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CORDIAL AND PRODUCT CORDIAL LABELING FOR THE EXTENDED DUPLICATE GRAPH OF SPLITTING GRAPH OF PATH

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ABSTRACT

In this paper, we prove that the extended duplicate graph of splitting graph of path admits cordial, total cordial, product cordial and total product cordial labeling.

AMS Subject Classification: *05C78.*

Keywords: *Graph labeling, Duplicate graph, Splitting graph, Cordial labeling, Product cordial labeling.*

1. INTRODUCTION

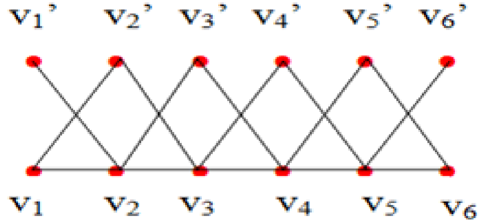
The origin of graph labeling can be attributed to Rosa. E.Sampthkumar [1, 2] introduced the concept of splitting graph and duplicate graph. Gallian [3] provide the literature on survey of different types of graph labeling. The idea of cordial labeling was introduced by Cahit [4]. P.Lawrence Rozario Raj and S.Koilraj [5] have proved the cordial labeling for the splitting graph of some standard graphs. The concept of product cordial labeling and total product labeling was introduced by Sundaram and Somasundaram [6, 7]. K.Thirusangu, B.Selvam and P.P. Ulaganathan have proved that the extended duplicate graph of twig graphs is cordial and total cordial [8].

2. PRELIMINARIES

In this section, we give some basic definitions which are relevant to this paper. Let G(V, E) be a finite, simple and undirected graph with p vertices and q edges.

Definition 2.1 Splitting graph: For each vertex v of a graph G, take a new vertex v'. Join v' to all the vertices of G adjacent to v. The graph Spl(G) thus obtained is called splitting graph of G.

Illustration 1: The Splitting graph of Path Spl (P_6)



Spl(P_6)

Definition 2.2 Duplicate Graph: Let G (V, E) be a simple graph and the duplicate graph of G is DG = (V_1, E_1), where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f: V \rightarrow V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as the edge ab is in E if and only if both ab' and $a'b$ are edges in E_1.

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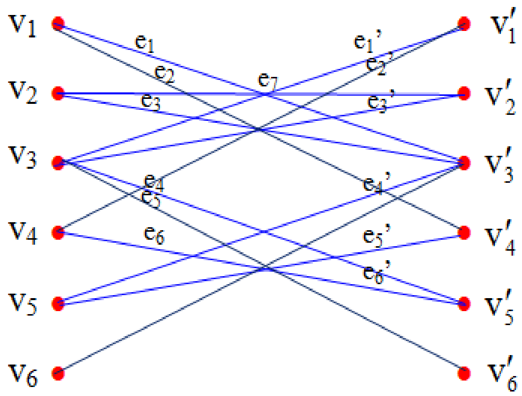
Definition 2.3 Extended duplicate graph of Splitting graph of Path: Let DG = (V_1, E_1) be a duplicate graph of splitting graph of path G(V,E). Extended duplicate graph of splitting graph of path is obtained by adding the edge v_2v_2' to the duplicate graph. It is denoted by EDG Spl(P_m). Clearly it has 4m vertices and 6m-5 edges, $m \geq 2$.

Illustration 2: Extended duplicate graph of Splitting graph of Path.



EDG(Spl(P_3))

Definition 2.4 Cordial **labeling:** A function f: V → {0, 1} such that each edge uv receives the label $|f(u) - f(v)|$ is said to be cordial labeling if the number of vertices labeled '0' and the number of vertices labeled '1' differ by at most one, and the number of edges labeled '0' and the number of edges labeled '1' differ by at most one.

Definition 2.5 Total cordial labeling: A function $f : V \rightarrow \{0,1\}$ such that each edge uv receives the label $|f(u) - f(v)|$ is said to be total cordial labeling if the number of vertices and edges labeled '0' and the number of vertices and edges labeled '1'differ by at most one.

