

INTUITIONISTIC FUZZY ASSIGNMENT PROBLEM  
WITH REPLACEMENT BASED ON INTUITIONISTIC FUZZY AGGREGATION

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ABSTRACT

The objective of this paper is to develop an intuitionistic fuzzy assignment model to draw attention to problems involving breakdowns while assigning. Corporate competitiveness is heavily influenced by the professional (skilled) staff involved in the decision making. The job performance of a professional may correlate to the time taken to task. Two issues being analysed in this approach. The first one is to develop an ideal priority of groups and the second one is to replace the person where we need to avoid breakdowns. Replacement is made in the form of maximum intuitionistic fuzzy scoring by utilizing the intuitionistic fuzzy aggregation operator to aggregate the given intuitionistic fuzzy information and by the weight vector. A numerical example is given to clarify the developed approach under intuitionistic fuzzy environment.

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Keywords – Intuitionistic fuzzy; break down; aggregate operator.

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1. INTRODUCTION :

The complexity of socioeconomic environments often makes it difficult for a single decision maker to consider all the important aspects of some decision problems. So the corporates employ groups of people in decision making. In the real world many decision making processes take place in group settings. In 1952 Votaw and Orden [16] first proposed the assignment problem. Lin and Wen [10] concentrate on the assignment problem where costs are not deterministic numbers but imprecise ones. Huang and Zhang [12] proposed a mathematical model for the fuzzy assignment problem with restriction on qualification. Chen [6] introduced a fuzzy assignment model that considers all individuals have same skills. Kuhn [9] developed the Hungarian algorithm for the assignment problem. Balinski and Gomory [4] introduced a labeling algorithm for solving assignment problem. Aggarwal *et al.* [1] developed an algorithm for bottleneck assignment problem. Liu and Gao [11] introduced fuzzy weighted equilibrium multi-job assignment problem and genetic algorithm. Yang and Liu [20] proposed a multi – objective fuzzy assignment problem. Mukherjee and Basu [13] proposed intuitionistic fuzzy assignment problem by using similarity measures and score functions. Sakawa *et al.* [15] dealt with problems on production and work force assignment in a firm.

In the information technology, construction work, military operation, election work etc for a single task different characteristics are considered, that is for a single job a team is assigned. In the usual situation the assignment is made with respect to one to one basis in such a way that the total time or total cost involved is minimized and the total sales or total profit is maximized. Some complexity may happen while assigning the jobs to the teams. The job performance of a worker may correlate to the time taken to finish the task. Each team needs a minimum time to perform task to maximize the profit. To reach our goal some replacement is made to avoid breakdowns. In such situation two issues being analysed. The first one is to develop ideal priorities of a group by utilizing intuitionistic fuzzy operators like intuitionistic fuzzy hybrid averaging (IFHA) operator, intuitionistic fuzzy hybrid weighted geometric (IFHWG) operator etc to aggregate the given intuitionistic fuzzy information. The second one is replacement. It is made from the pool of workers which is characterized by priorities by utilizing intuitionistic fuzzy aggregative operators.

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## 2. PRELIMINARIES ON INTUITIONISTIC FUZZY SETS

This section presents the basic concepts related to Intuitionistic Fuzzy Set, which was originally introduced by Atanassov and Gargov.

### 2.1. Intuitionistic Fuzzy Sets (IFS)

Let  $X = \{x_1, x_2, \dots, x_n\}$  be a universe of discourse. A fuzzy set  $A = \{(x_j, \mu_A(x_j)) \mid x_j \in X\}$ , defined by zadeh [21] is characterized by a membership function  $\mu_A: X \rightarrow [0,1]$  where  $\mu_A(x_j)$  denotes the degree of membership of the element  $x_j$  to the set  $A$ .

Atanassov [3] introduced a generalized fuzzy set called IFS as follows:

An Intuitionistic Fuzzy Set (IFS)  $A$  in  $X$  is an object having the form:  $A = \{(x_j, \mu_A(x_j), \vartheta_A(x_j)) \mid x_j \in X\}$  which is characterized by a membership function  $\mu_A$  and a nonmembership function  $\vartheta_A$  where  $\mu_A: X \rightarrow [0,1]$ ,  $\vartheta_A: X \rightarrow [0,1]$  with the condition  $\mu_A(x_j) + \vartheta_A(x_j) \leq 1$  for all  $x_j \in X$ . Atanassov defined  $\pi_A(x_j) = 1 - \mu_A(x_j) - \vartheta_A(x_j)$ , for all  $x_j \in X$  as the degree of indeterminacy or hesitancy of  $x_j$  to  $A$  where  $A$  is an IFS in  $X$ . Especially, if  $\pi_A(x_j) = 1 - \mu_A(x_j) - \vartheta_A(x_j) = 0$  for each  $x_j \in X$  then the IFS  $A$  is reduced to a fuzzy set .

### 2.2 Intuitionistic Fuzzy Number (IFN)

An Intuitionistic fuzzy number  $A$  is defined as follows:

- (i) intuitionistic fuzzy sub set of the real line.
- (ii) normal i.e. there is any  $x_0 \in \mathbb{R}$  such that  $\mu_A(x_0) = 1$  ( so  $\vartheta_A(x_0) = 0$ )
- (iii) a convex set for the membership function  $\mu_A(x)$   
 $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_A(x_1), (\mu_A(x_2)))$  for all  $x_1, x_2 \in \mathbb{R}$ ,  $\lambda \in [0, 1]$
- (iv) a concave set for the non membership function  $\vartheta_A(x)$  i.e  
 $\vartheta_A(\lambda x_1 + (1 - \lambda)x_2) \leq \max(\vartheta_A(x_1), (\vartheta_A(x_2)))$  for all  $x_1, x_2 \in \mathbb{R}$ ,  $\lambda \in [0,1]$

### 2.3 Ranking of Intuitionistic Fuzzy Number

Let  $a = (\mu_1, \vartheta_1)$  be an intuitionistic fuzzy number. Chen and Tan [7] introduced a score function  $S$  of an intuitionistic fuzzy value, which is represented as follows:

$$S(a) = \mu_1 - \vartheta_1 \text{ where } S(a) \in [-1,1]. \quad (1)$$

The larger the score  $S(a)$ , the greater the intuitionistic fuzzy value  $a$ . Hong and Choi [8] defined an accuracy function  $H$  to evaluate the degree of accuracy of the intuitionistic fuzzy value  $a$  where  $H(a) \in [0,1]$  and

$$H(a) = \mu_1 + \vartheta_1 \quad (2)$$

The larger the value of  $H(a)$ , the more the degree of accuracy of the degree of membership of the intuitionistic fuzzy value  $a$ .

