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A THEOREM ON DEGREE OF VERTICES WITH RESPECT TO A VERTEX SET

SATYANARAYANA BHAVANARI[1], SRINIVASULU DEVANABOINA*[2]

SRINIVAS THOTA[3], PASHAM NARASIMHA SWAMY[4] & MALLIKARJUN BHAVANARI[5]

[1]Department of Mathematics, Acharya Nagarjuna University,
Nagarjuna Nagar – 522 510, Andhra Pradesh, INDIA.

[2]Department of BSH, NRI Institute of Technology,
Agiripalli 521 212, Andhra Pradesh, INDIA.

[3]Department of Mathematics,
Kakatiya University, Warangal-506 009, Telangana, INDIA.

[4]Department of Mathematics, Gitam University, Hyderabad, INDIA.

[5]Institute of Energy Engineering, Department of Mechanical Engineering,
National Central University Jhongli, Taoyuan, TAIWAN – 32001, R.O.C.

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ABSTRACT

In this short paper, we considered the concept "degree of a vertex with respect to a given vertex set" in simple graphs. We included the necessary fundamentals and examples. Finally we obtained a theorem "If A is a proper subset of a vertex set V(G) of a simple graph G, then the following two conditions are equivalent: (i) $d_A(v)=d_G(v)$ for all $v \in V(G)$; and (ii) $d_G(w) = 0$ for all $w \in V(G) \setminus A$, where $d_A(v)$ denotes the degree of v with respect to the given vertex set A".

***Keywords**: Graph, Degree of a vertex, Degree of any vertex of a graph with respect to a vertex set, Prime graph of a Ring,*

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1. INTRODUCTION

Let G = (V, E) be a graph consist of a finite non-empty set V of vertices and finite set E of edges such that each edge e_k is identified as an unordered pair of vertices $\{v_i, v_j\}$, where v_i, v_j are called end points of e_k. The edge $e_k$is also denoted by either $v_i v_j$ or $\overline{v_i v_j}$. We also write $G(V,E)$for the graph. Vertex set and edge set of G are also denoted by $V(G)$ and $E(G)$respectively. An edge associated with a vertex pair $\{v_i, v_i\}$ is called a self-loop. The number of edges associated with the vertex is the degree of the vertex, and d(v) denotes the degree of the vertex v. If there is more than one edge associated with a given pair of vertices, then these edges are called parallel edges or multiple edges. A graph that does not have self-loop or parallel edges is called a simple graph. We consider simple graphs only.

Corresponding Author: Srinivasulu Devanaboina[2]*
[2]Department of BSH, NRI Institute of Technology,
Agiripalli 521 212, Andhra Pradesh, INDIA.

1.1 Definitions:

(i) A graph G (V, E) is said to be a **star graph** if there exists a fixed vertex v (called the center of the star graph) such that E = {vu / u ∈ V and u ≠ v}. A star graph is said to be an **n-star graph** if the number of vertices of the graph is n.

(ii) In a graph G, a subset S of V(G) is said to be a ***dominating set*** if every vertex not in S has a neighbour in S. The ***domination number***, denoted by γ(G) is defined as min {|S| / S is a dominating set in G}.

For other preliminary results and notations we use [18], [20] or [21]

SECTION -2: THE DEGREE OF VERTICES WITH RESPECT TO A VERTEX SET

2.1 Definition (Rajesh kanna, Dharmendra, Sridhara and Pradeep kumar [2]**):** Let G be a simple graph and A$\subseteq V(G)$. The degree of a vertex $\boldsymbol{v}$ ∈V of a graph G with respect to A is the number of vertices of A that are adjacent to v. This degree is denoted by $d_A(v)$. The degree of a vertex v in G is denoted by$d_G(v)$.

2.2 Example: Consider the graph given by Fig 2.2 where V(G) = $\{v_i\ /\ 1 \le i \le 5\}$

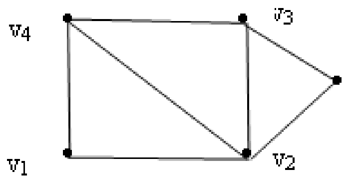

Fig 2.2

Let A = $\{v_1, v_3, v_5\}$. Then by our definition$d_A(v_1) = 0$, $d_A(v_2) = 3$, $d_A(v_3) = 1$, $d_A(v_4) = 2$, $d_A(v_5) = 1$.

2.3 Example: Consider the graph G given by Fig 2.3 where V(G) = $\{v_i\ /\ 0 \le i \le 6\}$

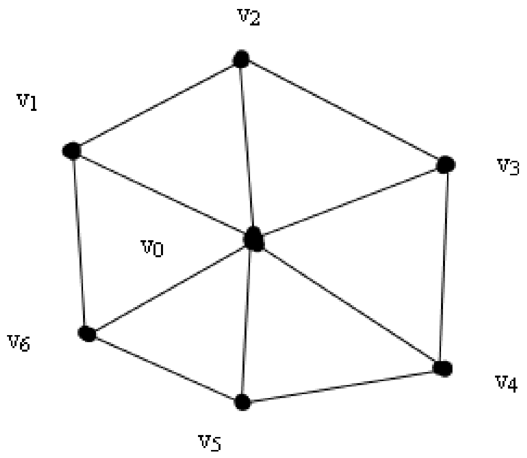

Fig 2.3

Write A = $\{v_0\}$, Then $d_A(v_0) = 0$ and $d_A(v_i) = 1$ for $1 \le i \le 6$.

