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EFFECT OF NON-LINEAR DENSITY TEMPERATURE ON HYDRO MAGNETIC MIXED CONVECTIVE HEAT AND MASS TRANSFER FLOW IN A CYLINDRICAL ANNULUS

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ABSTRACT

In this paper, we analyse non- darcy free and forced convection flow through a porous medium in a Co-axial cylindrical duct where the boundaries are maintained at temperature T_w and Concentration C_w with non-linear density temperature relation. The behaviour of velocity, temperature and concentration is analyzed at different axial positions. The shear stress and the rate of heat and mass transfer have also been obtained for variations in the governing parameters N,γ,N1, Pr, ak.

Keywords: *Circular duct-Non-linear mixed convection-Thermal radiation-Heat sources*

1. INTRODUCTION

Combined heat and mass transfer problems with chemical reaction are important in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one of in which electrical energy is extracted directly from a moving conducting fluid. Obviously, the understanding of this transport process is desirable in order to effectively control the overall transport characteristics.

Free convection in a vertical porous annulus has been extensively studied by several investigators Prasad and Kulacki [16], Prasad and Kulacki [17], Nanda and purushotham[13], Neeraja[14] Leppinen *et.al* [9] Jha.*et.al* [8] Saravannan *et.al* [21]. Charrier *et.al* [4].Chen and Yuh [6] have investigated the heat and mass transfer characteristics of natural convection flow along a vertical cylinder under the combined buoyancy effects of thermal and species diffusion, Sivanjaneya Prasad [20] has investigated the free convection flow of an incompressible, viscous fluid through a porous medium in the annulus between the porous concentric cylinders under the influence of a radial magnetic field. Antonio [2] has investigated the laminar flow,heat transfer in a vertical cylinder duct by taking into account both viscous dissipation and the effect of buoyancy, the limiting case of fully developed natural convection in porous annuli is solved analytically for steady and transient cases Al-Nimir (1). Chamkha *et al.* [4] studied the effect of radiation on combined heat and mass transfer by non-Darcy natural convection about an impermeable horizontal cylinder embedded in porous medium. Sreevani [23] has studied the convective heat and mass transfer through a porous medium in cylindrical annulus under radial magnetic field with Soret effect. Srinivasareddy [22] has discussed the Soret effect on mixed convective heat and mass transfer though a porous cylindrical annulus. For natural convection, the existence of large temperature differences between the surfaces is important. Sudheer Kumar *et.al* [*25*] have studied the effect of radiation on natural convection over a vertical cylinder in a porous media. Padmavathi [19] has analyzed the convective heat transfer in a cylindrical annulus by using finite element method.

Madhusudhan Reddy [11] has investigated the convective heat and mass transfer flow of a viscous fluid through a porous medium in a vertical annulus with cylinder maintained at constant temperature and constant concentration. Sudarsana Reddy *et.al* [24] have discussed the convective heat and mass transfer flow of a viscous fluid in a concentric cylindrical annulus with Soret and Dufour effect. Mallikarjuna *et.al* [12] have investigated the mixed convective heat and mass transfer flow through a porous medium in a vertical cylindrical annulus with Soret and Dufour effects.

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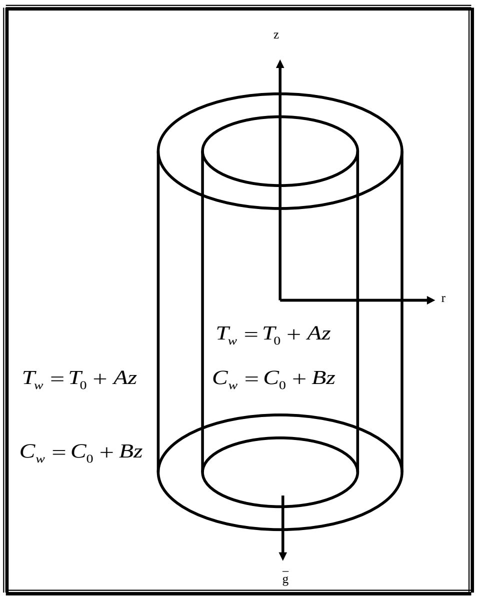

