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SOME FIXED POINT THEOREMS IN FUZZY METRIC SPACES

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ABSTRACT

In this paper we prove some fixed point theorems in two complete fuzzy metric spaces.

Key words and Phrases: *Fuzzy metric space; fixed point; complete fuzzy metric space*

Mathematics subject classification: *54H25, 47H10.*

INTRODUCTION

Fixed point theory is one of the most fruitful and effective tools in mathematics which has enormous applications within as well as outside the mathematics. Despite noted improvements in computer skill and its remarkable success in facilitating many areas of research, there still stands one major short coming: computers are not designed to handle situations wherein uncertainties are involved. To deal with uncertainty, we need techniques other than classical one wherein some specific logic is required. Fuzzy set theory is one of uncertainty approaches wherein topological structures are basic tools to develop mathematical models compatible to concrete real life situations. The concept of fuzzy set was introduced by Zadeh [7]. Kramosil and Michalek [4] built the fuzzy metric spaces in various ways. George and Veermani [2] modified the notion of fuzzy metrices spaces introduced by Kramosil and Michalek [4] in order to get a Hausdroff topology. Fixed point theory in fuzzy metric spaces was initiated by Grabiec [3]. In the literature many authors established the fixed point theorems for mapping satisfying different types conditions in fuzzy metric spaces. In this paper, we prove some new common fixed theorems in two complete fuzzy metric spaces.

Definition 1.1: A binary operation $*: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t-norm if it satisfies the following conditions:
(1) $*$ is associative and commutative,
(2) $*$ is continuous,
(3) $a * 1 = a$ for all $a \in [0, 1]$,
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Examples of continuous t-norm are $a * b = ab$ and $a * b = \min(a, b)$.

Definition 1.2[4]: The 3-tuple $(X, M, *)$ is called a Fuzzy Metric spaces (shortly, FM-space) if X is a n arbitrary set. $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions:
(FM-1) $M(x, y, 0) = 0$,
(FM-2) $M(x, y, t) = 1$, for all $t > 0$ if and only if $x = y$,
(FM-3) $M(x, y, t) = M(y, x, t)$,
(FM-4) $M(x, y, t) * M(y, z, s) \geq M(x, z, t + s)$,
(FM-5) $M(x, y, .) : [0, 1) \rightarrow [0, 1]$ is left continuous for all $x, y, z \in X$ and $s, t > 0$.

Note that M(x, y, t) can be thought of as the degree of nearness between x and y with respect to t. In the sequel, we will only consider FM-space verifying:
(FM-6) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$.

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Definition 1.3[2]: Let (X, M, $*$) be fuzzy metric space. Then

(a) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$

(b) A sequence $\{x_n\}$ in X is said to be converges $x \in X$ if, for all t > 0, $\lim_{n\to\infty} M(x_n, x, t) = 1$.

(c) (X, M, $*$) is said to be complete if and only if every Cauchy sequence in X is convergent.

Before proving our main results, we state some results which are used in proving our main results:

Lemma 1[5]: Let (X, M, $*$) be fuzzy metric space and for all $x, y \in X$, t > 0 and if for a number $k \in (0, 1)$, $M(x, y, kt) \geq M(x, y, t)$, then x = y.

Lemma 2[5]: Let (X, M, $*$) be fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in (0, 1)$, such that $M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t)$, for all t > 0 and n = 1,2,3,…… then $\{y_n\}$ is a Cauchy sequence in X.

MAIN RESULT

Theorem 2.1: Let (X, M, $*$) and (Y, M', $*$) be complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X satisfying the following conditions:

$$M(Sy, STx, t) \geq \min\{M(x, Sy, \frac{t}{c_1}), M'(y, Tx, \frac{t}{c_1})\} \tag{2.1}$$

$$M'(Tx, TSy, t) \geq \min\{M(x, Sy, \frac{t}{c_2}), M'(y, Tx, \frac{t}{c_2})\} \tag{2.2}$$

for all x in X and y in Y where $0 < c_1 < 1$ and $0 < c_2 < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Futher Tz = w and Sw = z.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X and a sequence $\{y_n\}$ in Y, as follows:

