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EFFECT OF POROUS MEDIUM ON UNSTEADY LAMINAR FREE CONVECTIVE FLOW OF DUSTY VISCOUS FLUID WITH THERMAL DIFFUSION, HEAT AND MASS TRANSFER

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ABSTRACT

The purpose of the present problem is to study the effect of porous medium on unsteady laminar free convective flow of dusty viscous fluid along a moving porous hot vertical plate in the presence of heat source and thermal diffusion with heat, mass transfer and hall currents. The governing equations of motions are solved by a regular perturbation technique. The velocity of dusty fluid and dust particles and skin friction are discussed with the help of tables and graphs. The velocity of dusty fluid and dust particles increase with the increase in K_1 (Porosity parameter) and Q (Velocity ratio parameter), but it decreases with the increase in M (Hartman number).

Keywords: *Dusty viscous fluid, Porous medium, Magnetic field, Convective Flow, Laminar Flow, Heat and Mass transfer.*

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1. INTRODUCTION

The problem of free convection flow of an electrically conducting fluid past a vertical plate under the influence of a magnetic field attracted many scientists, in view of its application in Aerodynamics, Astrophysics, Geophysics and Engineering.

Laminar natural convection and heat transfer in fluids flow with and without heat source in channels with constant wall temperature was discussed by OSTRACH [5]. An analysis of laminar free convective flow and heat transfer on a flat plate parallel to the direction of governing body force was studied by OSTRACH [6]. Combined natural and forced convection laminar flow and heat transfer in fluid, with and without source channels, with linearly varying wall temperature was discussed by OSTRACH [7]. SASTRI [10] dealt heat transfer in the flow over a flat plate with suction and constant heat source. Also SASTRI [11] studied a problem of heat transfer in the presence of temperature dependent heat source in the flow over a flat plate with suction. Forced and natural flows were discussed by SCHLITCHTING [13], ECKERT and DRAKE [3] and BANSAL [1]. Free convection effects on the stokes problem for an infinite vertical plate has been studied by SOUNDALGEKAR [15]. POP and SOUNDALGEKAR [8] investigated free convection flow past an accelerated vertical infinite plate. RAPTIS et al [9] studied effects of free convection currents on the flow of an electrically conducting fluid of an accelerated vertical infinite plate with variable suction. SHARMA [14] investigated free convection effect on the flow past an infinite vertical, porous plate with constant suction and heat flux. KUMAR and VARSHNEY [4] studied steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source with mass transfer.VARSHNEY and KUMAR [16] have discussed effect of thermal diffusion on steady laminar free convective flow along a moving porous hot vertical plate in the presence of heat source with mass transfer. VARSHNEY and SINGH [18] have analyzed the unsteady effect on laminar free convective flow along a moving porous hot vertical plate in the presence of heat source and thermal diffusion with mass transfer. SAXENA et al [12] have studied the effect of dusty viscous fluid on unsteady laminar free convective flow along a moving porous hot vertical plate with thermal diffusion and mass transfer. VARSHNEY et al [17] have studied effect of hall currents on unsteady laminar free convective flow of dusty viscous fluid with thermal diffusion and mass transfer.

In the present section we have considered the problem of VARSHNEY et al [17] with the effect of porous medium on laminar free convective flow of dusty viscous fluid.

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2. FORMULATION OF THE PROBLEM

An infinitely long non conducting hot vertical, thin porous plate is situated in an electrically conducting viscous fluid. The x*-axis is taken along the plate in the upwards direction and y*-axis is normal to it. A transverse constant magnetic field is applied i.e. in the direction of y*-axis. Since the motion is two dimensional and length of the plate is large, therefore, all the physical variables are independent of x* only. The governing equations of continuity, motion and energy for a free convective flow through porous medium of an electrically conducting fluid (BANSAL [2]) along a hot, non conducting porous vertical plate in the presence of heat source with mass transfer are given by:

$$\frac{dv^*}{dy^*}=0$$

i.e.:-

$$v^*=-v_0\left(constant\right) \tag{1}$$

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=g\beta\left(T^*-T_\infty\right)+g\beta'\left(C^*-C_\infty\right)+\nu\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{KN_0}{\rho}\left(V-u^*\right)-\sigma\frac{B_0^{\ 2}}{\rho\left(1+m^2\right)}u^*-\frac{\nu}{K}u^* \tag{2}$$