Fig.1 :CONFIGURATION OF THE PROBLEM

2. FORMULATION OF THE PROBLEM

We consider the free and forced convection flow in a vertical circular annulus through a porous medium whose walls are maintained at a constant heat flux and uniform concentration. The flow, temperature and concentration in the fluid are assumed to be fully developed. Both the fluid and porous region have constant physical properties and the flow is a mixed convection flow taking place under thermal and molecular buoyancies and uniform axial pressure gradient. The Boussinesque approximation is invoked so that the density variation is confined to the thermal and molecular buoyancy forces. The Brinkman-Forchhimer-Extended Darcy model which accounts for the inertia and boundary effects has been used for the momentum equation in the porous region. The momentum, energy and diffusion equations are coupled and non-linear. Also the flow is unidirectional along the axial direction of the cylindrical annulus. Making use of the above assumptions the governing equations in the non-dimensional form are

$$\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)=1+\delta\left(D^{-1}+\frac{M^2}{r^2}\right)u+\delta^2(D^{-1})^{1/2}\Lambda u^2-\delta G(\theta(1+ak\theta)+NC) \tag{1}$$

$$\left(1+\frac{4Rd}{3}\right)\left(\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right)=u+\frac{\alpha}{\Pr}\theta \tag{2}$$

$$\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right)-\gamma C=Sc\,u \tag{3}$$

where $\Lambda = FD^{-1}$ (Inertia parameter or Forchhimer number), $G=\frac{g\beta(T_e-T_i)a^3}{\nu^2}$ (Grashof number)

$D^{-1}=\frac{a^2}{k}$ (Inverse Darcy parameter), $M^2=\frac{\sigma\mu_e^2H_0^2}{a\nu}$ (Hartmann number)

$P_r=\frac{\mu C_p}{k_f}$ (Prandtl number), $Sc=\frac{\nu}{D_B}$ (Schmidt number), $\gamma=\frac{k_c^1a^2}{D_B}$ (Chemical Reaction parameter)

$Rd=\frac{4\sigma^\bullet T_e^3}{\beta_R k_f}$ (Radiation parameter) , $\alpha=\frac{qa^2}{C_p}$

The corresponding non-dimensional conditions are

$$u=0,\quad \theta=0,\quad C=0 \quad \text{at} \quad r=1 \text{ and } 1+s \tag{4}$$

3. METHOD OF SOLUTION

The Galerkinfinite element method has been implemented to obtain numerical solutions of coupled non-linear equations (1) to (3) of third-order in*f* and second order in *f*, θ, φ under boundary conditions (4). This technique is extremely efficient and allows robust solutions of complex coupled, nonlinear multiple degree differential equation systems. The fundamental steps comprising the method are

1) Discretization of the domain into elements
2) Derivation of element equations
3) Assembly of Element Equations
4) Imposition of boundary conditions
5) Solution of assembled equations

4. COMPARISON

In the absence of non-linear parameter (ak=0) the results are in good agreement with ***Madhusudana et al*** [10].

