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ON FIXED POINT THEORM IN A SPACE WITH TWO METRICES

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ABSTRACT

In the present paper, we shall proved a fixed point theorem for mapping in two metric spaces.

Keywords: *Two metric space, Continuous mapping, Fixed point.*

INTRODUCTION

In 1968 Maia (1) generalized the result of well known Banach, S. (2) contraction Principle by taking two metrics on a set X. During the past few years Maia's theorem was generalized and fixed point theorems proved in several directionsby Iseki,K.(3), Iyer,S.(4), Mishra (5) Ray, B.K.(6), Rus, I.A.(7), Singh,S.P. and Pant,S.P. (8) Bhore, S.K. (9) Phatak,H.K. (10) and others

OUR MAINRESULTS

Theorem 1: Let (X, d_1, d_2) be metric space such that the following are conditions holds,

(i) $d_1(x, y) \leq d_2(x, y)$ for all x, y ε X.

(ii) (X, d_1) is complete space.

(iii) Two mappings S, T : X → X be a mapping are continuous with respect to d_1 and satisfies inequality

$$[d_2(Sx, Ty)]^2 \leq C \frac{d_2(x,y)d_2(Sx,Ty)[1+2\{\sqrt{d_2(x,y)+d_2(y,Ty)}\}^2]}{[1+\{\sqrt{d_2(x,Ty)+d_2(y,Sx)}\}^2+\{\sqrt{d_2(x,Sx)+d_2(y,Ty)}\}^2]}$$

Then S and T have a common fixed point, further if $0< C < 1$ *then each of S and T has a unique fixed* point and these two fixed points coincide.

Proof: Let x_0 ε X be an arbitrary and define a sequence $\{x_n\}$ be a sequence of X then we defined as below $x_{2m-1} = Sx_{2m-2}$, $x_{2m} = Tx_{2m-1}$, m = 1, 2, 3, ...then we show that the sequence$\{x_n\}$of iterates at x_0 is a Cauchy sequence. If for some k be the positive integers such that $x_{k-1} = x_k$ then $\{x_n\}$ is a Cauchy sequence. Suppose that the positive $x_{m-1} \neq x_m$ for each m=1, 2, 3,…then for $x = x_{2m-2}$ and $y = x_{2m-1}$ using inequality (iv), we have

$$[d_2(x_{2m-1}, x_{2m})]^2 \leq C \frac{d_2(x_{2m-2}, x_{2m-1})d_2(x_{2m-1}, x_{2m})[1+2\{\sqrt{d_2(x_{2m-2}, x_{2m-1})+d_2(x_{2m-1}, x_{2m})}\}^2]}{[1+\{\sqrt{d_2(x_{2m-2}, x_{2m})+d_2(x_{2m-1}, x_{2m-1})}\}^2+\{\sqrt{d_2(x_{2m-2}, x_{2m-1})+d_2(x_{2m-1}, x_{2m})}\}^2]}$$

$$\leq C\frac{d_2(x_{2m-2}, x_{2m-1})d_2(x_{2m-1}, x_{2m})[1+2\{\sqrt{d_2(x_{2m-2}, x_{2m})}\}^2]}{[1+\{\sqrt{d_2(x_{2m-2}, x_{2m})}\}^2+\{\sqrt{d_2(x_{2m-2}, x_{2m})}\}^2]}$$

$$\leq C\frac{d_2(x_{2m-2}, x_{2m-1})d_2(x_{2m-1}, x_{2m})[1+2d_2(x_{2m-2}, x_{2m})]}{[1+2d_2(x_{2m-2}, x_{2m})]}$$

$$[d_2(x_{2m-1}, x_{2m})]^2 \leq Cd_2(x_{2m-2}, x_{2m-1})d_2(x_{2m-1}, x_{2m})$$

$$d_2(x_{2m-1}, x_{2m}) \leq Cd_2(x_{2m-2}, x_{2m-1})$$

Where C < 1. Proceeding in this way we get$d_2(x_{2m-1}, x_{2m}) \leq C^{2m-1}d_2(x_{2m-2}, x_{2m-1})$

By calculations we can easily seen that the following inequality hold for sequence $\{x_n\}$.

$$d_2(x_i, x_j) \leq \sum_{k=1}^{j-1} d_2(x_k, x_{k+1})$$

$$\leq \left(\frac{C^i}{1-C}\right)d_2(x_0, x_1) \quad j < i$$

Tends to zero as i→ ∞.

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It follows that $\{x_n\}$ is a Cauchy sequence with respect to d_2. Further by condition (i) sequence is a Cauchy under metricd_1. Since X is complete space there is some u in X such that $\lim_{n\to\infty} x_n$ = p in the metric d_1. Since S is continuous on X it is continuous at p also. Sp = S $\log_{m\to\infty}$ x_n= S $\log_{m\to\infty}$ x_{2m}= $\log_{m\to\infty}$ Sx_{2m}= $\log_{m\to\infty}$ x_{2m+1}= p

Similarly, by the continuity of T we have Tp = p. Hence p is the a common fixed point of S and T.

Let p and q be two distinct common fixed point of S and T then

$$[d_2(p,q)]^2 = [d_2(Sp,Tq)]^2$$
$$\leq C\,\frac{d_2(p,q)d_2(Sp,Tq)[\,1+2\{\sqrt{d_2(p,q)+d_2(q,Tq)}\}^2]}{[\,1+\{\sqrt{d_2(p,Tq)+d_2(q,Sp)}\}^2+\{\sqrt{d_2(p,Sp)+d_2(q,Tq)}\}^2]}$$
$$\leq C\,\frac{d_2(p,q)d_2\,(p,q)[\,1+2\{\sqrt{d_2(p,q)+d_2\,(q,q)}\}^2]}{[\,1+\{\sqrt{d_2(p,q)+d_2(q,p)}\}^2+\{\sqrt{d_2(p,p)+d_2\,(q,q)}\}^2]}$$
$$= C[\,d_2\,(p,q)]^2$$

Since 0 < C < 1 we aarive at a contradiction then p = q. Therefore S and T have unique common fixed point.

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