$$m_1\frac{\partial V}{\partial t^*}=K\left(u^*\right)-V \tag{3}$$

$$\frac{dp^*}{dy^*}=0$$

$$p^*=constant \tag{4}$$

$$\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{S^*}{\rho C_p}\left(T^*-T_\infty\right) \tag{5}$$

$$\frac{\partial C^*}{\partial t^*}+v^*\frac{\partial C^*}{\partial y^*}=D\frac{\partial^2 C^*}{\partial y^{*2}}+D_1\frac{\partial^2 T^*}{\partial y^{*2}} \tag{6}$$

where ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, β' is the coefficient of concentration expansion, ν is the Kinematics viscosity, T_∞ is the temperature of the fluid in the free stream, σ is the electric conductivity, B_o is the magnetic induction, K is the Stoke's resistance coefficient, N_o is the number density of the dust particles which is taken to be constant and m_1 is the mass of a dust particle, D is the chemical molecular diffusivity, k is the thermal conductivity, S* is the coefficient of heat source, C_∞ is the concentration at infinity, D_1 is the thermal diffusivity, C_p is the specific heat at constant pressure, m is the Hall parameter, K* is the permeability of the medium.

The boundary conditions of the problem are

$$u^*=u_w,\quad T^*=T_w,\quad C^*=C_w \qquad \text{at} \qquad y^*=0,\ t^*=0$$

$$u^*\to 0 \quad T^*\to T_\infty \quad C^*\to C_\infty \qquad \text{as} \qquad y^*\to\infty,\ t^*>0 \tag{7}$$

On introducing the following non dimensional quantities

$$y=\frac{v_0 y^*}{\nu},\quad u=\frac{u^*}{v_0},\quad t=\frac{t^* v_0^{\ 2}}{\nu},\quad \theta=\frac{T^*-T_\infty}{T_w-T_\infty},\quad \phi=\frac{C^*-C_\infty}{C_w-C_\infty},\quad v=\frac{V}{v_0}, \tag{8}$$

in equations (2), (3), (5) and (6), we get

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}-\frac{\partial u}{\partial t}+B_1(\nu-u)-\left(\frac{M}{1+m^2}-\frac{1}{K}\right)u=-G_r\theta-G_m\phi \tag{9}$$

$$B\frac{\partial \nu}{\partial t}=(u-\nu) \tag{10}$$

$$\frac{\partial^2\theta}{\partial y^2}+P_r\frac{\partial\theta}{\partial y}-P_r\frac{\partial\theta}{\partial t}+S\theta=0 \tag{11}$$

$$\frac{\partial^2\phi}{\partial y^2}+S_c\frac{\partial\phi}{\partial y}-S_c\frac{\partial\phi}{\partial t}+AS_c=0 \tag{12}$$

Corresponding boundary conditions are

$u = Q,\quad \theta = 1,\quad \phi = 1 \qquad$ at $y = 0,\quad t = 0$

$u \to 0,\ \theta \to 0,\ \phi \to 0 \qquad$ as $y \to \infty,\ t > 0$

Where

$$M=\frac{\sigma\nu B_0^{\ 2}}{\rho\nu_0^{\ 2}},\quad P_r=\frac{\mu C_p}{K},\quad Q=\frac{u_w}{\nu_0},\quad S_c=\frac{\nu}{D},\quad S=\frac{\nu^2 S^*}{k\nu_0^{\ 2}}$$

$$G_r=g\beta\nu\frac{(T_w-T_\infty)}{\nu_0^{\ 3}},\quad G_m=g\beta'\nu\frac{(C_w-C_\infty)}{\nu_0^{\ 3}}$$

$$B_1=\frac{\nu KN_0}{\nu_0^{\ 2}\rho},\quad B=\frac{m_1\nu_0^{\ 2}}{\nu K},\quad K=\frac{\nu_0^{\ 2}K'}{\nu^2}$$

