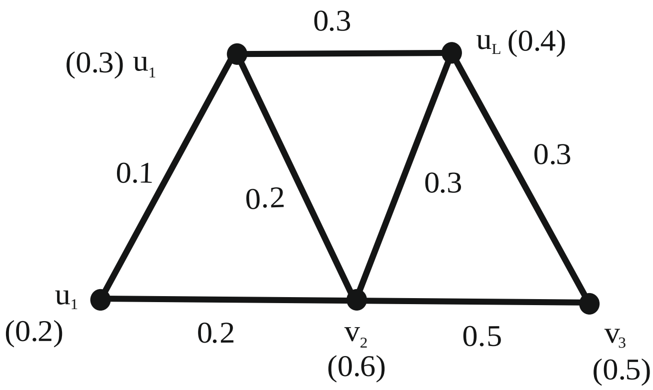

Figure-6: Square fuzzy graph of bipartite fuzzy graph, G (σ^2, μ^2) is not bipartite fuzzy graph.

Theorem 3.9: Let G (σ, μ) be a strong fuzzy graph and $\overline{G}(\overline{\sigma}, \overline{\mu})$ be the complement of G. Then $\overline{G}^{2^*}$ is a complete fuzzy graph.

Proof: Let, G (σ, μ) be the given graph

Then $\overline{G}(\overline{\sigma}, \overline{\mu})$ be a fuzzy graph with node set is same as in G and the edge set is

$$\overline{\mu}(u, v) = \begin{cases} 0 & \text{if } (u, v) \text{ is effective edge} \\ \sigma(u) \wedge \sigma(v), & \text{otherwise} \end{cases}$$

In $\overline{G}$, any two nonadjacent vertices in G can be made adjacent (even though they are at any distance)

Some of the edges can be removed in $\overline{G}$ if they are effective edges in G.

In $\overline{G}$, the distance between any two non adjacent vertices is length is equal to two.

They are made adjacent in $\overline{G}^2$. Since length is two.

$\therefore$ G^2 (σ^2, μ^2) has all the edges connecting any two points in $\overline{G}^{*2}(V^2, \overline{E}^2)$

$\therefore$ $(\overline{G}^2)^*$ is a complete fuzzy graph.

Remark 3.10: If G (σ, μ) is a strong fuzzy graph then $\overline{G}^2(\overline{\sigma}^2, \overline{\mu}^2)$ andG^2 (σ^2, μ^2) are a complete fuzzy graph.

Results 3.11:

1. Square fuzzy graph of complete fuzzy graph is fuzzy graph itself
2. If G (σ, μ) is fuzzy graph and G^* (V, E) is a cycle of length 3 then the square fuzzy graph G^2 (σ^2, μ^2) is G itself.
3. Fuzzy graph G (σ, μ) is an edge induced subgraph of G^2 (σ^2, μ^2).

4. CONCLUSION

In this paper we find the relation between the order and size of a given fuzzy graph and its square fuzzy graph. We find the condition that the square fuzzy graph is complete fuzzy graph. We find under what condition that the square fuzzy graph and its original fuzzy graph are equal. We find that given fuzzy graph is always weak isomorphic to its square fuzzy graph. Square fuzzy graph in very much useful is fuzzy neural networks.

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Source of support: Nil, Conflict of interest: None Declared.