Definition 2.6 Product **cordial labeling:** A function f: V →{0, 1} such that each edge uv receive the label $f(u) \times f(v)$ is said to be product cordial labeling if the number of vertices labeled '0' and the number of vertices labeled '1' differ by at most one, and the number of edges labeled '0' and the number of edges labeled '1'differ by at most one.

Definition 2.7 Total product cordial labeling: A function $f : V \rightarrow \{0,1\}$ such that each edge uv receives the label $f(u) \times f(v)$ is said to be total product cordial labeling if the number of vertices and edges labeled '0' and the number of vertices and edges labeled '1' differ by at most one.

3. MAIN RESULTS

3.1 Cordial labeling

In this section, we present an algorithm and prove the existence of cordial labeling for the extended duplicate graph of splitting graph of path P_m, m ≥ 2.

Algorithm: 3.1: Procedure [Cordial labeling for EDG Spl(P_m), m≥ 2]
V ← {v_1, v_2, … ,v_{2m}, v'_1, v'_2, … ,v'_{2m}}
E ← {e_1, e_2, ….,e_{3m-3}, e_{3m-2}, e'_1,e'_2,…..e'_{3m-3}}
$v_1 \leftarrow 1$, $v_2 \leftarrow 0$, $v'_1 \leftarrow 0$, $v'_2 \leftarrow 1$
for i = 0 to (m-2) do
$v_{3+2i} \leftarrow 1$
$v_{4+2i} \leftarrow 0$
end for
for i = 0 to [(m-2)/2] do
$v'_{3+4i} \leftarrow 0$
$v'_{4+4i} \leftarrow 1$
end for
for i = 0 to [(m-3)/2] do
$v'_{5+4i} \leftarrow 1$
$v'_{6+4i} \leftarrow 0$
end for
end procedure

Theorem 3.1: The extended duplicate graph of splitting graph of path Spl(P_m), $m \geq 2$ admits cordial labeling.

Proof: Let Spl(P_m), $m \geq 2$ be a splitting graph of path. Let EDG Spl(P_m), $m \geq 2$ be the extended duplicate graph of splitting graph of path.

To label the vertices, define a function $f : V \rightarrow \{0,1\}$ as given in algorithm 3.1.

The vertices v_1, v_2, v'_1 and v'_2 receive label '1', '0', '0' and ' 1' respectively**;**

for $0 \leq i \leq (m-2)$, the vertices v_{3+2i} receive label '1' and the vertices v_{4+2i} receive label '0'**;**

for $0 \leq i \leq [(m-2)/2]$, the vertices v'_{3+4i} receive label '0' and the vertices v'_{4+4i} receive label '1'**;**

for $0 \leq i \leq [(m-3)/2]$, the vertices v'_{5+4i} receive label '1' and the vertices v'_{6+4i} receive label '0'**.**

Hence the entire 4m vertices are labeled such that the number of vertices labeled '0' is 2m which is same as the number of vertices labeled '1' and satisfies the required condition.

To obtain the label for edges, we define the induced function $f^*: E \rightarrow \{0, 1\}$ such that

$$f^*(v_i v_j) = \;=\; | f(v_i) - f(v_j) |; \; v_i, v_j \in V$$

The induced function yields the label '0' for the edges e'_2, e'_3 and the label '1' for the edges e'_1 and e_{3m-2};

for $0 \leq i \leq [(m-2)/2]$, the edges e_{1+6i} receive label '1';

for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$, the edges e_{2+6i+j} receive label '0'**;**

for $0 \leq i \leq [(m-3)/2]$, the edges e_{4+6i} receive label '0' and the edges e'_{4+6i} receive label '1';

for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 1$, the edges e_{5+6i+j} receive label '1'and the edges e'_{5+6i+j} receive label '0'**;**