We assume the solution of

$$\left.\begin{aligned} u(y,t)&=u_o(y)e^{-nt}\\ \nu(y,t)&=\nu_o(y)e^{-nt}\\ \theta(y,t)&=\theta_o(y)e^{-nt}\\ \phi(y,t)&=\phi(y)e^{-nt}\end{aligned}\right] \tag{14}$$

Substituting eq.(14) in equations (9) to (12), we get

$$u_0''+u_0'+B_1(\nu_0-u_0)-u_0\left\{\left(\frac{M}{1+m^2}-\frac{1}{K}\right)\right\}+nu_0=-G_r\theta_0-G_m\phi_0 \tag{15}$$

$$\nu_0=\frac{u_0}{(1-nB)} \tag{16}$$

$$\theta_0''+P_r\theta_0'+(nP_r+S)\theta_0=0 \tag{17}$$

$$\phi_0''+S_c\phi_0'+nS_c\phi_0=-AS_c\theta_0'' \tag{18}$$

With corresponding boundary conditions

$$\begin{aligned} &u_o=Q,\quad \theta_o=1,\quad \phi_o=1 \qquad \text{at } y=0\\ &u_o=0,\quad \theta_o=0,\quad \phi_o=0 \qquad \text{at } y\to\infty \end{aligned} \tag{19}$$

On solving equations (15) to (18) which are ordinary linear differential equation in $u_0 . v_0 , \theta_0 , \phi_0$ with boundary conditions (19)? We get the value of u, v , θ, ϕ using eq. (14) as

$$u = \left[A_6 e^{-A_1 y} - G_r A_3 e^{-A_2 y} - (1 + A_5) G_m A_4 e^{-A_2^* y} + G_m A_3 A_5 e^{-A_2 y}\right] e^{-nt} \tag{20}$$

$$v = \left[A_6 e^{-A_1 y} - G_r A_3 e^{-A_2 y} - (1 + A_5) G_m A_4 e^{-A_2^* y} + G_m A_3 A_5 e^{-A_2 y}\right] e^{-nt} / (1 - nB) \tag{21}$$

$$\theta = e^{-A_2 y} e^{-nt} \tag{22}$$

$$\phi = \left[(1 + A_5) e^{-A_2 y} - A_5 e^{-A_2 y}\right] e^{-nt} \tag{23}$$

Where

$$A_1 = \frac{1 + \left[1 + 4\left\{\frac{M}{1 + m^2}\frac{1}{K} - \frac{nBB_1}{1 - nB}\right\}\right]^{1/2}}{2}$$

$$A_2 = \frac{P_r + \left\{P_r^{\,2} - 4(nP_r + S)\right\}^{1/2}}{2}$$

$$A_2^{\,*} = \frac{S_c + \left\{S_c^{\,2} - 4nS_c\right\}^{1/2}}{2}$$

$$A_3 = \frac{1}{A_2^2 - A_2 - \left(\frac{M}{1+m^2} - \frac{nBB_1}{1-nB} - n\right)}$$

$$A_4 = \frac{1}{A_2^{*2} - A_2^{*2} - \left(\frac{M}{1+m^2} - \frac{nBB_1}{1-nB} - n\right)}$$

$$A_5 = \frac{AS_c A_2^2}{\left(A_2^2 - A_2 S_c + nS_c\right)}$$

$$A_6 = G_r A_3 + (1 + A_5) G_m A_4 - G_m A_3 A_5 + Q$$

Skin Friction Coefficient at the plate is given by:

$$\tau = \left(\frac{\partial u}{\partial y}\right) = -A_1 A_6 + G_r A_2 A_3 + (1 + A_5) G_c A_2^{\,*} A_4 - G_m A_2 A_3 A_5$$

3. RESULT AND DISCUSSION

The velocity profiles for dusty fluid and dust particles are tabulated in Table-1 & 2 and plotted in Fig-1 & 2 having Graphs from 1 to 4 at $G_m = 4$, $G_r = 5$, $P_r = 0.71$, $S_c = 0.6$, S = 0.05, n = 0.1, t = 2, A = 1, B = 1, $B_1 = 1$, m = 1 and following different values of K_1 (Porosity parameter), Q (Velocity ratio parameter) and M (Hartman number).