Parameters			Madhusudana et al(10)				Present results(S_0=0)			
N	Rd	γ	Nu(1)	Nu(2)	Sh(1)	Sh(2)	Nu(1)	Nu(2)	Sh(1)	Sh(2)
1	0.5	0.5	0.10673	1.7335	12.3727	14.5232	0.10669	1.7336	12.36987	14.52289
2	0.5	0.5	0.11549	1.74131	12.3734	14.5233	0.11538	1.741299	12.37299	14.52319
-0.5	0.5	0.5	0.01152	1.74152	12.3718	14.5237	0.011492	1.741499	12.3714	14.52299
-1.5	0.5	0.5	0.01156	1.74162	12.3712	14.5138	0.01151	1.74155	12.3706	14.5136
1	1.5	0.5	0.2225	3.3518	12.3721	14.5237	0.22239	3.35168	12.3719	14.5231
1	5.0	0.5	0.30165	4.5736	12.3713	14.5239	0.30159	4.57345	12.3709	14.52301
1	0.5	1.5	0.11552	1.74131	13.0332	14.4276	0.115499	1.74131	13.03289	14.4266
1	0.5	-0.5	0.11554	1.741525	13.3226	14.4516	0.115035	1.741499	13.32189	14.4516
1	0.5	-1.5	0.115432	1.741625	12.1728	14.4526	0.115437	1.741587	12.17187	14.4522

5. SHEAR STRESS, NUSSELT NUMBER AND SHERWOOD NUMBER

The shear stress (τ) is evaluated using the formula $\tau = \left(\frac{du}{dr}\right)_{r=1,1+s}$

The rate of heat transfer (Nusselt number) is evaluated using the formula $Nu = -\left(\frac{d\theta}{dr}\right)_{r=1,1+s}$

The rate of mass transfer (Sherwood number) is evaluated using the formula $Sh = -\left(\frac{dC}{dr}\right)_{r=1,1+s}$

6. RESULTS AND DISCUSSION

In order to get physical Insight into the problem the coupled equations governing the flow, heat and mass transfer have been solved by using Galerkin finite method with quadratic approximation functions. The velocity, temperature and concentration have been analysed for different parametric variations N, γ, N_1, ak, Pr.

The important results of the analysis are

- The velocity and the temperature enhances and the concentration reduces in the flow region with increase in N>0 when the buoyancy forces are in the same directions and for the forces acting in opposite directions a reversed effect is noticed (fig.1, 6, 11). The Nusselt number and Sherwood number increases with increases in N >0 and a reversed behaviour is noticed with N<0 (table.1).
- With reference to the chemical reaction parameter γ,we find that the velocity, temperature and concentration enhance in the flow region in the degenerating chemical reaction case while in the generating chemical reaction case, they reduces in the flow region (2, 7, 12). The rate of heat transfer reduces for γ>0 and enhance on r=1 &2. The Sherwood number reduces on r=1 and enhances on r=2 for γ>0 and for γ<0,it enhances on r=1 and reduces on r=2 (table.1).
- An increase in the radiation parameter N_1 enhances |u| in the regions abutting the boundaries r=1 and 2 in the entire flow region. Actual temperature decreases in N_1, higher the radiative flux larger the magnitude of the velocity and actual temperature in the flow region. While the actual concentration enhances in the region 1.01 to 1.3 and reduces in the remaining flow region (fig.3, 8, 13).|τ |, |Sh| enhances at r=1 and 2, while |Nu| reduces at r=1 and 2 (table.1).
- The actual concentration enhances with increase in Pr, the actual temperature reduces in the regions adjacent to the boundaries and enhances in the central region. The magnitude of the velocity enhances with increase in Pr (figs.4, 9, 14).
- The axial velocity and temperature reduces with increase in ak, the velocity and the actual temperature decreases with increase in ak. The actual graphs of the actual concentration increases in ak with a dip in the central region of the annular flow. Thus the non-linearity in the density temperature variation leads to depreciation in the velocity and the actual temperature and enhancement in actual concentration (figs.5, 10, 15).