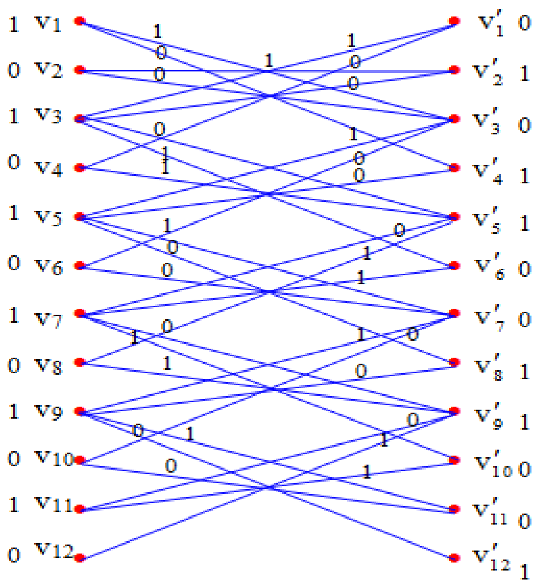
for $0 \leq i \leq [(m-4)/2]$, the edges e'_{7+6i} receive label '0'**;**

for $0 \leq i \leq [(m-4)/2]$ and $0 \leq j \leq 1$, the edges e'_{8+6i+j} receive label '1'**.**

Thus all the 6m-5 edges are labeled such that 3m-3 edges receive label '0' and 3m-2 edges receive label '1' which differ by at most one and satisfies the required condition.

Hence the extended duplicate graph of splitting graph of path Spl(P_m), $m \geq 2$ is cordial.

Illustration 3: *Cordial labeling for the extended duplicate graph of Splitting graph of Path* P_6



EDG(Spl(P_6))

Theorem 3.2: The extended duplicate graph of splitting graph of path $Spl(P_m)$, $m \geq 2$ admits total cordial labeling.

Proof: In theorem 3.1, 2m vertices were assigned the label '0' and 2m vertices were assigned the label '1' and it has been proved that the number of edges labeled '0' is (3m-3) and the number of edges labeled '1' is (3m-2). From this, we conclude that the number of vertices and edges labeled '0' is 2m + (3m-3) = 5m-3 and the number of vertices and edges labeled '1' is 2m+(3m-2) = 5m-2, which differ by one. Hence the extended duplicate graph of splitting graph of path $Spl(P_m)$, $m \geq 2$ is total cordial.

3.2 Product cordial labeling

In this section, we present an algorithm and prove the existence of product cordial labeling for the extended duplicate graph of splitting graph of path P_m, $m \geq 2$.

Algorithm: 3.2: Procedure [Product cordial labeling for EDG $Spl(P_m)$, $m\geq 2$]
$V \leftarrow \{v_1, v_2, \dots, v_{2m}, v'_1, v'_2, \dots, v'_{2m}\}$

$E \leftarrow \{e_1, e_2, \dots, e_{3m-3}, e_{3m-2}, e'_1, e'_2, \dots e'_{3m-3}\}$
        $v_1 \leftarrow 0$, $v_2 \leftarrow 0$, $v'_1 \leftarrow 1$, $v'_2 \leftarrow 1$
for i = 0 to [(m-2)/2] do
        for j = 0 to 1 do
            $v_{3+4i+j} \leftarrow 1$
            $v'_{3+4i+j} \leftarrow 0$
        end for
end for
for i = 0 to [(m-3)/2] do
        for j = 0 to 1 do
            $v_{5+4i+j} \leftarrow 0$
            $v'_{5+4i+j} \leftarrow 1$
        end for
end for
end procedure

Theorem 3.3: The extended duplicate graph of splitting graph of path $Spl(P_m)$, $m \geq 2$ admits product cordial labeling.

Proof: Let $Spl(P_m)$, $m \geq 2$ be a splitting graph of path. Let EDG $Spl(P_m)$, $m \geq 2$ be the extended duplicate graph of splitting graph of path.

