International Journal of Mathematical Archive-8(2), 2017, 99-106
Available online through www.ijma.info ISSN 2229 – 5046

COMPUTATION OF SOME TOPOLOGICAL INDICES OF CERTAIN NETWORKS

V. R. KULLI*

Department of Mathematics, Gulbarga University, Gulbarga 585106, India.

(Received On: 20-01-17; Revised & Accepted On: 18-02-17)

ABSTRACT

Chemical graph theory is a branch of graph theory whose focus of interest is to finding topological indices of chemical graphs which correlate well with chemical properties of the chemical molecules. In this paper, we compute the modified first and second Zagreb indices, harmonic index and augmented Zagreb index for certain networks like silicate networks and honeycomb networks.

Keywords: *silicate network, hexagonal network, oxide network, honeycomb network, modified first and second Zagreb indices, harmonic index, augmented Zagreb index.*

Mathematics Subject Classification: *05C05, 94C15.*

1. INTRODUCTION

In this paper, we consider finite simple, undirected graphs. Let $G = (V, E)$ be a graph. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. We refer to [1] for undefined term and notation.

A molecular graph or a chemical graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms.

A topological index is a numerical parameter mathematically derived from the graph structure. These indices are useful for establishing correlation between the structure of a molecular compound and its physico-chemical properties.

The modified first and second Zagreb indices [2] are respectively defined as

$$^{m}M_1(G)=\sum_{u\in V(G)}\frac{1}{d_G(u)^2}, \qquad ^{m}M_2(G)=\sum_{uv\in E(G)}\frac{1}{d_G(u)d_G(v)}.$$

Many other topological indices were studied, for example, in [3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15].

The harmonic index of a graph G is defined as

$$H(G)=\sum_{uv\in E(G)}\frac{2}{d_G(u)+d_G(v)}.$$

This index was studied by Favaron *et al.* [16] and Zhong [17].

The augmented Zagreb index of a graph G is defined as

$$AZI(G)=\sum_{uv\in E(G)}\left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3.$$

The augmented Zagreb index was introduced by Furtula *et al.* in [18] and was studied, for example, in [19].

In this paper, the modified first and second Zagreb indices, harmonic index and augmented Zagreb index for certain network. For Figures see [20].

*Corresponding Author: V. R. Kulli**

2 RESULTS FOR SILICATE NETWORKS

Silicates are obtained by fusing metal oxides or metal carbonates with sand. A silicate network is symbolized by SL_n, where n is the number of hexagons between the center and boundary of SL_n. A silicate network of dimension two is depicted in Figure 1.

We compute the exact values of ${}^mM_1(SL_n)$, ${}^mM_2(SL_n)$, $H(SL_n)$ and $AZI(SL_n)$ for silicate networks.

Theorem 2.1: Let SL_n be the silicate networks. Then

(1) ${}^mM_1(SL_n) = \frac{11}{12}n^2 + \frac{7}{12}n.$ (2) ${}^mM_2(SL_n) = \frac{3}{2}n^2 + \frac{2}{3}n.$

(3) $H(SL_n) = 7n^2 + \frac{4}{3}n.$ (4) $AZI(SL_n) = \left(\frac{1}{7^3} + \frac{1}{5^3}\right)18^4 n^2 + \left[\left(\frac{9}{4}\right)^3 + \left(\frac{18}{7}\right)^3 - 2\left(\frac{18}{5}\right)^3\right]6n.$

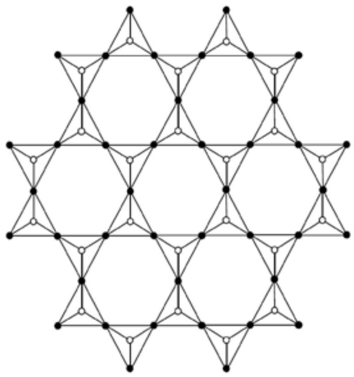

Figure-1: Silicate network of dimension two

Proof: Let G be the graph of silicate network SL_n with $|V(SL_n)| = 15n^2 + 3n$ and $|E(SL_n)| = 36n^2$. From Figure 1, it is easy to see that there are two partitions of the vertex set of SL_n as follows:

$V_3 = \{u \in V(G) \mid d_G(u) = 3\}$, $|V_3| = 6n^2 + 6n$.