Let  $b = (\mu_2, \vartheta_2)$  be another intuitionistic fuzzy number. Based on the score function  $S$  and the accuracy function  $H$ , in the following, Xu and Yager [17] give an order relation between two intuitionistic fuzzy value  $a$  and  $b$  which is defined as follows:

If  $S(a) < S(b)$  then  $a < b$

If  $S(a) > S(b)$  then  $a > b$

If  $S(a) = S(b)$  and

If  $H(a) = H(b)$  then  $a = b$

If  $H(a) < H(b)$  then  $a < b$

If  $H(a) > H(b)$  then  $a > b$ .

### 2.4 Operational Laws of Intuitionistic Fuzzy Number (Xu and Yager, 2006)

Let  $\alpha = (\mu_\alpha, \vartheta_\alpha)$ ,  $\alpha_1 = (\mu_{\alpha_1}, \vartheta_{\alpha_1})$  and  $\alpha_2 = (\mu_{\alpha_2}, \vartheta_{\alpha_2})$  be IFNs. Then

- (1)  $\bar{\alpha} = (\vartheta_{\alpha}, \mu_{\alpha})$  where  $\bar{\alpha}$  is the complement of  $\alpha$
- (2)  $\alpha_1 \wedge \alpha_2 = (\min\{\mu_{\alpha_1}, \mu_{\alpha_2}\}, \max\{\vartheta_{\alpha_1}, \vartheta_{\alpha_2}\})$ ;
- (3)  $\alpha_1 \vee \alpha_2 = (\max\{\mu_{\alpha_1}, \mu_{\alpha_2}\}, \min\{\vartheta_{\alpha_1}, \vartheta_{\alpha_2}\})$ ;
- (4)  $\alpha_1 \oplus \alpha_2 = (\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, \vartheta_{\alpha_1}\vartheta_{\alpha_2})$ ;
- (5)  $\alpha_1 \ominus \alpha_2 = \mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, \vartheta_{\alpha_1}\vartheta_{\alpha_2}$
- (6)  $\lambda\alpha = (1 - (1 - \mu_{\alpha})^{\lambda}, \vartheta_{\alpha}^{\lambda}), \lambda > 0$ ;
- (7)  $\alpha^{\lambda} = (\mu_{\alpha}^{\lambda}, 1 - (1 - \vartheta_{\alpha})^{\lambda}), \lambda > 0$ .

**Theorem 1 (Xu, 2007):** Let  $\dot{\alpha}_1 = \alpha_1 \oplus \alpha_2$ ,  $\dot{\alpha}_2 = \alpha_1 \otimes \alpha_2$ ,  $\dot{\alpha}_3 = \lambda \alpha$ ,  $\dot{\alpha}_4 = \alpha^{\lambda}$  and  $\lambda > 0$ . Then all  $\dot{\alpha}_i$  ( $i = 1, 2, 3, 4$ ) are IFNs.

**Definition (Xu, 2007):** Let  $\Theta$  be the set of all IFNs. An intuitionistic fuzzy hybrid averaging (IFHA) operator is a mapping IFHA:  $\Theta^n \rightarrow \Theta$ , such that

$$IFHA_{\omega, \omega}(\alpha_1, \alpha_2, \dots, \alpha_n) = \omega_1 \dot{\alpha}_{\sigma(1)} \oplus \omega_2 \dot{\alpha}_{\sigma(2)} \oplus \dots \oplus \omega_n \dot{\alpha}_{\sigma(n)} \quad (3)$$

where  $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$  is the weighting vector associated with the IFHA operator, with  $\omega_j \in [0, 1]$  ( $j = 1, 2, \dots, n$ ) and  $\sum_{j=1}^n \omega_j = 1$ ;  $\dot{\alpha}_j = n \omega_j \alpha_j$ ,  $j = 1, 2, \dots, n$ ,  $(\dot{\alpha}_{\sigma(1)}, \dot{\alpha}_{\sigma(2)}, \dots, \dot{\alpha}_{\sigma(n)})$  is any permutation of a collection of the weighted IFNs  $(\dot{\alpha}_1, \dot{\alpha}_2, \dots, \dot{\alpha}_n)$ , such that  $\dot{\alpha}_{\sigma(j)} \geq \dot{\alpha}_{\sigma(j+1)}$  ( $j = 1, 2, \dots, n-1$ );  $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$  is weight vector of  $\alpha_j$  ( $j = 1, 2, \dots, n$ ), with  $\omega_j \in [0, 1]$  ( $j = 1, 2, \dots, n$ ) and  $\sum_{j=1}^n \omega_j = 1$ , and  $n$  is the balancing coefficient.

Let  $\dot{\alpha}_{\sigma(j)} = (\mu_{\dot{\alpha}_{\sigma(j)}}, \vartheta_{\dot{\alpha}_{\sigma(j)}})$  ( $j = 1, 2, \dots, n$ ). Then by (Xu 2007), we have

$$IFHA_{\omega, \omega}(\alpha_1, \alpha_2, \dots, \alpha_n) = \left(1 - \prod_{j=1}^n (1 - \mu_{\dot{\alpha}_{\sigma(j)}})^{\omega_j}, \prod_{j=1}^n \vartheta_{\dot{\alpha}_{\sigma(j)}}^{\omega_j}\right) \quad (4)$$

and the aggregated value by using the IFHA operator is also an IFN.

### 3. MULTI-ATTRIBUTE DECISION MAKING PROBLEM

Let  $P = \{P_1, P_2, \dots, P_n\}$  be a finite set of alternatives,  $Q = \{Q_1, Q_2, \dots, Q_m\}$  a set of attributes, and  $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$  the weight vector of attributes, where  $\omega_j \in [0, 1]$  ( $j = 1, 2, \dots, m$ ) and  $\sum_{j=1}^m \omega_j = 1$ . Suppose that the characteristics of the alternatives  $P_i$  ( $i = 1, 2, \dots, n$ ) are represented by the IFNs:

$$P_i = \{< Q_j, \mu_{P_i}(Q_j), \vartheta_{P_i}(Q_j) > | Q_j \in Q\}, \quad i = 1, 2, \dots, n \quad (5)$$

where  $\mu_{P_i}(Q_j)$  indicates the degree that the alternative  $P_i$  satisfies the attributes  $Q_j$ ,  $\vartheta_{P_i}(Q_j)$  indicates the degree that the alternative  $P_i$  does not satisfy the attribute  $Q_j$ , and  $\mu_{P_i}(Q_j) \in [0, 1]$ ,  $\vartheta_{P_i}(Q_j) \in [0, 1]$ ,

$$\mu_{P_i}(Q_j) + \vartheta_{P_i}(Q_j) \leq 1 \quad (6)$$

Let  $r'_{ij} = (k_{ij}, l_{ij})$  denote the characteristic of the alternative  $P_i$  with respect to the attribute  $Q_j$ , where  $k_{ij}$  indicates the degree that the alternative  $P_i$  satisfies the attribute  $Q_j$ , and  $l_{ij}$  indicates the degree that the alternative  $P_i$  does not satisfy the attribute  $Q_j$ . Therefore, the characteristics of all the alternatives  $P_i$  ( $i = 1, 2, \dots, n$ ) with respect to the attributes  $Q_j$  ( $j = 1, 2, \dots, m$ ) can be contained in an intuitionistic fuzzy decision matrix  $R' = (r'_{ij})_{n \times m}$  where  $r'_{ij} = (k_{ij}, l_{ij})$ ,  $k_{ij} \in [0, 1]$ ,  $l_{ij} \in [0, 1]$  and  $k_{ij} + l_{ij} \leq 1$ .