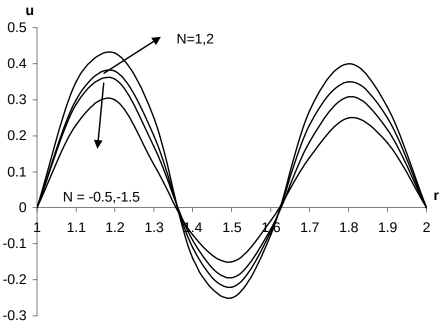

Fig. 1 : Variation of *u* with N
G=20, M=1, α=2, Sc=1.3,γ=0.5,
N_1=0.5, Pr=0.71

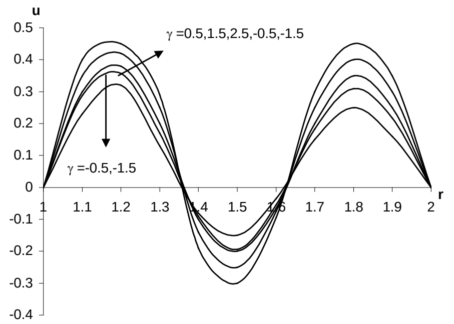

Fig. 2 : Variation of *u* with γ
G=20, M=1, α=2, N=1,Sc=1.3,
N_1=0.5, Pr=0.71

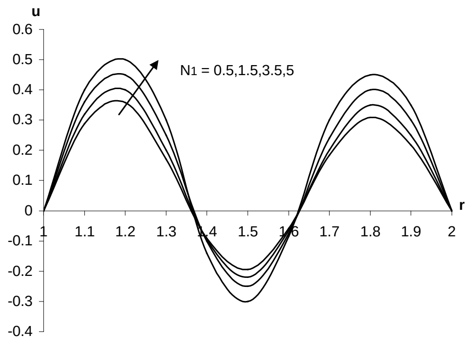

Fig. 3 : Variation of *u* with N_1
G=20, M=1, α=2, N=1,Sc=1.3,γ=0.5, Pr=0.71

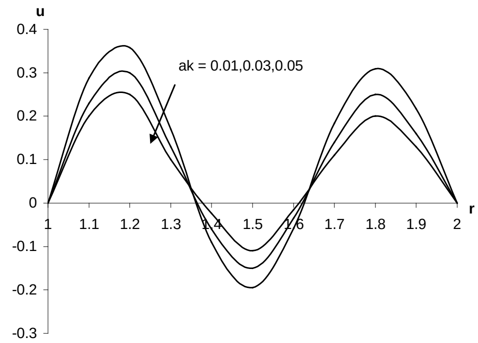

Fig. 4 : Variation of u with ak

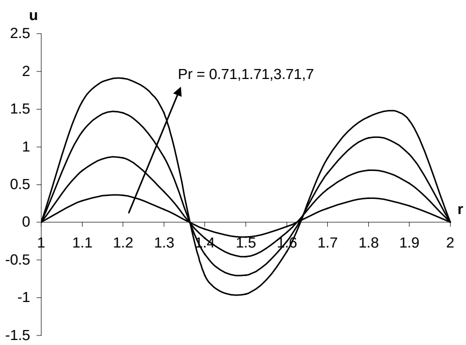

Fig. 5 : Variation of *u* with Pr
G=20, M=1, α=2, N=1,Sc=1.3,γ=0.5,N_1=0.5

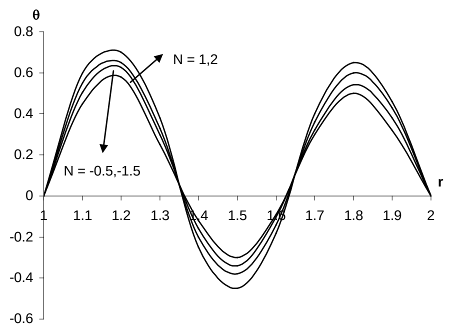

Fig. 6 : Variation of θ with N
G=20, M=1, D^{-1}=0.2, α=2, Sc=1.3,γ=0.5,
N_1=0.5, Pr=0.71