$V_6 = \{u \in V(G) \mid d_G(u) = 6\}$, $|V_6| = 9n^2 - 3n$.

By algebraic method, in SL_n there are three types of edges based on the degree of the vertices of each edge as follows:

$E_6 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_6| = 6n$.

$E_9 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_9| = 18n^2 + 6n$.

$E_{12} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}$, $|E_{12}| = 18n^2 - 12n$.

1) Now to compute ${}^mM_1(G)$, we see that

$${}^mM_1(G) = \sum_{u \in V(G)} \frac{1}{d_G(u)^2} = \sum_{u \in V_3} \frac{1}{d_G(u)^2} + \sum_{u \in V_6} \frac{1}{d_G(u)^2} = \frac{1}{3^2}(6n^2+6n) + \frac{1}{6^2}(9n^2-3n) = \frac{11}{12}n^2 + \frac{7}{12}n.$$

2) To compute ${}^mM_2(G)$, we see that

$${}^mM_2(G) = \sum_{uv \in E(G)} \frac{1}{d_G(u)d_G(v)} = \sum_{uv \in E_6} \frac{1}{d_G(u)d_G(v)} + \sum_{uv \in E_9} \frac{1}{d_G(u)d_G(v)} + \sum_{uv \in E_{12}} \frac{1}{d_G(u)d_G(v)}$$

$$= \left(\frac{1}{3\times3}\right)6n + \left(\frac{1}{3\times6}\right)(18n^2+6n) + \left(\frac{1}{6\times6}\right)(18n^2-12n) = \frac{3}{2}n^2 + \frac{2}{3}n.$$

3) To compute $H(G)$, we see that

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d_G(u)+d_G(v)} = \sum_{uv \in E_6} \frac{2}{d_G(u)+d_G(v)} + \sum_{uv \in E_9} \frac{2}{d_G(u)+d_G(v)} + \sum_{uv \in E_{12}} \frac{2}{d_G(u)+d_G(v)}$$

$$= \left(\frac{2}{3+3}\right)6n + \left(\frac{2}{3+6}\right)(18n^2+6n) + \left(\frac{2}{6+6}\right)(18n^2-12n) = 7n^2 + \frac{4}{3}n.$$

4) To compute $AZI(G)$, we see that

$$AZI(G) = \sum_{uv \in E(G)} \left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3$$

$$= \sum_{uv \in E_6} \left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3 + \sum_{uv \in E_9} \left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3 + \sum_{uv \in E_{12}} \left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3$$

$$= \sum_{uv\in E_6}\left(\frac{3\times 3}{3+3-2}\right)^3 6n + \sum_{uv\in E_6}\left(\frac{3\times 6}{3+6-2}\right)^3 \left(18n^2+6n\right) + \sum_{uv\in E_{12}}\left(\frac{6\times 6}{6+6-2}\right)^3 \left(18n^2-12n\right)$$

$$= \left(\frac{1}{7^3}+\frac{1}{5^3}\right)18^4 n^2 + \left[\left(\frac{9}{4}\right)^3 + \left(\frac{18}{7}\right)^3 - 2\left(\frac{18}{5}\right)^3\right]6n.$$

3. RESULTS FOR CHAIN SILICATE NETWORKS

We now consider a family of chain silicate networks. This network is symbolized by CS_n and is obtained by arranging n tetrahedral linearly, see Figure 2.

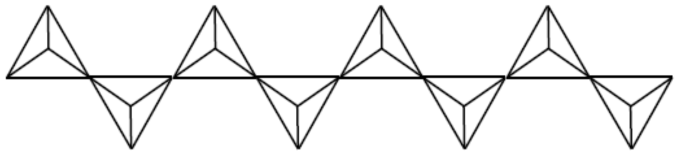

Figure-2: Chain silicate network

We compute the values of ${}^mM_1(CS_n)$, ${}^mM_2(CS_n)$, $H(CS_n)$, and $AZI(CS_n)$ for chain silicate networks.