**Table-1:** Intuitionistic fuzzy decision matrix  $R'$

	$Q_1$	$Q_2$	...	$Q_m$
$P_1$	$(k_{11}, l_{11})$	$(k_{12}, l_{12})$	...	$(k_{1m}, l_{1m})$
$P_2$	$(k_{21}, l_{21})$	$(k_{22}, l_{22})$	...	$(k_{2m}, l_{2m})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$P_n$	$(k_{n1}, l_{n1})$	$(k_{n2}, l_{n2})$	...	$(k_{nm}, l_{nm})$

If all the attributes  $Q_j$  ( $j = 1, 2, \dots, m$ ) are of the same type, then the attribute values do not need normalization. However, there are generally benefit attributes (the bigger the attribute values the better) and cost attributes (the smaller the attribute values the better) in multi-attribute decision making. In such cases, we may transform the attribute values of cost type into the attribute values of benefit type then  $R' = (r'_{ij})_{n \times m}$  can be transformed into the intuitionistic fuzzy decision matrix  $R = (r_{ij})_{n \times m}$ ,

where

$$r_{ij} = (\mu_{ij}, \vartheta_{ij}) = \begin{cases} r'_{ij}, & \text{for benefit attribute } Q_j, \\ \bar{r}'_{ij}, & \text{for cost attribute } Q_j, \quad i = 1, 2, \dots, n \end{cases} \quad (7)$$

where  $\bar{r}'_{ij}$  is the complement of  $r'_{ij}$ , such that  $\bar{r}'_{ij} = (l_{ij}, k_{ij})$ .

#### 4. MATHEMATICAL MODEL OF INTUITIONISTIC FUZZY ASSIGNMENT PROBLEM

An assignment problem is a special type of transportation problem which can be stated in the form of  $n \times n$  cost matrix  $[\tilde{c}_{ij}]$  of intuitionistic fuzzy numbers as follows:

**Table-2:** Cost matrix of an assignment problem

<b>Job</b>				
<b>Group</b>	1	2	...	n
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	...	$\tilde{c}_{1n}$
2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	...	$\tilde{c}_{2n}$
⋮	⋮	⋮	⋮	⋮
n	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	...	$\tilde{c}_{nn}$

The objective is to assign a number of origins to an equal number of destinations at a minimum cost or maximum profit. Each job must be done by exactly one group and one group can do, at most one job. Mathematically assignment problem can be denoted as

$$\text{Min } Z = \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, n \tag{8}$$

$$\sum_{i=1}^n x_{ij} = 1, \quad j = 1, 2, \dots, n \tag{9}$$

where  $x_{ij}$  is the decision variable defined as

$$x_{ij} = \begin{cases} 1, & \text{if the } i^{\text{th}} \text{ group is assigned to the } j^{\text{th}} \text{ job; where } i, j = 1, 2, \dots, n. \\ 0, & \text{otherwise} \end{cases}$$

The cost of a group  $i$  doing the job  $j$  is considered as an intuitionistic fuzzy number  $\tilde{c}_{ij} = \{(\mu_{ij}, \vartheta_{ij}), i, j = 1, 2, \dots, n\}$  where  $\mu_{ij}$  denotes the degree of acceptance and  $\vartheta_{ij}$  denotes the degree of rejection.

As our objective is to minimize the cost and maximize the profit, we should go for maximize the acceptance degree  $\mu_{ij}$ .

Then the objective function becomes a multi-objective function as

$$\text{Max } z_1 = \sum_{i=1}^n \sum_{j=1}^n \mu_{ij} x_{ij} \quad \text{and} \quad \text{Min } z_2 = \sum_{i=1}^n \sum_{j=1}^n \vartheta_{ij} x_{ij}$$

$$\text{subject to } (\mu_{ij} + \vartheta_{ij} - 1)x_{ij} \leq 0 \tag{10}$$

$$\mu_{ij} x_{ij} \geq \vartheta_{ij} x_{ij} \tag{11}$$

$$\vartheta_{ij} x_{ij} \geq 0 \tag{12}$$

Thus the model becomes

$$\text{max } Z = \sum_{i=1}^n \sum_{j=1}^n (\mu_{ij} - \vartheta_{ij}) x_{ij}.$$

subject to the conditions (8), (9), (10), (11) and (12).

#### 5. SOLUTION PROCEDURE

**Step-1:** Identify the teams with members and jobs. Fix the number of persons (alternatives) in a team and fix characteristics (attributes) to develop ideal priorities to each job.

**Step-2:** Represent attributes of the alternatives by the IFNs in an intuitionistic fuzzy decision matrix  $R' = (r'_{ij})_{n \times m}$

**Step-3:** Transform the attribute value of cost type of each job into attribute values of benefit type using equation (7) and represent the transformed matrix as  $R = (r_{ij})_{n \times m}$

**Step-4:** Weight all the alternative values  $r_{ij}$  by the weight vector  $\omega_j$  of the attributes and multiply these values by the coefficient values  $n$  and then get the weighted attribute values  $n\omega_j r_{ij}$  and represent in an intuitionistic fuzzy decision matrix  $\hat{R} = (n\omega_j r_{ij})_{n \times m}$

**Step-5:** Utilize the IFHA operator  $\hat{r}_i = IFHA_{\omega, \omega} (r_{i1}, r_{i2}, \dots, r_{im}), i = 1, 2, \dots, n$  calculate the aggregated values of  $\hat{r}_i$ .

**Step-6:** Utilize equation (1) calculate the scores of  $\hat{r}_i (i = 1, 2, \dots, n)$  and get the ideal priorities among persons of a team to each job according to the hierarchical score values. If two scores are equal we may use the accuracy function given in equation (2) to compare the two IFNs.

**Step-7:** If there is no breakdown find the maximum of acceptance and minimum of rejection of the cost to do each job by the persons of the teams. Assign these IFNs representing the cost in a matrix  $R_1$  to a particular team to do a particular job.

**Step-8:** Find the score values of the entries of the matrix  $R_1$  and represent in a matrix  $R_2$ . Using Hungarian method or any other software find the assignment.

**Step-9:** If there exist a break down in a team with a particular person then the assignment will be affected. Replace the person corresponding to the break down from the pool whose priority is higher than the priority of the relieved person.