	K_1	Q	M
Graph-1	1	2	4
Graph-2	10	2	4
Graph-3	1	4	4
Graph-4	1	2	8

From all the Graphs of Fig.-1, it is found that the velocity of dusty fluid increases till y = 1.2 after it velocity of dusty fluid decreases sharply till y = 3.5 then after it velocity of dusty fluid decreases continuously with the increase in y. It is also noticed that velocity of dusty fluid increases with the increase in K_1 and Q, but it decreases with the increase in M.

From all the Graphs of Fig.-2, it is found that the velocity of dust particles increases till y = 1.2 after it velocity of dust particles decreases sharply till y = 3.5 then after it velocity of dust particles decreases continuously with the increase in y. It is also noticed that velocity of dust particles increases with the increase in K_1 and Q, but it decreases with the increase in M.The temperature and concentration do not change with the change in K_1, Q and M.

Skin friction profile is tabulated in Table-3 and plotted in Fig.-3 at the same value as taken for velocity profile. From this Fig.-3 it is observed that skin friction increases with the increase in K_1, but it decreases with the increase in Q and M.

4. PARTICULAR CASE

When K_1 is equal to zero, this problem reduces to the problem of VARSHNEY et al [17].

5. CONCLUSION

1. Velocity of dusty viscous fluid increases with the increase in K_1.
2. Velocity of dust particles increases with the increase in K
3. Skin friction increases with the increase in K_1.

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Table - 1:- Values of velocity of dusty fluid at G_r = 5, G_m = 4, P_r = 0.71, S_c = 0.6, S = 0.05, n = 0.1, A = 2, t = 2, B = 1, B_1 =1, m = 1 and different values of K_1, Q and M

Y	Graph-1	Graph-2	Graph-3	Graph-4
0	1.63746	1.63746	3.27492	1.63746
1	1.69789	2.27728	1.87221	1.08590
2	1.28121	1.79754	1.29977	0.77891
3	0.92099	1.30525	0.92297	0.55611
4	0.65165	0.92568	0.65186	0.39307
5	0.45667	0.64909	0.45669	0.27541

Table - 2:- Values of velocity of dust particles at G_r= 5, G_m= 4, P_r= 0.71, S_c= 0.6, S = 0.05, n = 0.1, A = 2, t = 2, B = 1, B_1 =1, m = 1 and different values of K_1, Q and M

Y	Graph-1	Graph-2	Graph-3	Graph-4
0	1.81940	1.81940	3.63880	1.81940
1	1.88654	2.53031	2.08023	1.20655
2	1.42357	1.99726	1.44419	0.86545
3	1.02333	1.45027	1.02552	0.61790
4	0.72406	1.02854	0.72429	0.43675
5	0.50741	0.72121	0.50744	0.30601

Table - 3 :-Values of Skin friction at G_r = 5, G_m = 4, P_r = 0.71, S_c = 0.6, S = 0.05, n = 0.1, A = 2, t = 2, B = 1, B_1 =1, m = 1 and different values of K_1, Q and M

T	Graph-1	Graph-2	Graph-3	Graph-4
0	1.25982	3.06054	-3.22018	-1.12700
2	1.03145	2.50576	-2.63646	-0.92271
4	0.84448	2.05154	-2.15855	-0.75545
6	0.69140	1.67966	-1.76727	-0.61851