Theorem 3.1: Let CS_n be the chain silicate networks. Then

(1) ${}^mM_1\left(CS_n\right)=\frac{1}{4}n+\frac{7}{36}$ (2) ${}^mM_2\left(CS_n\right)=\frac{13}{36}n+\frac{5}{18}$

(3) $H\left(CS_n\right)=\frac{25}{18}n+\frac{5}{9}.$ (4) $AZI\left(CS_n\right)=\left[\left(\frac{9}{4}\right)^3+\left(\frac{18}{7}\right)^3 4-\left(\frac{18}{5}\right)^3\right]n+\left[\left(\frac{9}{4}\right)^3 4-\left(\frac{18}{7}\right)^3 2-\left(\frac{18}{5}\right)^3 2\right].$

Proof: Let G be the graph of chain silicate networks CS_n with $|V(CS_n)| = 3n + 1$ and $|E(CS_n)| = 6n$. From Figure 2, it is easy to see that there are two partitions of the vertex set of CS_n as follows:

$V_3 = \{u\in V(G) \mid d_G(u) = 3\}$, $|V_3| = 2n + 2$.
$V_6 = \{u\in V(G) \mid d_G(u) = 6\}$, $|V_6| = n - 1$.

By algebraic method, in CS_n, $n \geq 2$, there are three types of edges based on the degree of the vertices of each edge as follows:

$E_6 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_6| = n + 4$.
$E_9 = \{uv\in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_9| = 4n - 2$.
$E_{12} = \{uv\in E(G) \mid d_G(u) = d_G(v) = 6\}$, $|E_{12}| = n - 2$.

(1) Now to compute ${}^mM_1(G)$ we see that

$${}^mM_1(G) = \sum_{u\in V(G)}\frac{1}{d_G(u)^2} = \sum_{u\in V_3}\frac{1}{d_G(u)^2} + \sum_{u\in V_6}\frac{1}{d_G(u)^2} = \frac{1}{3^2}(2n+2)+\frac{1}{6^2}(n-1) = \frac{1}{4}n+\frac{7}{36}.$$

(2) To compute ${}^mM_2(G)$, we see that

$${}^mM_2(G) = \sum_{uv\in E(G)}\frac{1}{d_G(u)d_G(v)} = \sum_{uv\in E_6}\frac{1}{d_G(u)d_G(v)} + \sum_{uv\in E_9}\frac{1}{d_G(u)d_G(v)} + \sum_{uv\in E_{12}}\frac{1}{d_G(u)d_G(v)}$$

$$= \left(\frac{1}{3\times 3}\right)(n+4)+\left(\frac{1}{3\times 6}\right)(4n-2)+\left(\frac{1}{6\times 6}\right)(n-2) = \frac{13}{36}n+\frac{5}{18}.$$

(3) To compute $H(G)$, we see that

$$H(G) = \sum_{uv\in E(G)}\frac{2}{d_G(u)+d_G(v)} = \sum_{uv\in E_6}\frac{2}{d_G(u)+d_G(v)} + \sum_{uv\in E_9}\frac{2}{d_G(u)+d_G(v)} + \sum_{uv\in E_{12}}\frac{2}{d_G(u)+d_G(v)}$$

$$= \left(\frac{2}{3+3}\right)(n+4)+\left(\frac{2}{3+6}\right)(4n-2)+\left(\frac{2}{6+6}\right)(n-2) = \frac{25}{18}n+\frac{5}{9}.$$

(4) To compute $AZI(G)$, we see that

$$AZI(G) = \sum_{uv\in E(G)}\left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3$$

$$= \sum_{uv\in E_6}\left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3 + \sum_{uv\in E_9}\left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3 + \sum_{uv\in E_{12}}\left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3$$

$$= \left(\frac{3\times 3}{3+3-2}\right)^3 (n+4) + \left(\frac{3\times 6}{3+6-2}\right)^3 (4n-2) + \left(\frac{6\times 6}{6+6-2}\right)^3 (n-2)$$

$$= \left[\left(\frac{9}{4}\right)^3 + \left(\frac{18}{7}\right)^3 4 - \left(\frac{18}{5}\right)^3\right] n + \left[\left(\frac{9}{4}\right)^3 4 - \left(\frac{18}{7}\right)^3 2 - \left(\frac{18}{5}\right)^3 2\right]$$

Theorem 3.2: Let CS_n (n = 1) be the chain silicate network. Then

(1) ${}^m M_1(CS_1) = \frac{4}{9}$. (2) ${}^m M_2(CS_1) = \frac{2}{3}$.

