International Journal of Mathematical Archive-8(2), 2017, 65-68
Available online through www.ijma.info ISSN 2229 – 5046

A THEOREM ON DEGREE OF VERTICES WITH RESPECT TO A VERTEX SET

SATYANARAYANA BHAVANARI[1], SRINIVASULU DEVANABOINA*[2]

SRINIVAS THOTA[3], PASHAM NARASIMHA SWAMY[4] & MALLIKARJUN BHAVANARI[5]

[1]Department of Mathematics, Acharya Nagarjuna University,
Nagarjuna Nagar – 522 510, Andhra Pradesh, INDIA.

[2]Department of BSH, NRI Institute of Technology,
Agiripalli 521 212, Andhra Pradesh, INDIA.

[3]Department of Mathematics,
Kakatiya University, Warangal-506 009, Telangana, INDIA.

[4]Department of Mathematics, Gitam University, Hyderabad, INDIA.

[5]Institute of Energy Engineering, Department of Mechanical Engineering,
National Central University Jhongli, Taoyuan, TAIWAN – 32001, R.O.C.

(Received On: 31-01-17; Revised & Accepted On: 14-02-17)

ABSTRACT

In this short paper, we considered the concept "degree of a vertex with respect to a given vertex set" in simple graphs. We included the necessary fundamentals and examples. Finally we obtained a theorem "If A is a proper subset of a vertex set V(G) of a simple graph G, then the following two conditions are equivalent: (i) $d_A(v)$=$d_G(v)$ *for all* $v \in V(G)$*; and (ii)* $d_G(w) = 0$ *for all* $w \in V(G) \setminus A$*, where* $d_A(v)$ *denotes the degree of v with respect to the given vertex set A".*

***Keywords**: Graph, Degree of a vertex, Degree of any vertex of a graph with respect to a vertex set, Prime graph of a Ring,*

***Mathematics Subject Classification**: 05C07, 05C20, 05C76, 05C99, 13E15, 68R10.*

1. INTRODUCTION

Let G = (V, E) be a graph consist of a finite non-empty set V of vertices and finite set E of edges such that each edge e_k is identified as an unordered pair of vertices $\{v_i, v_j\}$, where v_i, v_j are called end points of e_k. The edge $e_k$is also denoted by either $v_i v_j$ or $\overline{v_i v_j}$. We also write $G(V,E)$for the graph. Vertex set and edge set of G are also denoted by $V(G)$ and $E(G)$respectively. An edge associated with a vertex pair $\{v_i, v_i\}$ is called a self-loop. The number of edges associated with the vertex is the degree of the vertex, and d(v) denotes the degree of the vertex v. If there is more than one edge associated with a given pair of vertices, then these edges are called parallel edges or multiple edges. A graph that does not have self-loop or parallel edges is called a simple graph. We consider simple graphs only.

Corresponding Author: Srinivasulu Devanaboina[2]*
[2]Department of BSH, NRI Institute of Technology,
Agiripalli 521 212, Andhra Pradesh, INDIA.

1.1 Definitions:

(i) A graph G (V, E) is said to be a **star graph** if there exists a fixed vertex v (called the center of the star graph) such that E = {vu / u ∈ V and u ≠ v}. A star graph is said to be an **n-star graph** if the number of vertices of the graph is n.

(ii) In a graph G, a subset S of V(G) is said to be a ***dominating set*** if every vertex not in S has a neighbour in S. The ***domination number***, denoted by γ(G) is defined as min {|S| / S is a dominating set in G}.

For other preliminary results and notations we use [18], [20] or [21]

SECTION -2: THE DEGREE OF VERTICES WITH RESPECT TO A VERTEX SET

2.1 Definition (Rajesh kanna, Dharmendra, Sridhara and Pradeep kumar [2]**):** Let G be a simple graph and A$\subseteq V(G)$. The degree of a vertex $\boldsymbol{v}$ ∈V of a graph G with respect to A is the number of vertices of A that are adjacent to v. This degree is denoted by $d_A(v)$. The degree of a vertex v in G is denoted by$d_G(v)$.