$x_n = (ST)^n x_0$, $y_n = T(x_{n-1})$ for n=1,2,……

We have,

$$\begin{aligned} M(x_n, x_{n+1}, t) &= M((ST)^n x_0, (ST)^{n+1} x_0, t) \\ &= M(S(Tx_{n-1}), STx_n, t) \\ &= M(Sy_n, STx_n, t) \\ &\geq \min\{M(x_n, Sy_n, \frac{t}{c_1}), M'(y_n, Tx_n, \frac{t}{c_1})\} \\ &= \min\{M(x_n, STx_{n-1}, \frac{t}{c_1}), M'(y_n, Tx_n, \frac{t}{c_1})\} \\ &= \min\{M(x_n, x_n, \frac{t}{c_1}), M'(y_n, y_{n+1}, \frac{t}{c_1})\} \\ &= M'(y_n, y_{n+1}, \frac{t}{c_1}) \end{aligned} \tag{2.3}$$

Now,

$$\begin{aligned} M'(y_n, y_{n+1}, t) &= M'(Tx_{n-1}, Tx_n, t) \\ &= M'(Tx_{n-1}, TSy_n, t) \\ &\geq \min\{M(x_{n-1}, Sy_n, \frac{t}{c_2}), M'(y_n, Tx_{n-1}, \frac{t}{c_2})\} \\ &= \min\{M(x_{n-1}, x_n, \frac{t}{c_2}), M'(y_n, y_n, \frac{t}{c_2})\} \\ &= M(x_{n-1}, x_n, \frac{t}{c_2}) \end{aligned} \tag{2.4}$$

From (2.3) and (2.4),

$$M(x_n, x_{n+1}, t) \geq M(x_{n-1}, x_n, \frac{t}{c_1 c_2}) \text{………}$$

$$\geq M\left(x_{n-1}, x_n, \frac{t}{(c_1c_2)^n}\right) \rightarrow 0 \text{ as } n \rightarrow \infty \text{ (since } c_1c_2 \in (0,1))$$

Thus $\{x_n\}$ is a Cauchy sequence in (X, M. $*$). Since (X, M, $*$) is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also Cauchy sequence in (Y, M', $*$) . Since (Y, M', $*$) is complete, it converges to a point w in Y.

Now, we prove that Tz = w

Consider,

$$\begin{aligned} M'(Tz, w, c_2t) &= \lim_{n\to\infty} M'(Tz, y_{n+1}, c_2t) \\ &= \lim_{n\to\infty} M'(Tz, TSy_n, c_2t) \\ &\geq \lim_{n\to\infty} \min\{M(z, Sy_n, t), M'(y_n, Tz, t)\} \\ &= \lim_{n\to\infty} \min\{M(z, x_n, t), M'(y_n, Tz, t)\} \\ &= M'(w, Tz, t), \end{aligned}$$

So, Tz = w

Now, we prove that Sw = z

Consider,

$$\begin{aligned} M(Sw, z, c_1t) &= \lim_{n\to\infty} M(Sw, x_{n+1}, c_1t) \\ &= \lim_{n\to\infty} M(Sw, STx_n, c_1t) \\ &\geq \lim_{n\to\infty} \min\{M(x_n, Sw, t), M'(w, Tx_n, t)\} \\ &= \lim_{n\to\infty} \min\{M(x_n, Sw, t), M'(w, y_{n+1}, t)\} \\ &= M(z, Sw, t) \end{aligned}$$

So, Sw = z

We have STz = Sw = z. Thus the point z is a fixed point of ST in X and the point w is fixed point of Ts in Y.