(3) $H(CS_1) = 2$. (4) $AZI(CS_1) = \frac{2187}{32}$.

Proof: Let CS_1 be the graph of a chain silicate network. Then $CS_1 = K_4$. Clearly $|V(CS_1)| = 4$ and $|E(CS_1)| = 6$. Also $d_{CS_1}(u) = 3$ for every $u \in V(CS_1)$.

To compute ${}^mM_1(CS_1)$, ${}^mM_2(CS_1)$, $H(CS_1)$, and $AZI(CS_1)$, we see that

1) $${}^m M_1(CS_1) = \sum_{u\in V(CS_1)} \frac{1}{d(u)^2} = \frac{1}{3^2}\times 4 = \frac{4}{9}.$$

2) $${}^m M_2(CS_1) = \sum_{uv\in E(CS_1)} \frac{1}{d(u)d(v)} = \frac{1}{3\times 3}\times 6 = \frac{2}{3}.$$

3) $$H(CS_1) = \sum_{uv\in E(CS_1)} \frac{2}{d(u)+d(v)} = \frac{2}{3+3}\times 6 = 2.$$

4) $$AZI(CS_1) = \sum_{uv\in E(CS_1)} \left(\frac{d(u)d(v)}{d(u)+d(v)-2}\right)^3 = \left(\frac{3\times 3}{3+3-2}\right)^3 \times 6 = \frac{2187}{32}.$$

4. RESULTS FOR HEXAGONAL NETWORKS

It is known that there exist three regular plane tilings with composition of same kind of regular polygons such as triangular, hexagonal and square. Triangular tiling is used in the construction of hexagonal networks. This network is symbolized by HX_n, where n is the number of vertices in each side of hexagon. A hexagonal network of dimension six in shown in Figure 3.

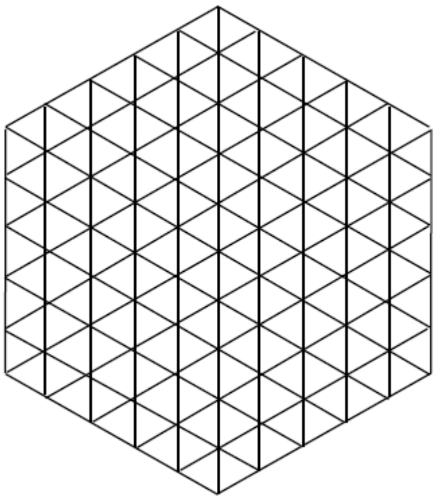

Figure-3: Hexagonal network of dimension six

Now we compute the values of ${}^mM_1(HX_n)$, ${}^mM_2(HX_n)$, $H(HX_n)$ and $AZI(HX_n)$ for hexagonal networks.

Theorem 4.1: Let HX_n be the hexagonal networks. Then

1) ${}^m M_1(HX_n) = \frac{1}{12}n^2 + \frac{1}{8}n + \frac{17}{18}$. 2) ${}^m M_2(HX_n) = \frac{1}{24}(6n^2 - n + 1)$.

3) $H(HX_n) = \frac{3}{2}n^2 - \frac{8}{5}n + \frac{97}{210}$. 4) $AZI(HX_n) \approx 419.904n^2 - 1101.870n + 678.252$.

Proof: Let G be the graph of hexagonal network HX_n with $|V(HX_n)|=3n^2-3n+1$ and $|E(HX_n)|=9n^2-15n+6$. From Figure 3, it is easy to see that there are three partitions of the vertex set of HX_n as follows:

$V_3=\{u\in V(G)\mid d_G(u)=3\}$, $|V_3|=6$.
$V_4=\{u\in V(G)\mid d_G(u)=4\}$, $|V_4|=6n-2$.
$V_6=\{u\in V(G)\mid d_G(u)=6\}$, $|V_6|=3n^2-9n+7$.