2.2 Example: Consider the graph given by Fig 2.2 where V(G) = $\{v_i\ /\ 1 \leq i \leq 5\}$

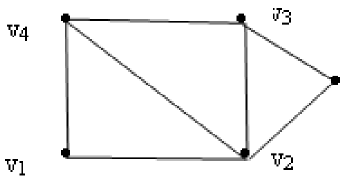

Fig 2.2

Let A = $\{v_1, v_3, v_5\}$. Then by our definition$d_A(v_1) = 0$, $d_A(v_2) = 3$, $d_A(v_3) = 1$, $d_A(v_4) = 2$, $d_A(v_5) = 1$.

2.3 Example: Consider the graph G given by Fig 2.3 where V(G) = $\{v_i\ /\ 0 \leq i \leq 6\}$

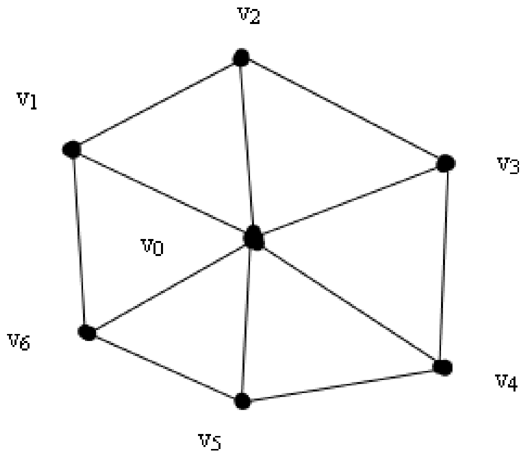

Fig 2.3

Write A = $\{v_0\}$, Then $d_A(v_0) = 0$ and $d_A(v_i) = 1$ for $1 \leq i \leq 6$.

2.4 Example: Consider the graph G given by Fig 2.4 where V(G) = $\{x_1, x_2, x_3, y_1, y_2, y_3, y_4\}$.

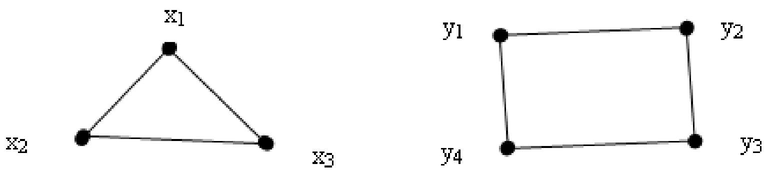

Fig 2.4

(i) If A = $\{x_1, x_2, x_3\}$ then $d_A(x_i)$ = $d_G(x_i)$ for $1 \leq i \leq 3$; and $d_A(y_i)$ = 0 for $1 \leq i \leq 4$.

(i) If B = $\{y_1, y_2, y_3, y_4\}$ then $d_B(y_i)$ = $d_G(y_i)$ for $1 \leq i \leq 4$; and $d_B(x_i)$ = 0 for $1 \leq i \leq 3$.

2.5 Example: Consider the prime graph PG(R) where R =$\mathbb{Z}_2 \times \mathbb{Z}_2$= {(0, 0), (0, 1), (1, 0), (1, 1)} (Considered in the Note 1.2(ii) of Satyanarayana, Srinivasulu[8]). The graph PG(R) is given by Fig. 2.5

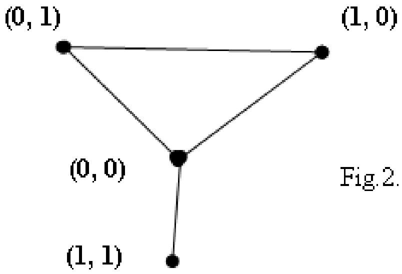

Fig.2.5

Write A = {(0,0)},

Then $d_A((0,0)) = 0$, $d_A((0,1) = d_A((1,0) = d_A((1,1) = 1$.

3. A THEOREM

3.1 Lemma: If A⊊V(G) and $d_A(v)$ = $d_G(v)$ for all v ∈ V(G), then $d_G(w)$ = 0 for all $w \in V(G) \setminus A$.

Proof: Suppose that A⊊V(G) and $d_A(v)$ = $d_G(v)$ for all v ∈ V(G).

In a contrary way, suppose $w \in V(G) \setminus A$ and $d_G(w)$ = k> 0.