**Step-10:** Find the score values of the matrix  $R_1$  and represent in a matrix  $R_2$ . Using Hungarian method find the assignment.

### 6. ILLUSTRATIVE EXAMPLE

Let us consider five teams (alternatives)  $Y_i (i = 1, 2, \dots, 5)$  to do five jobs. Each team consists of four skilled persons. By considering the following five attributes to decide the priorities of the persons in the team:  $w_1$ : capacity,  $w_2$ : performance,  $w_3$ : cost to do job,  $w_4$ : time management and  $w_5$ : Experience. Assume that the characteristics of the alternatives  $Y_i (i = 1, 2, \dots, 5)$  and the attribute values corresponding to the person of the teams are represented by the IFNs.

#### Team $Y_1$

a) Let us consider team  $Y_1$  correspondings to job  $J_1$  and it has four alternatives  $P_i (i = 1, 2, 3, 4)$  and five attributes  $w_j (j = 1, 2, \dots, 5)$ . The weight vector of the attributes  $\omega = (0.19, 0.16, 0.24, 0.21, 0.20)^T$ . Assume that the characteristics of the alternatives  $P_i (i = 1, 2, 3, 4)$  are represented by the IFNs, as shown in the intuitionistic fuzzy decision matrix  $R' = (r'_{ij})_{4 \times 5}$

**Table-3:** Intuitionistic fuzzy decision matrix  $R'$

	$w_1$	$w_2$	$J_1$	$w_3$	$w_4$
$P_1$	(0.3 , 0.5)	(0.5 , 0.2)	(0.1 , 0.4)	(0.4 ,0.2)	(0.6 , 0.4)
$P_2$	(0.4 ,0.3)	(0.7 ,0.2)	(0.6 ,0.2)	(0.6 ,0.3)	(0.2 ,0.4)
$P_3$	(0.8 ,0.2)	(0.2 ,0.6)	(0.5 ,0.5)	(0.7 ,0.2)	(0.9 ,0.0)
$P_4$	(0.1 ,0.2)	(0.3 ,0.2)	(0.1 ,0.8)	(0.6 ,0.1)	(0.3 ,0.3)

$$J_1: \max(0.1, 0.6, 0.5, 0.1) = 0.6 \text{ and } \min(0.4, 0.2, 0.5, 0.8) = 0.2$$

Transform the attribute values of cost type in the attribute values of benefit type by using equation (7). Then

$$R' = (r'_{ij})_{4 \times 5} \text{ is transformed into } R = (r_{ij})_{4 \times 5}$$

**Table-4:** Intuitionistic fuzzy decision matrix  $R$

	$w_1$	$w_2$	$J_1$	$w_3$	$w_4$
$P_1$	(0.3 , 0.5)	(0.5 , 0.2)	(0.4 , 0.1)	(0.4 ,0.2)	(0.6 , 0.4)
$P_2$	(0.4 ,0.3)	(0.7 ,0.2)	(0.2 ,0.6)	(0.6 ,0.3)	(0.2 ,0.4)
$P_3$	(0.8 ,0.2)	(0.2 ,0.6)	(0.5 ,0.5)	(0.7 ,0.2)	(0.9 ,0.0)
$P_4$	(0.1 ,0.2)	(0.3 ,0.2)	(0.8 ,0.1)	(0.6 ,0.1)	(0.3 ,0.3)

To get the ideal priorities the following steps are followed:

First weight all the attribute values  $r_{ij} (i = 1, 2, 3, 4; j = 1, 2, \dots, 5)$  by weight vector  $\omega = (0.19, 0.16, 0.24, 0.21, 0.20)^T$  of the attributes  $w_j (j = 1, 2, \dots, 5)$  and multiply these values by the balancing coefficient  $m = 5$ , and then get the weighted attribute values  $5\omega_j r_{ij} (i = 1, 2, 3, 4; j = 1, 2, \dots, 5)$ , as listed in the weighted intuitionistic fuzzy decision matrix  $\hat{R} = (5\omega_j r_{ij})_{4 \times 5}$ .

From table 4 we utilize the operational laws (4) and (6) in section 2.4 to get weighted IFNs:

$$\begin{aligned} \hat{r}_{11} &= (1 - (1 - 0.3)^{5 \times 0.19}, 0.5^{5 \times 0.19}) = (0.288, 0.518), \\ \hat{r}_{12} &= (1 - (1 - 0.5)^{5 \times 0.16}, 0.2^{5 \times 0.16}) = (0.426, 0.276), \\ \hat{r}_{13} &= (1 - (1 - 0.4)^{5 \times 0.24}, 0.1^{5 \times 0.24}) = (0.459, 0.064), \\ \hat{r}_{14} &= (1 - (1 - 0.4)^{5 \times 0.21}, 0.2^{5 \times 0.21}) = (0.416, 0.185), \\ \hat{r}_{15} &= (1 - (1 - 0.6)^{5 \times 0.20}, 0.4^{5 \times 0.20}) = (0.6, 0.4) \end{aligned}$$

Similarly  $\hat{r}_{2j}$ ,  $\hat{r}_{3j}$  and  $\hat{r}_{4j}$  where  $j = 1, 2, \dots, 5$ .

**Table-5:** The weighted intuitionistic fuzzy decision matrix  $\hat{R}$

	$w_1$	$w_2$	$J_1$	$w_3$	$w_4$
$P_1$	(0.288 , 0.518)	(0.426 , 0.276)	(0.459 , 0.064)	(0.416 , 0.185)	(0.6 , 0.4)
$P_2$	(0.385 , 0.319)	(0.619 , 0.276)	(0.235 , 0.542)	(0.618 , 0.283)	(0.2 , 0.4)
$P_3$	(0.784 , 0.217)	(0.164 , 0.665)	(0.565 , 0.436)	(0.718 , 0.185)	(0.9 , 0.0)
$P_4$	(0.096 , 0.217)	(0.249 , 0.276)	(0.856 , 0.064)	(0.618 , 0.09)	(0.3 , 0.3)

The scores of  $\hat{r}_{ij}$  ( $i = 1, 2, 3, 4$ ) and ( $j = 1, 2, \dots, 5$ ) by equation (1) are

$$\begin{aligned} s(\hat{r}_{11}) &= (0.288 - 0.518) = -0.23, \quad s(\hat{r}_{12}) = (0.426 - 0.276) = 0.15, \\ s(\hat{r}_{13}) &= (0.459 - 0.064) = 0.395, \quad s(\hat{r}_{14}) = (0.416 - 0.185) = 0.231 \text{ and} \\ s(\hat{r}_{15}) &= (0.6 - 0.4) = 0.2 \end{aligned}$$

since  $s(\hat{r}_{13}) > s(\hat{r}_{14}) > s(\hat{r}_{15}) > s(\hat{r}_{12}) > s(\hat{r}_{11})$  we have

$$\begin{aligned} \hat{\alpha}_{\sigma(1)} &= (0.459, 0.064), \quad \hat{\alpha}_{\sigma(2)} = (0.416, 0.276), \quad \hat{\alpha}_{\sigma(3)} = (0.6, 0.2) \\ \hat{\alpha}_{\sigma(4)} &= (0.426, 0.276), \quad \hat{\alpha}_{\sigma(5)} = (0.288, 0.518). \end{aligned}$$

Similarly we can calculate the values of other rows.