In HX_n, by algebraic method, there are five types of edges based on the degree of the vertices of each edge as follows:

$E_7=\{uv\in E(G)\mid d_G(u)=3,\ d_G(v)=4\}$, $|E_7|=12$.
$E_9=\{uv\in E(G)\mid d_G(u)=3,\ d_G(v)=6\}$, $|E_9|=6$.
$E_8=\{uv\in E(G)\mid d_G(u)=d_G(v)=4\}$, $|E_8|=6n-18$.
$E_{10}=\{uv\in E(G)\mid d_G(u)=4,\ d_G(v)=6\}$, $|E_{10}|=12n-24$.
$E_{12}=\{uv\in E(G)\mid d_G(u)=d_G(v)=6\}$, $|E_{12}|=9n^2-33n+30$.

(1) Now compute ${}^mM_1(HX_n)$, we see that

$$^mM_1(G)=\sum_{u\in V(G)}\frac{1}{d_G(u)^2}=\sum_{u\in V_3}\frac{1}{d_G(u)^2}+\sum_{u\in V_4}\frac{1}{d_G(u)^2}+\sum_{u\in V_6}\frac{1}{d_G(u)^2}$$

$$=\frac{1}{3^2}(6)+\frac{1}{4^2}(6n-12)+\frac{1}{6^2}\left(3n^2-9n+7\right)=\frac{1}{12}n^2+\frac{1}{8}n+\frac{17}{18}.$$

(2) To compute ${}^mM_2(G)$, we see that

$$^mM_2(G)=\sum_{uv\in E(G)}\frac{1}{d_G(u)d_G(v)}$$

$$=\sum_{uv\in E_7}\frac{1}{d_G(u)d_G(v)}+\sum_{uv\in E_9}\frac{1}{d_G(u)d_G(v)}+\sum_{uv\in E_8}\frac{1}{d_G(u)d_G(v)}+\sum_{uv\in E_{10}}\frac{1}{d_G(u)d_G(v)}+\sum_{uv\in E_{12}}\frac{1}{d_G(u)d_G(v)}$$

$$=\left(\frac{1}{3\times4}\right)(12)+\left(\frac{1}{3\times6}\right)(6)+\left(\frac{1}{4\times4}\right)(6n-18)+\left(\frac{1}{4\times6}\right)(12n-24)+\left(\frac{1}{6\times6}\right)\left(9n^2-33n+30\right)$$

$$=\frac{1}{24}\left(6n^2-n+1\right).$$

(3) To compute $H(G)$, we see that

$$H(G)=\sum_{uv\in E(G)}\frac{2}{d_G(u)+d_G(v)}$$

$$=\sum_{uv\in E_7}\frac{2}{d_G(u)+d_G(v)}+\sum_{uv\in E_9}\frac{2}{d_G(u)+d_G(v)}+\sum_{uv\in E_8}\frac{2}{d_G(u)+d_G(v)}+\sum_{uv\in E_{10}}\frac{2}{d_G(u)+d_G(v)}$$
$$+\sum_{uv\in V_{12}}\frac{2}{d_G(u)+d_G(v)}$$

$$=\left(\frac{2}{3+4}\right)(12)+\left(\frac{2}{3+6}\right)(6)+\left(\frac{2}{4+4}\right)(6n-18)+\left(\frac{2}{4+6}\right)(12n-24)+\left(\frac{2}{6+6}\right)\left(9n^2-33n+30\right)=$$

$$=\frac{3}{2}n^2-\frac{8}{5}n+\frac{97}{210}.$$

(4) To compute AZI(G), we see that

$$AZI(G)=\sum_{uv\in E(G)}\left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3$$

$$=\left(\frac{3\times4}{3+4-2}\right)^3(12)+\left(\frac{3\times6}{3+6-2}\right)^3(6)+\left(\frac{4\times4}{4+4-2}\right)^3(6n-18)+\left(\frac{4\times6}{4+6-2}\right)^3(12n-24)$$
$$+\left(\frac{6\times6}{6+6-2}\right)^3\left(9n^2-33n+30\right)$$

$$\approx 419.904n^2-1101.870n+678.252.$$

5. RESULTS FOR OXIDE NETWORKS

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by OX_n. A 5-dimensional oxide network is shown in Figure 4.