Now $d_A(w)$=$d_G(w)$ = k ≥ 1. So there exist a vertex $u \in A$ with$\overline{wu} \in E(G)$.

Now consider$d_A(u)$. Since $w \in V(G) \setminus A$ we have that
$\overline{wu} \in \{\overline{xu}/x \in V(G)\} \setminus \{\overline{xu}/x \in A\}$.

This implies that
$|\{\overline{xu}/x \in V(G)\}| > |\{\overline{xu}/x \in A\}|$ (Since $\{\overline{xu}/x \in A\}$ is a subset of $\{\overline{xu}/x \in V(G)\}$)

This implies that $d_G(u) > d_A(u)$, a contradiction.

Hence $d_G(w)$ = 0 for all w∈ $V(G) \setminus A$.

3.2 Lemma: If A ⊊ V(G) and $d_G(w) = 0$ for all $w \in V(G) \setminus A$, then $d_A(v)$ = $d_G(v)$ for all v ∈ V(G).

Proof: Let v ∈ V(G), Since $\{\overline{xv}/x \in A\} \subseteq \{\overline{xv}/x \in V(G)\}$ we have that $d_A(v) \leq d_G(v)$ (as mentioned on page 126 of Rajesh kanna *et al.* [2]).

In a contrary way, suppose that there exists $w \in V(G)$ such that $d_A(w) \neq d_G(w)$.
⇒ $|\{\overline{xw}/x \in A\}| \lneqq |\{\overline{xw}/x \in V(G)\}|$
⇒ $\{\overline{xw}/x \in A\} \subsetneq \{\overline{xw}/x \in V(G)\}$
⇒ there exists$u \in V(G)$ such that$\overline{uw} \in \{\overline{xw}/x \in V(G)\} \setminus \{\overline{xw}/x \in A\}$
⇒ $u \in V(G) \setminus A$ and $d(u) \geq 1$, a contradiction.

Combining lemmas 1 and 2, we have the following Theorem:

3.3 Theorem: Suppose A⊊V(G), then the following two conditions are equivalent:
(i) $d_A(v)$ = $d_G(v)$ for all v ∈ V(G)
(ii) $d_G(w) = 0$ for all $w \in V(G) \setminus A$

REFERENCES

1. Kedukodi B.S., Kuncham S.P. and Satyanarayana Bhavanari, "Nearring Ideals, Graphs and Cliques", International Mathematical Forum, 8 (2) (2013) PP 73-83.
2. M.R.Rajesh kanna, B.N. Dharmendra, G.Sridhara and R. Pradeep kumar, " Some Results on the Degree of a Vertex set", Int. J. Contemp. Math. Sciences, Vol. 8, 2013, PP125-131, HIKARI Ltd, www.m-hikari.com
3. Satyanarayana Bhavanari, Godloza L., and Nagaraju D., "Some results on Principal Ideal graph of a ring", African Journal of Mathematics and Computer Science Research Vol.4 (6), 2011. PP 235-241. (ISSN: 2006-9731).