Let  $\omega = (0.1117, 0.2365, 0.3036, 0.2365, 0.1117)^T$  be its weighting vector as derived by the normal distribution based method of Xu (2005). Utilize the IFHA operator (4) derive the overall attribute values  $\hat{r}_i$  ( $i = 1, 2, 3, 4$ ) of alternatives  $P_i$  ( $i = 1, 2, 3, 4$ )

$$\begin{aligned} \hat{r}_1 &= (1 - (1 - 0.459)^{0.1117} \times (1 - 0.416)^{0.2365} \times (1 - 0.6)^{0.3036} \times (1 - 0.426)^{0.2365} \\ &\quad \times (1 - 0.288)^{0.1117}, 0.064^{0.1117} \times 0.276^{0.2365} \times 0.2^{0.3036} \times 0.276^{0.2365} \times 0.518^{0.1117}) \\ &= (0.474, 0.256). \end{aligned}$$

We obtained

$$\hat{r}_1 = (0.474, 0.256), \quad \hat{r}_2 = (0.432, 0.342), \quad \hat{r}_3 = (0.705, 0), \quad \hat{r}_4 = (0.468, 0.18)$$

The scores of  $\hat{r}_i$  ( $i = 1, 2, 3, 4$ ) by equation (1) are

$$\begin{aligned} s(\hat{r}_1) &= 0.474 - 0.256 = 0.218, \quad s(\hat{r}_2) = 0.432 - 0.342 = 0.09, \\ s(\hat{r}_3) &= 0.705 - 0 = 0.705 \text{ and } s(\hat{r}_4) = 0.468 - 0.18 = 0.288. \end{aligned}$$

Since  $s(\hat{r}_3) > s(\hat{r}_4) > s(\hat{r}_1) > s(\hat{r}_2)$ , we represent the ideal priorities of the persons of team  $Y_1$  corresponding to job  $J_1$  as  $P_3 > P_4 > P_1 > P_2$ .

**b)** Consider the cost values of the persons of the team  $Y_1$  to the job  $J_2$  as  $P_1: (0.1, 0.3)$ ,  $P_2: (0.4, 0.3)$ ,  $P_3: (0.3, 0.6)$  and  $P_4: (0.1, 0.5)$  and the remaining values of attributes are as in Table 3. The ideal priorities of the persons of team  $Y_1$  corresponding to job  $J_2$  represented by  $P_3 > P_4 > P_1 > P_2$ .

**c)** Consider the cost values of the person of the team  $Y_1$  to the job  $J_3$  as  $P_1: (0.7, 0.2)$ ,  $P_2: (0.5, 0.4)$ ,  $P_3: (0.6, 0.3)$  and  $P_4: (0.2, 0.6)$  and the remaining values of attributes are as in Table 3. The ideal priorities of the persons of team  $Y_1$  corresponding to job  $J_3$  represented by  $P_3 > P_4 > P_2 > P_1$ .

**d)** Consider the cost values of the person of the team  $Y_1$  to the job  $J_4$  as  $P_1: (0.1, 0.5)$ ,  $P_2: (0.9, 0.1)$ ,  $P_3: (0.7, 0.1)$  and  $P_4: (0.4, 0.3)$  and the remaining values of attributes are as in Table 3. The ideal priorities of the persons of team  $Y_1$  corresponding to job  $J_4$  represented by  $P_3 > P_1 > P_4 > P_2$ .

**e)** Consider the cost values of the person of the team  $Y_1$  to the job  $J_5$  as  $P_1: (0.2, 0.5)$ ,  $P_2: (0.4, 0.6)$ ,  $P_3: (0.1, 0.7)$  and  $P_4: (0.3, 0.4)$  and the remaining values of attributes are as in Table 3. The ideal priorities of the persons of team  $Y_1$  corresponding to job  $J_5$  represented by  $P_3 > P_2 > P_1 > P_4$ .

**Team Y<sub>2</sub>**

a) Let us consider team Y<sub>2</sub> correspondings to job J<sub>1</sub> and it has four alternatives P<sub>i</sub> (i = 1,2,3,4) and five attributes w<sub>j</sub> (j = 1,2, ... 5). The weight vector of the attributes ω = (0.15, 0.19, 0.18, 0.26, 0.22)<sup>T</sup>. Assume that the characteristics of the alternatives P<sub>i</sub> (i = 1,2,3,4) are represented by the IFNs, as shown in the intuitionistic fuzzy decision matrix R' = (r'ij)<sub>4x5</sub>.

**Table-6:** Intuitionistic fuzzy decision matrix R'

	w <sub>1</sub>	w <sub>2</sub>	J <sub>1</sub>	w <sub>3</sub>	w <sub>4</sub>
P <sub>1</sub>	(0.4 , 0.2)	(0.7 , 0.1)	(0.3 , 0.4)	(0.1 ,0.6)	(0.1 , 0.8)
P <sub>2</sub>	(0.3 ,0.5)	(0.2 ,0.6)	(0.6 ,0.1)	(0.2 ,0.4)	(0.2 ,0.6)
P <sub>3</sub>	(0.1 ,0.7)	(0.1 ,0.8)	(0.2 ,0.5)	(0.5 ,0.4)	(0.7 ,0.3)
P <sub>4</sub>	(0.6 ,0.3)	(0.5 ,0.3)	(0.4 ,0.5)	(0.3 ,0.5)	(0.8 ,0.1)

J<sub>1</sub>: max(0.3, 0.6, 0.2, 0.5) = 0.6 and min(0.4, 0.1,0.5,0.5) = 0.1

Following similar process of team Y<sub>1</sub> we get the ideal priorities of the persons of team Y<sub>2</sub> corresponding to job J<sub>1</sub> represented by P<sub>4</sub> > P<sub>3</sub> > P<sub>1</sub> > P<sub>2</sub>.

b) Consider the cost values of the persons of the team Y<sub>2</sub> to the job J<sub>2</sub> as P<sub>1</sub>: (0.5, 0.3), P<sub>2</sub>: (0.2, 0.6), P<sub>3</sub>: (0.4, 0.6) and P<sub>4</sub>: (0.1, 0.8) and the remaining values of attributes are as in Table 6. The ideal priorities of the persons of team Y<sub>2</sub> corresponding to job J<sub>2</sub> represented by P<sub>4</sub> > P<sub>3</sub> > P<sub>1</sub> > P<sub>2</sub>.