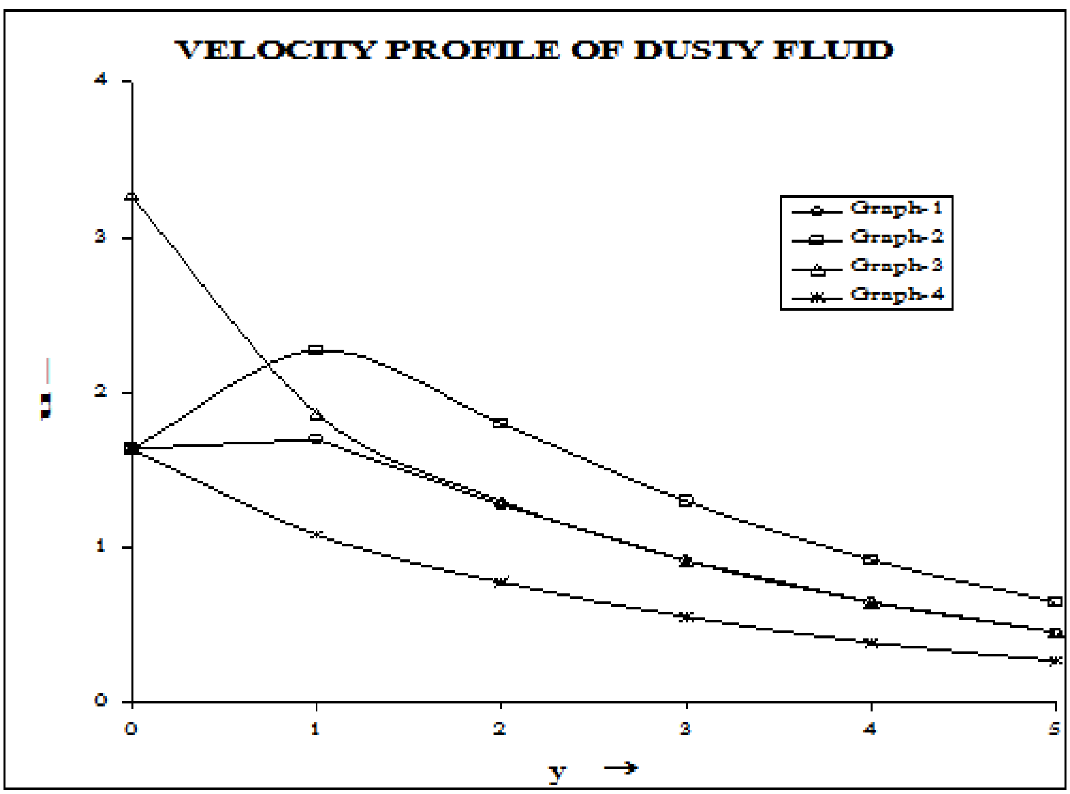

Fig.:1

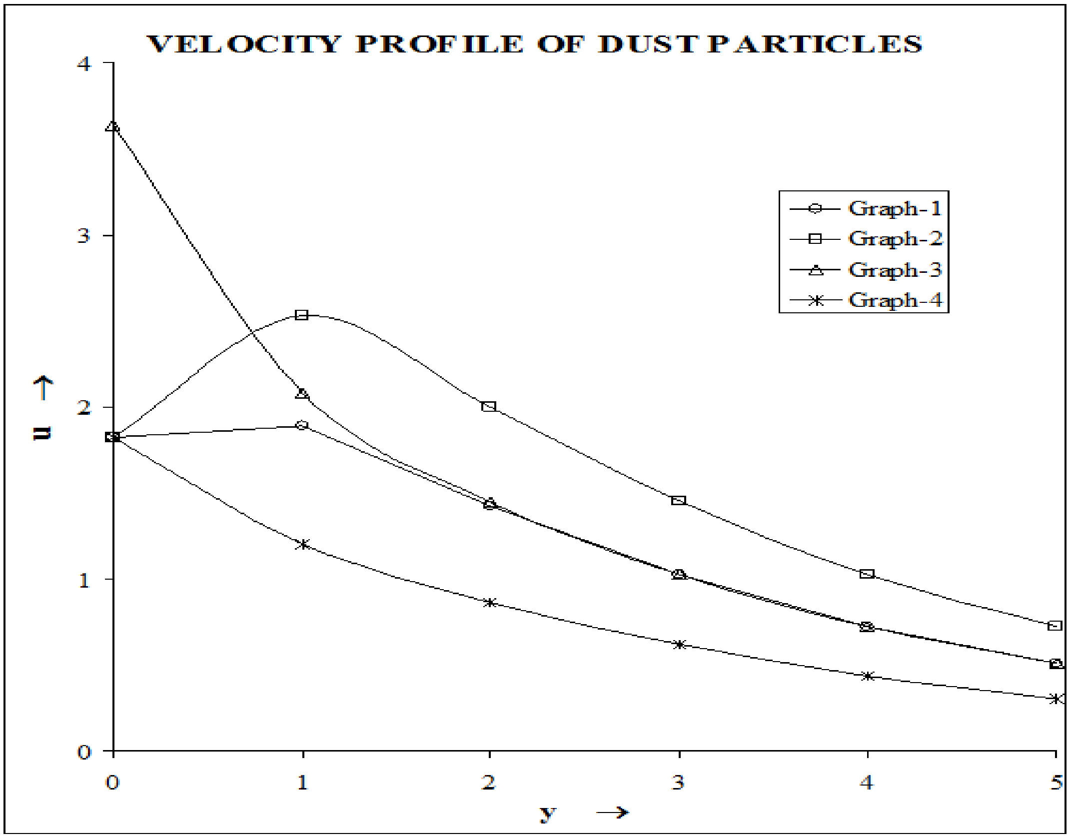
VELOCITY PROFILE OF DUST PARTICLES
Graph-1
Graph-2
Graph-3
Graph-4
u →
y →
0
1
2
3
4
5

