

BIANCHI TYPE V ELECTROMAGNETIC STRING DUST COSMOLOGICAL MODEL

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ABSTRACT

We have investigated Bianchi type V cosmological model with the matter string dust coupled with electromagnetic field in general theory of relativity. And will obtain the deterministic solution of Einstein's field equation, Further some physical and geometrical properties of the model are also discussed.

Key Words: Bianchi type V, String dust, electromagnetic field, general theory of relativity.

I. INTRODUCTION

Bianchi type cosmological models are important in the sense that these models are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view, anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogeneous and anisotropic cosmologies. Hence, these models are to be known as very much suitable models of our universe. Therefore, studies of these models create much more interest.

Adhav, K.S., Nimkar, A.S. [1] have investigated Non-Existence of Bianchi Type-V String Cosmological Model with Bulk Viscous fluid in General Relativity. Adhav, K.S.; Katore, S.D. [2] investigated Higher Dimensional Bianchi Type-V Universe in Creation Field Cosmology. Adhav, K.S.; Dawande, M.V. [3] have studied LRS Bianchi Type-V Universe in Creation Field Cosmology. Baghel, Prashant Singh; Singh, Jagdish Prasad. [4] studied Bianchi Type-V universe with bulk viscous matter and time varying gravitational and cosmological constants. Bali, Raj; Swati Singh [5] have obtained Bianchi Type-V Inflationary universe with decaying vacuum energy (Λ). Bali [6-10] have studied Bianchi Type-V fluid cosmological models in presence of decaying vacuum energy and also studied Bianchi Type-V Magnetized String Dust Universe With Variable Magnetic Permeability and Conformally Flat Tilted Bianchi Type-V Cosmological Models in General Relativity, Bianchi Type-V Bulk Viscous Barotropic Fluid Cosmological Model with variable G & Λ and also Bianchi Type-V Magnetized string dust cosmological models with petrov-type degenerate. Camci, U.; Yavuz, I.; Baysal, H. [11] studied Generation of Bianchi Type V Universes Filled with A Perfect Fluid. Chauhan, D.S.; Singh, R.P.[12] have investigated Bianchi Type-V Cosmological Models in Scalar Tensor Theory of Gravitation with some observable quantities. Chirde, V.R.; Rahate, P.N. [13] investigated Bianchi Type-V Isotropic cosmological model with strange Quark matter attached to cosmic string. Coley, A.A. [14] Bianchi V Imperfect Fluid Cosmology. Deo, Karan Singh, Rewar [15] pointed Bianchi Type-V string Dust Cosmological model in general relativity. Dubey, R.K.; Tripathi, S.K.[16] have studied On Mathematical Analysis for Bianchi Type-V cosmological Model with time varying Gravitational term G . Dwivedi, Uttam Kumar, [17-19] investigated Bulk Viscous Bianchi Type-V Cosmological models with stiff fluid and time dependent cosmological term Λ and also Bianchi Type-V Models with decaying cosmological term Λ and Bianchi Type-V Cosmological Models with time varying Λ . Fransworth, D.L.[20] have pointed Generation of Bianchi Type-V Bulk Viscous Cosmological Model. Kandalkar, S.P.; Wasnik, A.P.; Gawande, S.P. [21] investigated Bianchi Type-V String dust Cosmological Model with Bulk Viscous

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Magnetic Field. Katore, S.D., Shaikh, A.Y. [22] studied Bianchi Type-V Magnetized Anisotropic Dark Energy Models with constant Deceleration Parameter. Mishra, B.; Mohanty, G. [23] have investigated Bianchi Type-V Cosmological Model in Scale Invariant Theory. Mohd. Zeyauddin; Saha, Bijan [24] have investigated Bianchi type-V bulk viscous cosmological models with particle creation in general relativity. Deo S. D. and Roughe A K [25] have studied Non Existence of Bianchi type I, V and VI_0 cosmic string in Bimetric relativity.

In this paper, we have investigated Bianchi type V string dust coupled with electromagnetic field in general theory of relativity and obtained the solution of field equations. Further we have discussed the physical and geometrical behaviour of bianchi type V cosmological model.

II. THE METRIC AND FIELD EQUATIONS

Here we consider the spatially homogeneous Bianchi type-V metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2\alpha x} (B^2 dy^2 + C^2 dz^2) \quad (2.1)$$

Where α is non-zero constant and A, B and C are functions of t only.

The energy momentum tensor of the source string dust coupled with electromagnetic field is denoted by

$$T_i^j = \varepsilon v_i v^j - \lambda x_i x^j + E_i^j \quad (2.2)$$

Here, ε is the rest energy density of the system of string, λ the string tension density, The four velocity vector of the fluid satisfies $v_i v^i = -1 = -x_i x^i$.