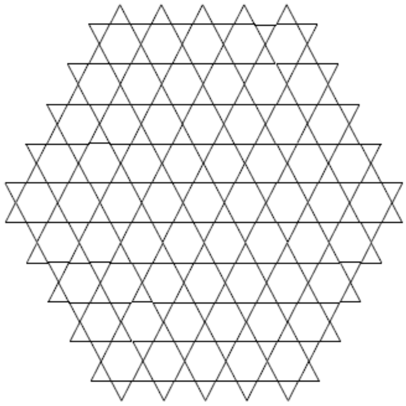

Figure-4: Oxide network of dimension 5

We compute the values of ${}^mM_1(OX_n)$, ${}^mM_2(OX_n)$, $H(OX_n)$ and $AZI(OX_n)$ for oxide networks.

Theorem 5.1: Let OX_n be the oxide network. Then

(1) ${}^mM_1(OX_n) = \frac{3}{4}n^2 + \frac{21}{16}n.$ (2) ${}^mM_2(OX_n) = \frac{9}{8}n^2 + \frac{3}{4}n$

(3) $H(OX_n) = \frac{9}{2}n^2 + n.$ (4) $AZI(OX_n) = \frac{1024}{3}n^2 - \frac{1184}{9}n.$

Proof: Let G be the graph of oxide network OX_n with $|V(OX_n)| = 9n^2 + 3n$ and $|E(OX_n)| = 18n^2$. From Figure 4, it is easy to see that there are two partitions of the vertex set of OX_n as follows:

$V_2 = \{u \in V(G) \mid d_G(u) = 2\}$, $|V_2| = 6n$.
$V_4 = \{u \in V(G) \mid d_G(u) = 4\}$, $|V_4| = 9n^2 - 3n$.

In OX_n, by algebraic method, there are two types of edges based on the degree of the vertices of each edge as follows;

$E_6 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4\}$, $|E_6| = 12n$.
$E_8 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$, $|E_8| = 18n^2 - 12n$.

Now to compute ${}^mM_1(OX_n)$, we see that

$${}^mM_1(G) = \sum_{u \in V(G)} \frac{1}{d_G(u)^2} = \sum_{u \in V_2} \frac{1}{d_G(u)^2} + \sum_{u \in V_4} \frac{1}{d_G(u)^2} = \left(\frac{1}{2^2}\right)6n + \left(\frac{1}{4^2}\right)\left(9n^2 - 3n\right) = \frac{3}{4}n^2 + \frac{21}{16}n.$$

1. To compute ${}^mM_2(OX_n)$, we see that

$${}^mM_2(G) = \sum_{uv \in E(G)} \frac{1}{d_G(u)d_G(v)} = \sum_{uv \in E_6} \frac{1}{d_G(u)d_G(v)} + \sum_{uv \in E_8} \frac{1}{d_G(u)d_G(v)}$$
$$= \left(\frac{1}{2 \times 4}\right)12n + \left(\frac{1}{4 \times 4}\right)\left(18n^2 - 12n\right) = \frac{9}{8}n^2 + \frac{3}{4}n.$$

2. To compute $H(OX_n)$, we see that

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d_G(u) + d_G(v)} = \sum_{uv \in E_6} \frac{2}{d_G(u) + d_G(v)} + \sum_{uv \in V_8} \frac{2}{d_G(u) + d_G(v)}$$
$$= \left(\frac{2}{2+4}\right)12n + \left(\frac{2}{4+4}\right)\left(18n^2 - 12n\right) = \frac{9}{2}n^2 + n.$$

3. To compute $AZI(OX_n)$, we see that

$$AZI(G) = \sum_{uv \in E(G)} \left(\frac{d_G(u)d_G(v)}{d_G(u) + d_G(v) - 2}\right)^3 = \sum_{uv \in E_6} \left(\frac{d_G(u)d_G(v)}{d_G(u) + d_G(v) - 2}\right)^3 + \sum_{uv \in E_8} \left(\frac{d_G(u)d_G(v)}{d_G(u) + d_G(v) - 2}\right)^3$$
$$= \left(\frac{2 \times 4}{2+4-2}\right)^3 12n + \left(\frac{4 \times 4}{4+4-2}\right)^3 \left(18n^2 - 12n\right) = \frac{1024}{3}n^2 - \frac{1184}{9}n$$

6. RESULTS FOR HONEYCOMB NETWORKS

If we recursively use hexagonal tiling in a particular pattern, honeycomb networks are formed. These networks are very useful in chemistry and also in computer graphics. A honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 5.