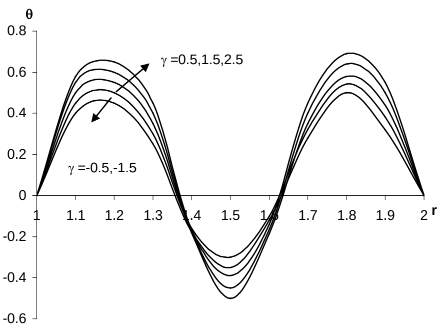

Fig. 7 : Variation of θ with γ
G=20, M=1, D^{-1}=0.2, α=2, N=1,Sc=1.3
N_1=0.5, Pr=0.71

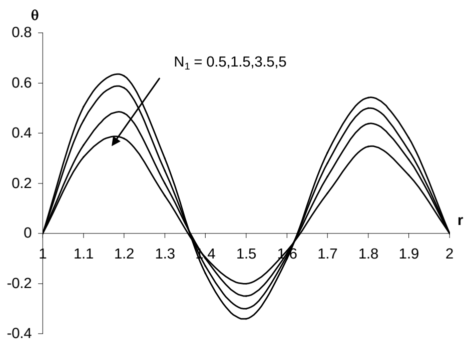

Fig. 8 : Variation of θ with N_1
G=20, M=1, D^{-1}=0.2, α=2, N=1,Sc=1.3,γ=0.5, Pr=0.71

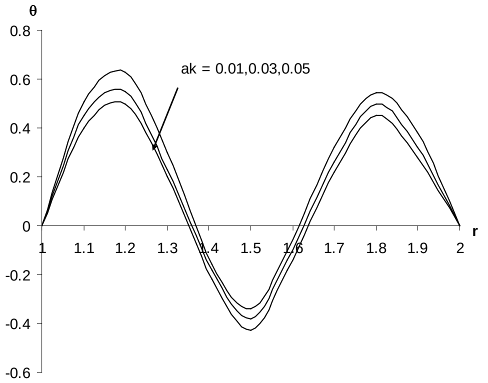

Fig. 9 : Variation of θ with ak

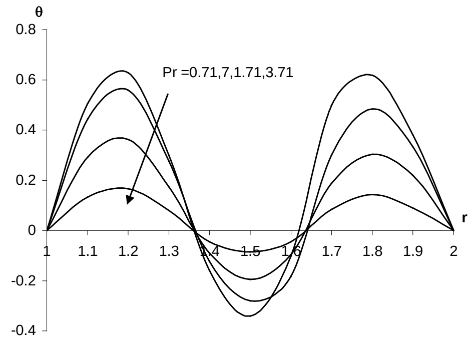

Fig. 10 : Variation of θ with Pr
G=20, M=1, D^{-1}=0.2, α=2, N=1,Sc=1.3,γ=0.5,N_1=0.5

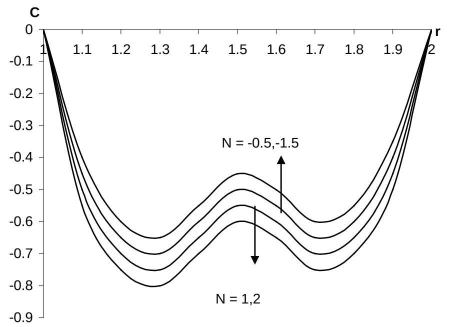

Fig. 11 : Variation of C with N
G=20, M=1, D^{-1}=0.2, α=2, Sc=1.3,γ=0.5,
N_1=0.5, Pr=0.71

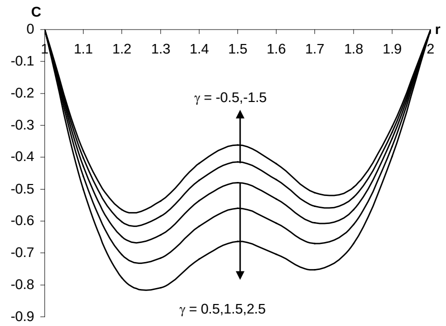

Fig. 12 : Variation of C with γ
G=20, M=1, D^{-1}=0.2, α=2, N=1,Sc=1.3
N_1=0.5, Pr=0.71