c) Consider the cost values of the person of the team Y<sub>2</sub> to the job J<sub>3</sub> as P<sub>1</sub>: (0.9, 0.1), P<sub>2</sub>: (0.5, 0.4), P<sub>3</sub>: (0.6, 0.3) and P<sub>4</sub>: (0.4, 0.5) and the remaining values of attributes are as in Table 6. The ideal priorities of the persons of team Y<sub>2</sub> corresponding to job J<sub>3</sub> represented by P<sub>4</sub> > P<sub>3</sub> > P<sub>1</sub> > P<sub>2</sub>.

d) Consider the cost values of the person of the team Y<sub>2</sub> to the job J<sub>4</sub> as P<sub>1</sub>: (0.6, 0.4), P<sub>2</sub>: (0.2, 0.3), P<sub>3</sub>: (0.3, 0.6) and P<sub>4</sub>: (0.4, 0.5) and the remaining values of attributes are as in Table 6. The ideal priorities of the persons of team Y<sub>2</sub> corresponding to job J<sub>4</sub> represented by P<sub>4</sub> > P<sub>3</sub> > P<sub>1</sub> > P<sub>2</sub>.

e) Consider the cost values of the person of the team Y<sub>2</sub> to the job J<sub>5</sub> as P<sub>1</sub>: (0.3, 0.6), P<sub>2</sub>: (0.3, 0.4), P<sub>3</sub>: (0.4, 0.2) and P<sub>4</sub>: (0.2, 0.7) and the remaining values of attributes are as in Table 6. The ideal priorities of the persons of team Y<sub>2</sub> corresponding to job J<sub>5</sub> represented by P<sub>4</sub> > P<sub>1</sub> > P<sub>3</sub> > P<sub>2</sub>.

**Team Y<sub>3</sub>**

a) Let us consider team Y<sub>3</sub> correspondings to job J<sub>1</sub> and it has four alternatives P<sub>i</sub> (i = 1,2,3,4) and five attributes w<sub>j</sub> (j = 1,2, ... 5). The weight vector of the attributes ω = (0.18, 0.23, 0.17, 0.22, 0.20)<sup>T</sup>. Assume that the characteristics of the alternatives P<sub>i</sub> (i = 1,2,3,4) are represented by the IFNs, as shown in the intuitionistic fuzzy decision matrix R' = (r'ij)<sub>4x5</sub>.

**Table-7:** Intuitionistic fuzzy decision matrix R'

	w <sub>1</sub>	w <sub>2</sub>	J <sub>1</sub>	w <sub>3</sub>	w <sub>4</sub>
P <sub>1</sub>	(0.6 , 0.2)	(0.3 , 0.5)	(0.6 , 0.4)	(0.1 ,0.7)	(0.8 , 0.1)
P <sub>2</sub>	(0.2 ,0.7)	(0.4 ,0.3)	(0.7 ,0.3)	(0.3 ,0.4)	(0.1 ,0.2)
P <sub>3</sub>	(0.4 ,0.5)	(0.3 ,0.6)	(0.1 ,0.9)	(0.5 ,0.1)	(0.4 ,0.4)
P <sub>4</sub>	(0.5 ,0.4)	(0.7 ,0.1)	(0.2 ,0.1)	(0.2 ,0.3)	(0.3 ,0.4)

J<sub>1</sub>: max(0.6, 0.7, 0.1, 0.2) = 0.7 and min(0.4, 0.3,0.9,0.1) = 0.1

Following similar process of the team Y<sub>1</sub> we get the ideal priorities of the persons of team Y<sub>3</sub> corresponding to job J<sub>1</sub> as P<sub>3</sub> > P<sub>4</sub> > P<sub>1</sub> > P<sub>2</sub>.

b) Consider the cost values of the person of the team Y<sub>3</sub> to the job J<sub>2</sub> as P<sub>1</sub>: (0.1, 0.2), P<sub>2</sub>: (0.3, 0.5), P<sub>3</sub>: (0.6, 0.3) and P<sub>4</sub>: (0.7, 0.1) and the remaining values of attributes are as in Table 7. The ideal priorities of the persons of team Y<sub>3</sub> corresponding to job J<sub>2</sub> represented by P<sub>1</sub> > P<sub>4</sub> > P<sub>2</sub> > P<sub>3</sub>.

c) Consider the cost values of the person of the team  $Y_3$  to the job  $J_3$  as  $P_1: (0.4, 0.6)$ ,  $P_2: (0.5, 0.4)$ ,  $P_3: (0.7, 0.1)$  and  $P_4: (0.2, 0.7)$  and the remaining values of attributes are as in Table 7. The ideal priorities of the persons of team  $Y_3$  corresponding to job  $J_3$  represented by  $P_4 > P_3 > P_1 > P_2$ .

d) Consider the cost values of the person of the team  $Y_3$  to the job  $J_4$  as  $P_1: (0.7, 0.2)$ ,  $P_2: (0.3, 0.6)$ ,  $P_3: (0.5, 0.4)$  and  $P_4: (0.6, 0.3)$  and the remaining values of attributes are as in Table 7. The ideal priorities of the persons of team  $Y_3$  corresponding to job  $J_4$  represented by  $P_4 > P_1 > P_2 > P_3$ .

e) Consider the cost values of the person of the team  $Y_3$  to the job  $J_5$  as  $P_1: (0.3, 0.7)$ ,  $P_2: (0.4, 0.3)$ ,  $P_3: (0.7, 0.2)$  and  $P_4: (0.5, 0.4)$  and the remaining values of attributes are as in Table 7. The ideal priorities of the persons of team  $Y_3$  corresponding to job  $J_5$  represented by  $P_1 > P_4 > P_3 > P_2$ .

**Team  $Y_4$**

a) Let us consider team  $Y_4$  correspondings to job  $J_1$  and it has four alternatives  $P_i (i = 1,2,3,4)$  and five attributes  $w_j (j = 1,2, \dots 5)$ . The weight vector of the attributes  $\omega = (0.16, 0.21, 0.19, 0.27, 0.17)^T$ . Assume that the characteristics of the alternatives  $P_i (i = 1,2,3,4)$  are represented by the IFNs, as shown in the intuitionistic fuzzy decision matrix  $R' = (r'_{ij})_{4 \times 5}$ .