Fig.:2

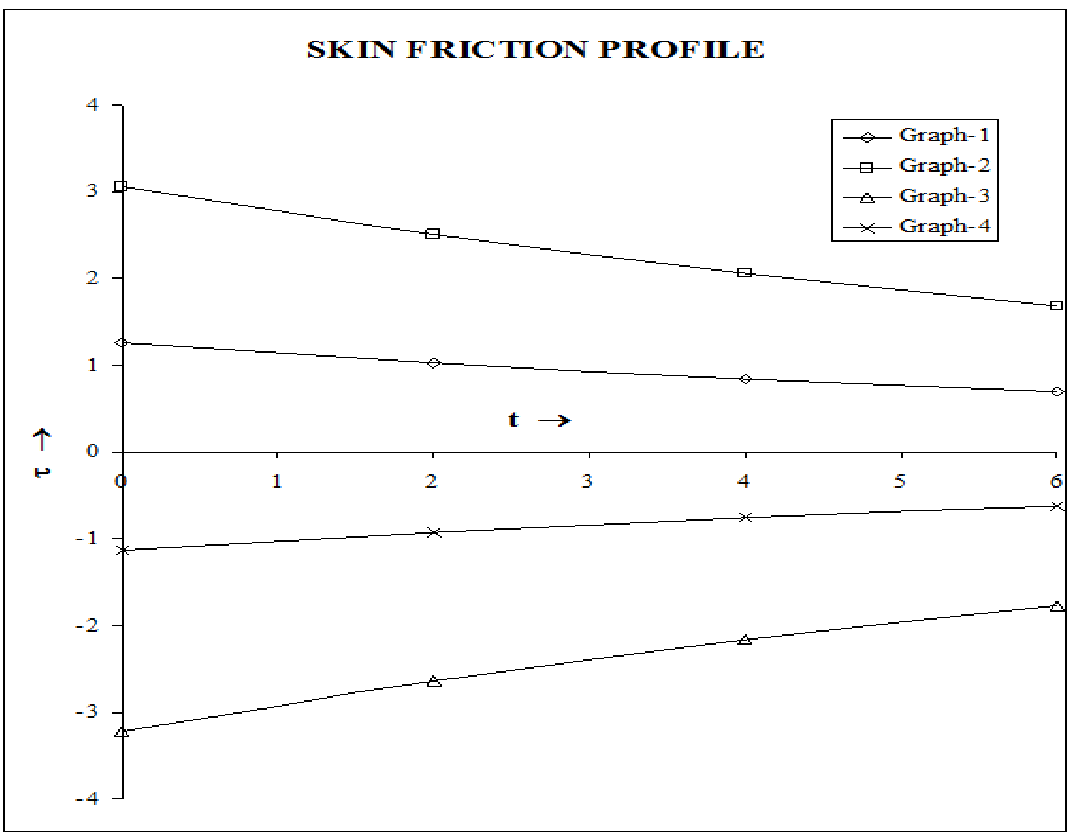
SKIN FRICTION PROFILE
Graph-1
Graph-2
Graph-3
Graph-4
τ →
t →
4
3
2
1
0
-1
-2
-3
-4
5
6

Fig.:3

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