4. Satyanarayana Bhavanari, Mohiddin Shaw, Mallikarjun Bhavanari and T.V.Pradeep Kumar, "On a Graph related to the Ring of Integers Modulo n", Proceedings of the International Conference on Challenges and Applications of Mathematics in Science and Technology (CAMIST) January 11-13 2010. (Publisher: Macmillan Research Series, 2010) PP.688-697. (India). (ISBN: 978 – 0230 – 32875 – 4).
5. Satyanarayana Bhavanari, Mohiddin Shaw and Venkata Vijaya Kumari Arava, "Prime Graph of an Integral Domain", Proceedings of the National Seminar on Present Trends in Mathematics and its Applications, November 11-12 (2010) PP 124-134.
6. Satyanarayana Bhavanari and Nagaraju D., "Dimension and Graph Theoretic Aspects of Rings,"VDM verlag Dr Muller, Germany, 2011. (ISBN: 978-3-639-30558-6).
7. Satyanarayana Bhavanari., Pradeep kumar T.V, Sk. Mohiddin Shaw. "Mathematical Foundations of Computer Science", BS publications, Hyderabad, A.P, India, 2016. (ISBN: 978-93-83-635-81-8).
8. Satyanarayana Bhavanari., Srinivasulu D. "Cartesian Product of Graphs Vs.Prime Graphs of Rings", Global Journal of Pure and Applied Mathematics (GJPAM), Volume 11, Number 2 (2015) PP 199-205. (ISSN 0973-1768)
9. Satyanarayana Bhavanari, Srinivasulu D., Hymavathi T and Mallikarjun Bhavanari "One Sided Zero Divisor Graphs Of Rings with a Total Order", Asian Journal of Mathematics and Computer Research, Volume 15, Issue 1, (2017), PP 9-17.(ISSN:2395-4205)
10. Satyanarayana Bhavanari, Srinivasulu D., and Mallikarjun Bhavanari "A Theorem on the Prime Graph of 2×2 matrix ring of Z2", International Journal on Recent and Innovation Trends in Computing and Communication, Volume 4, Issue 5, (2016) PP 571-573. (ISSN: 2321 - 8169)
11. Satyanarayana Bhavnagar, Srinivasulu D., and Mallikarjun Bhavanari "A Theorem on the Zero Divisor Graph of the ring of 2×2 - matrices over Z2", International Educational Scientific and Research journal Volume 2, issue 6, (June 2016), PP 45-46. (E – ISSN: 2455 – 295X)
12. Satyanarayana Bhavanari, Srinivasulu D., and Mallikarjun Bhavanari "Left Zero Divisor Graphs of Totally Ordered Rings", International Journal of Advanced Engineering, Management and Science(IJAEMS) Vol-2, Issue-6, (June 2016) PP 877-880. (ISSN: 2454 – 1311)
13. Satyanarayana Bhavanari, Srinivasulu D., and Mallikarjun Bhavanari "Star Number of a Graph", The International Journal of Research Publication's Research Journal of Science & IT Management, Volume 05, Number:11, September – 2016, PP 18-22.
14. Satyanarayana Bhavanari , Srinivasulu D., and Mallikarjun Bhavanari "Prime Graph Vs. zero Divisor Graph" IOSR Journal of Mathematics, Vol. 12, Issue 5 Ver.VI (Sep. – oct.2016), PP75-78.. (ISSN: 2319 – 765X)
15. Satyanarayana Bhavanari., Srinivasulu D., and Syam Prasad K. "Some Results on Degree of Vertices in Semitotal Block Graph and Total – Block Graph", International Journal of computer Applications Vol. 50, No.9 (July 2012) PP19-22. (ISSN: 0975 – 8887)
16. Satyanarayana Bhavanari., Srinivasulu D., Syam Prasad K. "Line Graphs& Quasi-Block Graphs", International Journal of Computer Applications Vol. 105, No.3, (November 2014) PP 12-16. (ISSN: 0975 – 8887)
17. Satyanarayana Bhavanari. and Syam Prasad K. "An Isomorphism Theorem on Directed Hypercubes of Dimension n", Indian J. Pure & Appl. Math 34 (10) (2003) PP 1453-1457.
18. Satyanarayana Bhavanari. and Syam Prasad K. "Discrete Mathematics and Graph Theory", Prentice Hall India Pvt. Ltd., New Delhi, (2009) (ISBN 978-81-203-3842-5).
19. Satyanarayana Bhavanari and Syam Prasad K., "Dimension of N-groups and Fuzzy ideals in Gamma Near–rings", VDM verlag Dr Muller, Germany, 2011. (ISBN: 978-3-639-36838)
20. Satyanarayana Bhavanari. and Syam Prasad K. "Nearrings, Fuzzy Ideals and Graph Theory" CRC Press (Taylor & Francis Group, London, New York), (2013) (ISBN 13: 9781439873106).
21. Satyanarayana Bhavanari. and Syam Prasad K. "Discrete Mathematics and Graph Theory", Prentice Hall India Pvt. Ltd., New Delhi, 2014 (Second Edition) (ISBN 978-81-203-4948-3).
22. Satyanarayana Bhavanari., Syam Prasad K and Nagaraju D. "Prime Graph of a Ring", J. Combinatorics, Informations & System Sciences 35 (2010) PP 27-42.

Source of support: Nil, Conflict of interest: None Declared.