**Table-8:** Intuitionistic fuzzy decision matrix  $R'$

	$w_1$	$w_2$	$J_1$	$w_3$	$w_4$
$P_1$	(0.1 , 0.2)	(0.2 , 0.3)	(0.9 , 0.1)	(0.4 ,0.6)	(0.6 , 0.3)
$P_2$	(0.2 ,0.4)	(0.5 ,0.2)	(0.6 ,0.3)	(0.3 ,0.5)	(0.7 ,0.2)
$P_3$	(0.6 ,0.3)	(0.4 ,0.5)	(0.1 ,0.4)	(0.2 ,0.7)	(0.2 ,0.8)
$P_4$	(0.4 ,0.6)	(0.3 ,0.7)	(0.2 ,0.5)	(0.5 ,0.4)	(0.5 ,0.3)

$J_1: \max(0.9, 0.6, 0.1, 0.2) = 0.9$  and  $\min(0.1, 0.3, 0.4, 0.5) = 0.1$

Following similar process of the team  $Y_1$  we get the ideal priorities of the persons of team  $Y_4$  corresponding to job  $J_1$  as  $P_2 > P_4 > P_3 > P_1$ .

b) Consider the cost values of the person of the team  $Y_4$  to the job  $J_2$  as  $P_1: (0.1, 0.3)$ ,  $P_2: (0.4, 0.5)$ ,  $P_3: (0.6, 0.2)$  and  $P_4: (0.9, 0.1)$  and the remaining values of attributes are as in Table 8. The ideal priorities of the persons of team  $Y_4$  corresponding to job  $J_2$  represented by  $P_2 > P_1 > P_4 > P_3$ .

c) Consider the cost values of the person of the team  $Y_4$  to the job  $J_3$  as  $P_1: (0.4, 0.3)$ ,  $P_2: (0.7, 0.1)$ ,  $P_3: (0.1, 0.4)$  and  $P_4: (0.3, 0.5)$  and the remaining values of attributes are as in Table 8. The ideal priorities of the persons of team  $Y_4$  corresponding to job  $J_3$  represented by  $P_2 > P_4 > P_1 > P_3$ .

d) Consider the cost values of the person of the team  $Y_4$  to the job  $J_4$  as  $P_1: (0.7, 0.2)$ ,  $P_2: (0.8, 0.1)$ ,  $P_3: (0.2, 0.5)$  and  $P_4: (0.4, 0.6)$  and the remaining values of attributes are as in Table 8. The ideal priorities of the persons of team  $Y_4$  corresponding to job  $J_4$  represented by  $P_2 > P_4 > P_1 > P_3$ .

e) Consider the cost values of the person of the team  $Y_4$  to the job  $J_5$  as  $P_1: (0.6, 0.1)$ ,  $P_2: (0.3, 0.4)$ ,  $P_3: (0.5, 0.3)$  and  $P_4: (0.1, 0.9)$  and the remaining values of attributes are as in Table 8. The ideal priorities of the persons of team  $Y_4$  corresponding to job  $J_5$  represented by  $P_4 > P_2 > P_3 > P_1$ .

**Team  $Y_5$**

a) Let us consider team  $Y_5$  correspondings to job  $J_1$  and it has four alternatives  $P_i (i = 1,2,3,4)$  and five attributes  $w_j (j = 1,2, \dots 5)$ . The weight vector of the attributes  $\omega = (0.15, 0.22, 0.21, 0.26, 0.16)^T$ . Assume that the characteristics of the alternatives  $P_i (i = 1,2,3,4)$  are represented by the IFNs, as shown in the intuitionistic fuzzy decision matrix  $R' = (r'_{ij})_{4 \times 5}$ .

**Table-9:** Intuitionistic fuzzy decision matrix  $R'$

	$w_1$	$w_2$	$J_1$	$w_3$	$w_4$
$P_1$	(0.5 , 0.4)	(0.3 , 0.7)	(0.1 , 0.7)	(0.6 ,0.2)	(0.1 , 0.4)
$P_2$	(0.4 ,0.5)	(0.5 ,0.3)	(0.4 ,0.6)	(0.9 ,0.1)	(0.7 ,0.2)
$P_3$	(0.7 ,0.1)	(0.2 ,0.4)	(0.3 ,0.4)	(0.5 ,0.2)	(0.6 ,0.1)
$P_4$	(0.3 ,0.6)	(0.1 ,0.2)	(0.5 ,0.5)	(0.4 ,0.6)	(0.2 ,0.3)



$$J_1: \max(0.1, 0.4, 0.3, 0.5) = 0.5 \text{ and } \min(0.7, 0.6, 0.4, 0.5) = 0.4$$

Following similar process of the team  $Y_1$  we get the ideal priorities of the persons of team  $Y_5$  corresponding to job  $J_1$  as  $P_2 > P_3 > P_1 > P_4$ .

**b)** Consider the cost values of the person of the team  $Y_5$  to the job  $J_2$  as  $P_1: (0.9, 0.1), P_2: (0.4, 0.6), P_3: (0.7, 0.2)$  and  $P_4: (0.1, 0.4)$  and the remaining values of attributes are as in Table 9. The ideal priorities of the persons of team  $Y_5$  corresponding to job  $J_2$  represented by  $P_2 > P_3 > P_4 > P_1$ .

**c)** Consider the cost values of the person of the team  $Y_5$  to the job  $J_3$  as  $P_1: (0.3, 0.6), P_2: (0.5, 0.2), P_3: (0.6, 0.3)$  and  $P_4: (0.2, 0.5)$  and the remaining values of attributes are as in Table 9. The ideal priorities of the persons of team  $Y_5$  corresponding to job  $J_3$  represented by  $P_2 > P_3 > P_1 > P_4$ .

**d)** Consider the cost values of the person of the team  $Y_5$  to the job  $J_4$  as  $P_1: (0.2, 0.4), P_2: (0.3, 0.6), P_3: (0.8, 0.1)$  and  $P_4: (0.7, 0.2)$  and the remaining values of attributes are as in Table 9. The ideal priorities of the persons of team  $Y_5$  corresponding to job  $J_4$  represented by  $P_2 > P_3 > P_1 > P_4$ .

**e)** Consider the cost values of the person of the team  $Y_5$  to the job  $J_5$  as  $P_1: (0.4, 0.1), P_2: (0.2, 0.5), P_3: (0.5, 0.4)$  and  $P_4: (0.8, 0.1)$  and the remaining values of attributes are as in Table 9. The ideal priorities of the persons of team  $Y_5$  corresponding to job  $J_5$  represented by  $P_2 > P_3 > P_1 > P_4$ .

**Team pool  $Y_6$**

**a)** Let us consider team  $Y_6$  correspondings to job  $J_1$  and it has four alternatives  $P_i (i = 1, 2, 3, 4)$  and five attributes  $w_j (j = 1, 2, \dots, 5)$ . The weight vector of the attributes  $\omega = (0.17, 0.21, 0.23, 0.24, 0.15)^T$ . Assume that the characteristics of the alternatives  $P_i (i = 1, 2, 3, 4)$  are represented by the IFNs, as shown in the intuitionistic fuzzy decision matrix  $R' = (r'_{ij})_{4 \times 5}$ .