To label the vertices, define a function $f : V \rightarrow \{0, 1\}$ as given in algorithm 3.2.

The vertices v_1, v_2, v'_1 and v'_2 receive label '0', '0', '1' and ' 1' respectively**;**

for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$, the vertices v_{3+4i+j} receive label '1' and the vertices v'_{3+4i+j} receive label '0'**;**

for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 1$, the vertices v_{5+4i+j} receive label '0' and the vertices v'_{5+4i+j} receive label '1'**.**

Hence the entire 4m vertices are labeled such that the number of vertices labeled '0' is 2m which is same as the number of vertices labeled '1' and satisfies the required condition.

To obtain the label for edges, we define the induced function $f^* : E \rightarrow \{0,1\}$ such that

$$f^*(v_i v_j) = f(v_i) \times f(v_j); \quad v_i, v_j \in V$$

The induced function yields the label '0' for the edge e_{3m-2};

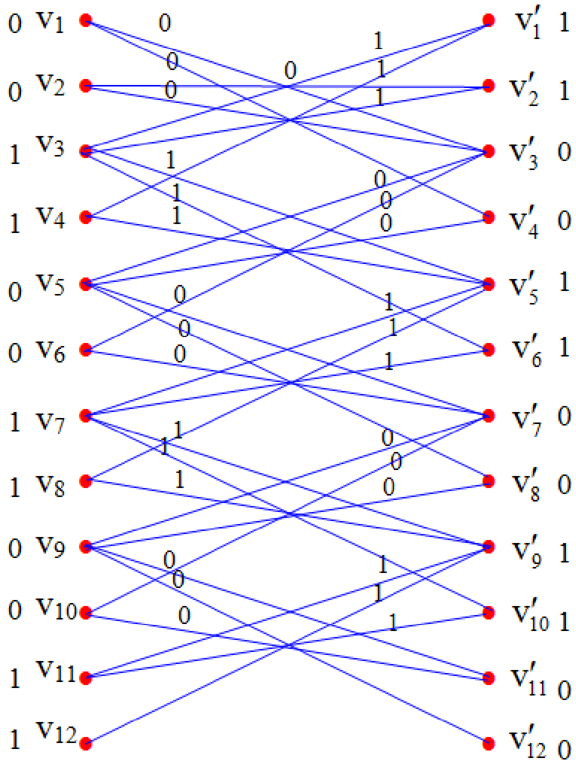
for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 2$, the edges e_{1+6i+j} receive label '0'and the edges e'_{1+6i+j} receive label '1'**;**

for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 2$, the edges e_{4+6i+j} receive label '1'and the edges e'_{4+6i+j} receive label '0' **.**

Thus all the 6m-5 edges are labeled such that 3m-2 edges receive label '0' and 3m-3 edges receive label '1' which differ by at most one and satisfies the required condition.

Hence the extended duplicate graph of splitting graph of path $Spl(P_m)$, $m \geq 2$ is product cordial.

Illustration 4: Product cordial labeling for the extended duplicate graph of Splitting graph of Path P_6



EDG(Spl(P₆))

Theorem 3.4: The extended duplicate graph of splitting graph of path $Spl(P_m)$, $m \geq 2$ admits total product cordial labeling.

Proof: In theorem 3.3, 2m vertices were assigned the label '0' and 2m vertices were assigned the label '1' and it has been proved that the number of edges labeled '0' is (3m-2) and the number of edges labeled '1' is (3m-3) . From this, we conclude that the number of vertices and edges labeled '0' is 2m + (3m-2) = 5m-2 and the number of vertices and edges labeled '1' is 2m+(3m-3) = 5m-3, which differ by one. Hence the extended duplicate graph of splitting graph of path $Spl(P_m)$, $m \geq 2$ is total product cordial.

4. CONCLUSION

In this paper, we presented algorithms and prove that the extended duplicate graph of splitting graph of path P_m, $m \geq 2$ admits cordial , total cordial, product cordial and total product cordial labeling .

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