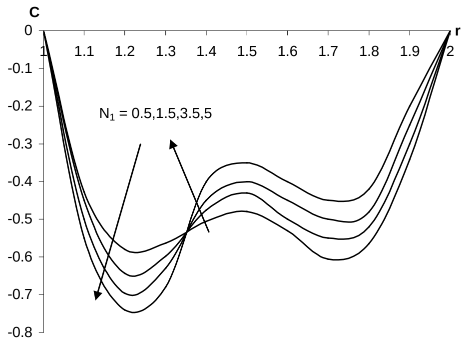

Fig. 13 : Variation of C with N_1
G=20, M=1, D^{-1}=0.2, α=2, N=1,Sc=1.3,γ=0.5, Pr=0.71

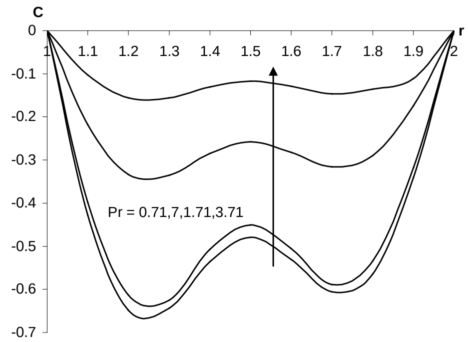

Fig. 14 : Variation of C with Pr
G=20, M=1, D^{-1}=0.2, α=2, N=1,Sc=1.3,γ=0.5, N_1=0.5

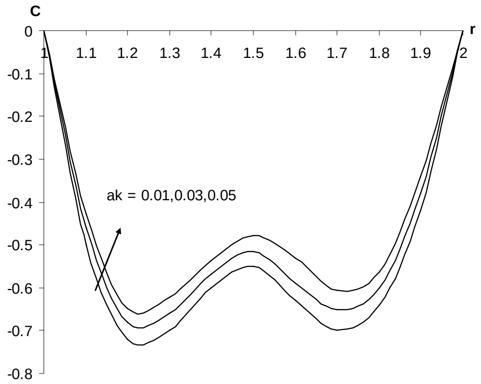

Fig. 15 : Variation of C with ak

Table: Values of Nusselt number, Sherwood number at η=1 and 2

Parameter		Nu(1)	Nu(2)	Sh(1)	Sh(2)
N	1 2 -0.5 -1.5 -4	0.00696551 0.00697059 0.00695781 0.00695275	-0.00492285 -0.00492665 -0.00491709 -0.0049133	-0.000530933 -0.000531328 -0.000530335 -0.000529942	0.000399956 0.000401198 0.000400461 0.000400169
γ	0.5 1.5 -0.5 -1.5	0.00696551 0.00696563 0.00696541 0.00696532	-0.00492285 -0.00492294 -0.00492278 -0.00492271	-0.000530933 -0.000566472 -0.000501852 -0.000477517	0.000399956 0.00042687 0.000379564 0.000361628
N_1	0.5 3.5 5.0 10.0	0.00696551 0.00695616 0.0069529 0.0069522	-0.00492285 -0.00491625 -0.00491395 -0.00491345	-0.000530933 -0.00053094 -0.000530942 -0.000530943	0.000399956 0.000400911 0.000400912 0.000400913
ak	0.01 0.03 0.05	0.00696552 0.006899765 0.00688567	-0.00492286 -0.0048765 -0.0048758	-0.000530933 -0.000529890 -0.0005287965	0.000400905 0.000399967 0.000399956
Pr	0.71 1.71 3.71 7.0	0.00696551 0.0401592 0.182058 0.608469	-0.00492285 -0.0283452 -0.128153 -0.426305	-0.000530933 -0.00127223 -0.00269864 -0.00490735	0.000399956 0.000960696 0.00203793 0.00370586

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