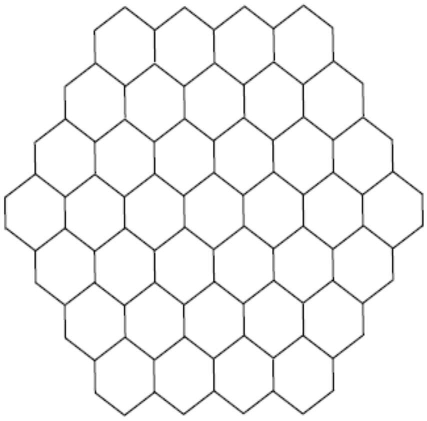

Figure-5: Honeycomb network of dimension four

In the following theorem, we compute the values of ${}^mM_1(HC_n)$, ${}^mM_2(HC_n)$, $H(HC_n)$ and $AZI(HC_n)$ for honey comb networks.

Theorem 5.1: Let HC_n be the honeycomb networks. Then

(1) ${}^mM_1(HC_n) = \frac{3}{2}n^2 + \frac{5}{6}n.$ (2) ${}^mM_2(HC_n) = n^2 + \frac{1}{3}n + \frac{1}{6}.$

(3) $H(HC_n) = 3n^2 - \frac{1}{5}n + \frac{1}{5}.$ (4) $AZI(HC_n) = (6561n^2 - 4791n + 1302)\frac{1}{4^3}.$

Proof: Let G be the graph of honeycomb network HC_n with $|V(HC_n)|=6n^2$ and $|E(HC_n)| = 9n^2 - 3n$. From Figure 5, it is easy to see that there are two partitions of the vertex set of HC_n as follows:

$V_2 = \{u\in V(G) \mid d_G(u) = 2\}$, $|V_2| = 6n$.

$V_3 = \{u\in V(G) \mid d_G(u) = 3\}$, $|V_3| = 6n^2 - 3n$.

In HC_n, by algebraic method, there are three types of edges based on the degree of the vertices of each edge as follows:

$E_4 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_4| = 6$.

$E_5 = \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_5| = 12n - 12$.

$E_6 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_6| = 9n^2 - 15n + 6$.

(1) Now compute ${}^mM_1(HC_n)$, we see that

$${}^mM_1(G) = \sum_{u\in V(G)} \frac{1}{d_G(u)^2} = \sum_{u\in V_2} \frac{1}{d_G(u)^2} + \sum_{u\in V_3} \frac{1}{d_G(u)^2} = \left(\frac{1}{2^2}\right)6n + \left(\frac{1}{3^2}\right)\left(6n^2 - 6n\right) = \frac{3}{2}n^2 + \frac{5}{6}n.$$

(2) To compute ${}^mM_2(HC_n)$, we see that

$${}^mM_2(G) = \sum_{uv\in E(G)} \frac{1}{d_G(u)d_G(v)} = \sum_{uv\in E_4} \frac{1}{d_G(u)d_G(v)} + \sum_{uv\in E_5} \frac{1}{d_G(u)d_G(v)} + \sum_{uv\in E_6} \frac{1}{d_G(u)d_G(v)}$$

$$= \left(\frac{1}{2\times 2}\right)(6) + \left(\frac{1}{2\times 3}\right)(12n-12) + \left(\frac{1}{3\times 3}\right)\left(9n^2 - 15n + 6\right) = n^2 + \frac{1}{3}n + \frac{1}{6}.$$

(3) To compute $H(HC_n)$, we see that

$$H(G) = \sum_{uv\in E(G)} \frac{2}{d_G(u)+d_G(v)} = \sum_{uv\in E_4} \frac{2}{d_G(u)+d_G(v)} + \sum_{uv\in E_5} \frac{2}{d_G(u)+d_G(v)} + \sum_{uv\in E_6} \frac{2}{d_G(u)+d_G(v)}$$

$$= \left(\frac{2}{2+2}\right)6 + \left(\frac{2}{2+3}\right)(12n-12) + \left(\frac{2}{3+3}\right)\left(9n^2 - 15n + 6\right) = 3n^2 - \frac{1}{5}n + \frac{1}{5}.$$