2.4 Example: Consider the graph G given by Fig 2.4 where V(G) = $\{x_1, x_2, x_3, y_1, y_2, y_3, y_4\}$.

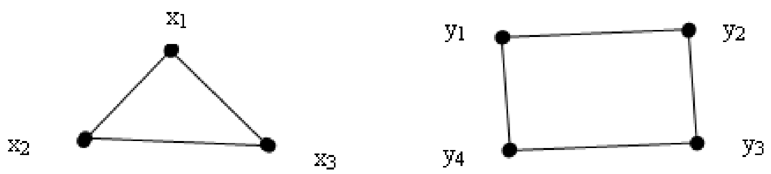

Fig 2.4

(i) If A = $\{x_1, x_2, x_3\}$ then $d_A(x_i)$ = $d_G(x_i)$ for $1 \le i \le 3$; and $d_A(y_i)$ = 0 for $1 \le i \le 4$.

(i) If B = $\{y_1, y_2, y_3, y_4\}$ then $d_B(y_i)$ = $d_G(y_i)$ for $1 \le i \le 4$; and $d_B(x_i)$ = 0 for $1 \le i \le 3$.

2.5 Example: Consider the prime graph PG(R) where R =$\mathbb{Z}_2 \times \mathbb{Z}_2$= {(0, 0), (0, 1), (1, 0), (1, 1)} (Considered in the Note 1.2(ii) of Satyanarayana, Srinivasulu[8]). The graph PG(R) is given by Fig. 2.5

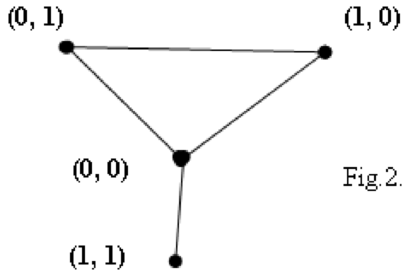

Fig.2.5

Write A = {(0,0)},

Then $d_A((0,0)) = 0$, $d_A((0,1) = d_A((1,0) = d_A((1,1) = 1$.

3. A THEOREM

3.1 Lemma: If A⊊V(G) and $d_A(v) = d_G(v)$ for all $v \in$ V(G), then $d_G(w) = 0$ for all $w \in V(G) \setminus A$.

Proof: Suppose that A⊊V(G) and $d_A(v) = d_G(v)$ for all $v \in$ V(G).

In a contrary way, suppose $w \in V(G) \setminus A$ and $d_G(w)$ = k> 0.

Now $d_A(w)$=$d_G(w)$ = k ≥ 1. So there exist a vertex $u \in A$ with$\overline{wu} \in E(G)$.

Now consider$d_A(u)$. Since $w \in V(G) \setminus A$ we have that
$\overline{wu} \in \{\overline{xu}/x \in V(G)\} \setminus \{\overline{xu}/x \in A\}$.

This implies that
$|\{\overline{xu}/x \in V(G)\}| > |\{\overline{xu}/x \in A\}|$ (Since $\{\overline{xu}/x \in A\}$ is a subset of $\{\overline{xu}/x \in V(G)\}$)

This implies that $d_G(u) > d_A(u)$, a contradiction.

Hence $d_G(w)$ = 0 for all w∈ $V(G) \setminus A$.

3.2 Lemma: If A ⊊ V(G) and $d_G(w) = 0$ for all $w \in V(G) \setminus A$, then $d_A(v) = d_G(v)$ for all $v \in$ V(G).

Proof: Let $v \in$ V(G), Since $\{\overline{xv}/x \in A\} \subseteq \{\overline{xv}/x \in V(G)\}$ we have that $d_A(v) \leq d_G(v)$ (as mentioned on page 126 of Rajesh kanna *et al.* [2]).

In a contrary way, suppose that there exists $w \in V(G)$ such that $d_A(w) \neq d_G(w)$.
⇒ $|\{\overline{xw}/x \in A\}| \lneqq |\{\overline{xw}/x \in V(G)\}|$
⇒ $\{\overline{xw}/x \in A\} \subsetneq \{\overline{xw}/x \in V(G)\}$
⇒ there exists$u \in V(G)$ such that$\overline{uw} \in \{\overline{xw}/x \in V(G)\} \setminus \{\overline{xw}/x \in A\}$
⇒ $u \in V(G) \setminus A$ and $d(u) \geq 1$, a contradiction.

Combining lemmas 1 and 2, we have the following Theorem:

3.3 Theorem: Suppose A⊊V(G), then the following two conditions are equivalent:
(i) $d_A(v) = d_G(v)$ for all $v \in$ V(G)
(ii) $d_G(w) = 0$ for all $w \in V(G) \setminus A$

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