Electromagnetic field is defined as

$$E_i^j = -F_{ir} F^{jr} + \frac{1}{4} F_{ab} F^{ab} g_i^j \quad (2.3)$$

Where, E_i^j is electromagnetic energy tensor and F_i^j is the electromagnetic field tensor.

We assume that F_{13} is the only non-vanishing component of F_{ij} which corresponds to the presence of magnetic field along y-direction.

The Einstein field equation in the general theory of relativity is given by

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G T_i^j \quad (2.4)$$

Where, R_i^j is known as Ricci tensor and $R = g^{ij} R_{ij}$ is the Ricci scalar and T_i^j is the energy momentum tensor for matter.

The field equations (2.4) together with the line element (2.1) with equations (2.2) and (2.3) we get

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = 8\pi G \left[\lambda - \frac{(F_{23})^2}{2B^2 C^2 e^{4\alpha x}} \right] \quad (2.5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = 8\pi G \left[\frac{(F_{23})^2}{2B^2 C^2 e^{4\alpha x}} \right] \quad (2.6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = 8\pi G \left[\frac{(F_{23})^2}{2B^2 C^2 e^{4\alpha x}} \right] \quad (2.7)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3\alpha^2}{A^2} = 8\pi G \left[\varepsilon - \frac{(F_{23})^2}{2B^2 C^2 e^{4\alpha x}} \right] \quad (2.8)$$

$$\left(\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) = 0 \quad (2.9)$$

After integration, we have

$$A^2 = k^2 (BC) \tag{2.10}$$

Where k is a constant of integration.

III. SOLUTION OF THE FIELD EQUATIONS

From equation (2.6) and (2.7) with (2.9) we obtain

$$\frac{C\ddot{B} - \ddot{C}B}{BC} = -\frac{1}{2} \left[\frac{C\dot{B} - \dot{C}B}{BC} \right] \tag{3.1}$$

After integration we have

$$C^2 \frac{d}{dt} \left(\frac{B}{C} \right) = \frac{k_1}{\sqrt{BC}} \tag{3.2}$$

where k_1 is constant of integration.

We assume that

$$BC = \xi, \frac{B}{C} = \tau \tag{3.3}$$

Using equation (3.3) in equation (3.2) we have

$$\frac{\dot{\xi}}{\xi} = \frac{k_1}{\tau^{3/2}} \tag{3.4}$$

To get deterministic model of the universe, we apply string dust condition $\varepsilon = \lambda$. Thus equation (2.5) and (2.8) with (2.10) we have

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{1}{2} \left[\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right]^2 + \frac{2}{k^2(BC)} = 0 \tag{3.5}$$

Using equation (3.3) and (3.4) in equation (3.5) we have

$$2\ddot{\xi} - 2\frac{\dot{\xi}^2}{\xi} = \frac{-k_1^2}{\xi^2} - \frac{4}{k^2} \tag{3.6}$$

Now assuming $\dot{\xi} = f(\xi)$, equation (3.6) takes the form

$$\frac{d}{d\xi} (f^2) - \frac{2}{\xi} f^2 = \frac{-k_1^2}{\xi^2} - \frac{4}{k^2} \tag{3.7}$$

Equation (3.7) has the general solution

$$f^2 = \left(\frac{d\xi}{dt} \right)^2 = \frac{a^2 + b^2\xi^2 + \xi^3 k_2}{\xi} \tag{3.8}$$

Where $a^2 = \frac{k_1^2}{3}$, $b^2 = \frac{4}{k^2}$ and k_2 is the constant of integration.

The Bianchi type -V model in this case reduces to the form, substituting, $\xi = T$, $k^2 x = X$, $y=Y$, $z=Z$ in (2.1) we get

$$ds^2 = - \left[\frac{TdT^2}{a^2 + b^2T^2 + k_2T^3} \right] + TdX^2 + Te^{2\alpha X} \left[\tau dY^2 + \tau^{-1} dZ^2 \right] \tag{3.9}$$

PHYSICAL AND GEOMETRICAL BEHAVIOUR OF THE MODEL

It is clear that, the expansion θ , The shear σ , The spatial volume R^3 , The deceleration parameter q for the model (3.9) are given by

$$\theta = \frac{3}{2T} \left[\sqrt{\frac{a^2}{T} + b^2T + k_2T^2} \right] \tag{3.10}$$

$$\sigma = \frac{k_1}{2T^{3/2}} \tag{3.11}$$

$$R^3 = \sqrt{kT}^{3/2} \tag{3.12}$$

and

$$q = 2 - \frac{2b^2T^2 + 3T^3}{a^2 + b^2T^2 + k_2T^3} \tag{3.13}$$

IV. CONCLUSION

In the present study, we have investigated the effect of string dust with electromagnetic field in bianchi type V Universe. Einstein’s field equations have been solved exactly with suitable physical and kinematical parameters. However for large value of T, it isotropizes. The deceleration parameter $q > 0$, hence the model decelerates.

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