$$= \frac{3}{2}n^2 - \frac{8}{5}n + \frac{97}{210}.$$

(4) To compute AZI(G), we see that

$$AZI(G) = \sum_{uv\in E(G)} \left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3 = \left(\frac{2\times 2}{2+2-2}\right)^3 6 + \left(\frac{2\times 3}{2+3-2}\right)^3 (12n-12) + \left(\frac{3\times 3}{3+3-2}\right)^3 \left(9n^2 - 15n + 6\right)$$

$$= \left(6561n^2 - 4791n + 1302\right)\frac{1}{4^3}.$$

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. S. Nikolić, G. Kovaćević, A. Milićević and N. Trinajstić, The Zagreb indices 30 years after, *Croatica Chemica Acta*, 76(2), (2003) 113-124.
3. V.R. Kulli, The first and second κ_a indices and coindices of graphs, International Journal of Mathematical Archive, 7(5), (2016) 71-77.
4. V.R. Kulli, On K edge index and coindex of graphs, *International Journal of Fuzzy Mathematical Archive*, 10(2), (2016) 111-116.
5. V.R. Kulli, On K-edge index of some nanostructures, *Journal of Computer and Mathematical Sciences*, 7(7), (2016) 373-378.
6. V.R.Kulli, Multiplicative hyper-Zagreb indices and coindices of graphs: Computing these indices of some nanostructures, *International Research Journal of Pure Algebra*, 6(7), (2016) 342-347.
7. V.R.Kulli, Multiplicative connectivity indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 12(2) (2016) 169-176. DOI: http://dx.doi.org/10.22457/apam.v12n2a8.
8. V.R. Kulli, General multiplicative Zagreb indices of $TUC_4C_8[m, n]$ and $TUC_4[m, n]$ nanotubes, *International Journal of Fuzzy Mathematical Archive,* 11(1) (2016) 39-43. DOI. http://dx.doi.org/10.22457/ijfma.v11n1a6.
9. V.R. Kulli, Multiplicative connectivity indices of $TUC_4C_8[m,n]$ and $TUC_4[m, n]$ nanotubes, *Journal of Computer and Mathematical Sciences*, 7(11) (2016) 599-605.
10. V.R.Kulli, Multiplicative connectivity indices of nanostructures, *Journal of Ultra Scientist of Physical Sciencs*, A 29(1) (2017) 1-10. http://dx.doi.org/10.22147/jusps-A/290101.
11. V.R.Kulli, Computation of general topological indices for titania nanotubes, *International Journal of Mathematical Archive*, 7(12) (2016) 33-38. .
12. V.R.Kulli, Some new multiplicative geometric-arithmetic indices, *Journal of Ultra Scientist of Physical Sciencs*, A, 29(2) (2017) http://dx.doi.org/10.22147/jusps-A/290201.
13. V.R.Kulli, Two new multiplicative atom bond connectivity indices, *Annals of Pure and Applied Mathematics,* 13(1) (2017) 1-7. DOI: http://dx.doi.org/10.22147/apam.v12n1a1.
14. V.R.Kulli, Some topological indices of certain nanotubes, Journal of Computer and Mathematical Sciences,8(1)(2017),1-7..
15. V.R.Kulli, F-index and reformulated Zagreb index of certain nanostructures, International Research Journal of Journal of Algebra, 7(1) (2017), 489-495.
16. O. Favaron, M. Maho and J.F. Sacle, Some eigenvalue properties in graphs (conjectures of Graffiti II), *Discrete Math.* 111(1-3) (1993) 197-220.
17. L. Zhong, The harmonic index for graphs, *Appl. Math. Lett.* 25(3) (2012) 561-566.
18. B. Furtula, A. Graovac and D.Vukičević, Augmented Zagreb index, *J. Math. Chem* 48(2010) 370-380.
19. Y. Huang, B. Liu and L. Gan, Augmented Zagreb index of connected graphs, *MATCH Commun. Math. Comput. Chem.* 67(2012) 483-494.
20. S.Wang, J.B.Liu, C.Wang and S. Hayat, Further results on computation of topological indices of certain networks, *arXiv: 1605.00253v2 [math. CO]* 5 may 2016.

Source of support: Nil, Conflict of interest: None Declared.