**Table-10: Intuitionistic fuzzy decision matrix  $R'$**

	$w_1$	$w_2$	$J_1$	$w_3$	$w_4$
$P_1$	(0.3 , 0.4)	(0.7 , 0.1)	(0.4 , 0.4)	(0.8 ,0.2)	(0.4 ,0.5)
$P_2$	(0.5 ,0.3)	(0.5 ,0.2)	(0.1 ,0.7)	(0.6 ,0.1)	(0.6 ,0.2)
$P_3$	(0.4 ,0.5)	(0.6 ,0.1)	(0.1 ,0.5)	(0.6 ,0.2)	(0.7 ,0.3)
$P_4$	(0.3,0.4)	(0.4 ,0.2)	(0.1 ,0.8)	(0.8 ,0.1)	(0.3 ,0.4)

$$J_1: \max(0.4, 0.1, 0.1, 0.1) = 0.4 \text{ and } \min(0.4, 0.7, 0.5, 0.8) = 0.4$$

Following similar process of the team  $Y_1$  we get the ideal priorities of the persons of team  $Y_6$  corresponding to job  $J_1$  as  $P_2 > P_4 > P_3 > P_1$ .

**b)** Consider the cost values of the person of the team  $Y_6$  to the job  $J_2$  as  $P_1: (0.2, 0.5), P_2: (0.2, 0.7), P_3: (0.1, 0.8)$  and  $P_4: (0.1, 0.7)$  and the remaining values of attributes as in Table 10. The ideal priorities of the persons of team  $Y_6$  corresponding to job  $J_2$  represented by  $P_3 > P_2 > P_1 > P_4$ .

**c)** Consider the cost values of the person of the team  $Y_6$  to the job  $J_3$  as  $P_1: (0.5, 0.1), P_2: (0.6, 0.3), P_3: (0.5, 0.4)$  and  $P_4: (0.5, 0.2)$  and the remaining values of attributes are as in Table 10. The ideal priorities of the persons of team  $Y_6$  corresponding to job  $J_3$  represented by  $P_3 > P_2 > P_1 > P_4$ .

**d)** Consider the cost values of the person of the team  $Y_6$  to the job  $J_4$  as  $P_1: (0.4, 0.5), P_2: (0.3, 0.4), P_3: (0.2, 0.3)$  and  $P_4: (0.1, 0.7)$  and the remaining values of attributes are as in Table 10. The ideal priorities of the persons of team  $Y_6$  corresponding to job  $J_4$  represented by  $P_4 > P_1 > P_3 > P_2$ .

**e)** Consider the cost values of the person of the team  $Y_6$  to the job  $J_5$  as  $P_1: (0.3, 0.5), P_2: (0.6, 0.4), P_3: (0.3, 0.6)$  and  $P_4: (0.1, 0.8)$  and the remaining values of attributes are as in Table 10. The ideal priorities of the persons of team  $Y_6$  corresponding to job  $J_5$  represented by  $P_4 > P_3 > P_1 > P_2$ .

**Final Table**

**Table-11:**  $R_1$ : Cost matrix of the teams corresponding to jobs

	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$
$Y_1$	(0.6,0.2)	(0.4,0.3)	(0.7,0.2)	(0.9,0.1)	(0.4,0.4)
$Y_2$	(0.6,0.1)	(0.5,0.3)	(0.9,0.1)	(0.6,0.3)	(0.4,0.2)
$Y_3$	(0.7,0.1)	(0.7,0.1)	(0.7,0.1)	(0.7,0.2)	(0.7,0.2)
$Y_4$	(0.9,0.1)	(0.9,0.1)	(0.7,0.1)	(0.8,0.1)	(0.6,0.1)
$Y_5$	(0.5,0.4)	(0.9,0.1)	(0.6,0.2)	(0.8,0.1)	(0.8,0.1)

Suppose the person  $P_2$  is absent the team  $Y_2$ . Choose the person from the pool whose priority is higher than  $P_2$  corresponding to job  $J_1$  and replace the value of above said person in the team  $Y_2$  with the place  $P_2$  corresponding to job  $J_1$ . Find the maximum of acceptance and minimum of rejection of the cost to do job  $J_1$  by the person of the team  $Y_2$ . Put the value in the table  $R_1$  with respect to the team  $Y_2$  corresponding to the job  $J_1$ . Similarly we do for other values of  $J_2, J_3, J_4$  and  $J_5$  by the team  $Y_2$ . Suppose more than one person are absent replacement is made above.

**Table-12:** After break down the cost matrix  $R_1$

	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$
$Y_1$	(0.6,0.2)	(0.4,0.3)	(0.7,0.2)	(0.9,0.1)	(0.4,0.4)
$Y_2$	(0.4,0.4)	(0.5,0.3)	(0.9,0.1)	(0.6,0.4)	(0.4,0.2)
$Y_3$	(0.7,0.1)	(0.7,0.1)	(0.7,0.1)	(0.7,0.2)	(0.7,0.2)
$Y_4$	(0.9,0.1)	(0.9,0.1)	(0.7,0.1)	(0.8,0.1)	(0.6,0.1)
$Y_5$	(0.5,0.4)	(0.9,0.1)	(0.6,0.2)	(0.8,0.1)	(0.8,0.1)

Utilizing equation (1) calculate the score matrix of  $R_1$

**Table-13:** Score matrix of  $R_1$

	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$
$Y_1$	0.4	0.1	0.5	0.8	0
$Y_2$	0	0.2	0.8	0.2	0.2
$Y_3$	0.6	0.6	0.6	0.5	0.5
$Y_4$	0.8	0.8	0.6	0.7	0.5
$Y_5$	0.1	0.8	0.4	0.7	0.7

The table 13 is the cost matrix of the assignment problem in the maximization form and it can be solved by Hungarian method or by using any standard software.

**The optimal assignment without break down is**

- 1<sup>st</sup> job is assigned to the 5<sup>th</sup> team.
- 2<sup>nd</sup> job is assigned to the 2<sup>nd</sup> team.
- 3<sup>rd</sup> job is assigned to the 4<sup>th</sup> team.
- 4<sup>th</sup> job is assigned to the 3<sup>rd</sup> team.
- 5<sup>th</sup> job is assigned to the 1<sup>st</sup> team.

**The optimal assignment after break down is**

- 1<sup>st</sup> job is assigned to the 2<sup>nd</sup> team.
- 2<sup>nd</sup> job is assigned to the 1<sup>st</sup> team.
- 3<sup>rd</sup> job is assigned to the 5<sup>th</sup> team.
- 4<sup>th</sup> job is assigned to the 3<sup>rd</sup> team.
- 5<sup>th</sup> job is assigned to the 4<sup>th</sup> team.

**7. CONCLUSION**

In this paper a new real life intuitionistic fuzzy assignment model with replacement is proposed. Two stages of solution procedure are discussed with intuitionistic fuzzy aggregation operators. By an example ideal priorities of the professionals of teams are found and replacement is made where it is necessary by means of the priorities to do the job in time.

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