Uniqueness: Let z $\neq$ z' be another fixed point of ST in X. We have

$$\begin{aligned} M(z, z', c_1 c_2 t) &= M(STz, STz', c_1 c_2 t) \\ &\geq \min\{M(z', STz, c_2 t), M'(Tz, Tz', c_2 t)\} \\ &= \min\{M(z', z, c_2 t), M'(Tz, Tz', c_2 t)\} \\ &= M'(Tz, Tz', c_2 t) \end{aligned} \tag{2.5}$$

Also,

$$\begin{aligned} M'(Tz, Tz', c_2 t) &= M'(Tz, TSTz', c_2t) \\ &\geq \min\{M(z, STz', t), M'(Tz', Tz, t)\} \\ &= \min\{M(z, z', t), M'(Tz', Tz, t)\} \\ &= M(z, z', t) \end{aligned} \tag{2.6}$$

From (2.5) and (2.6) we have,

$$M(z, z', c_1 c_2 t) \geq M(z, z', t)$$

Thus z = z'.

So the point z is a unique fixed point of ST. Similarly, we prove that the point w is a unique fixed point of TS.

Corollary 2.2: Let (X, M, $*$) be complete fuzzy metric spaces. If T and S are mapping from X into X satisfying the following conditions:

$$M(Sy, STx, t) \geq \min\{M(x, Sy, \frac{t}{c_1}), M(y, Tx, \frac{t}{c_1})\}$$

$$M(Tx, TSy, t) \geq \min\{M(x, Sy, \frac{t}{c_2}), M(y, Tx, \frac{t}{c_2})\}$$

for all x in X and y in Y where $0 < c_1 < 1$ and $0 < c_2 < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in X. Futher Tz = w and Sw = z and if z=w, then z is the unique common fixed point of S and T.

Proof: If (X, M, $*$) and (Y, M', $*$) are same fuzzy metric spaces, then by above theorem 2.1, we get the required result.

Theorem 2.3: Let (X, M, $*$) and (Y, M', $*$) be complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X satisfying the following conditions:

$$M(Sy, STx, t) \geq \min\{M(x, Sy, \frac{t}{c_1}), M'(y, Tx, \frac{t}{c_1}), M(x, Sy, \frac{t}{c_1}) * M(x, STx, \frac{t}{c_1})\}$$

$$M'(Tx, TSy, t) \geq \min\{M(x, Sy, \frac{t}{c_2}), M'(y, Tx, \frac{t}{c_2}), M'(y, Tx, \frac{t}{c_2}) * M'(y, TSy, \frac{t}{c_2})\}$$

for all x in X and y in Y where $c_1, c_2 \in (0,1)$, then ST has a unique fixed point in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X and a sequence $\{y_n\}$ in Y, as follows:

$x_n = (ST)^n x_0$, $y_n = T(x_{n-1})$ for n=1,2,……

We have,

$$\begin{aligned}
M(x_n, x_{n+1}, t) &= M((ST)^n x_0, (ST)^{n+1} x_0, t) \\
&= M(S(Tx_{n-1}), STx_n, t) = M(Sy_n, STx_n, t) \\
&\geq \min\{M(x_n, Sy_n, \frac{t}{c_1}), M'(y_n, Tx_n, \frac{t}{c_1}), M(x_n, Sy_n, \frac{t}{c_1}) * M(x_n, STx_n, \frac{t}{c_1})\} \\
&= \min\{M(x_n, x_n, \frac{t}{c_1}), M'(y_n, y_{n+1}, \frac{t}{c_1}), M(x_n, x_n, \frac{t}{c_1}) * M(x_n, x_{n+1}, \frac{t}{c_1})\} \\
&= \min\{1, M'(y_n, y_{n+1}, \frac{t}{c_1}), M(x_n, x_{n+1}, \frac{t}{c_1})\} \\
&= M'(y_n, y_{n+1}, \frac{t}{c_1}) \qquad (2.7)
\end{aligned}$$

Now,

$$\begin{aligned}
M'(y_n, y_{n+1}, t) &= M'(Tx_{n-1}, Tx_n, t) = M'(Tx_{n-1}, TSy_n, t) \\
&\geq \min\{M(x_{n-1}, Sy_n, \frac{t}{c_2}), M'(y_n, Tx_{n-1}, \frac{t}{c_2}), M'(y_n, Tx_{n-1}, \frac{t}{c_2}) * M'(y_n, TSy_n, \frac{t}{c_2})\} \\
&= \min\{M(x_{n-1}, x_n, \frac{t}{c_2}), M'(y_n, y_n, \frac{t}{c_2}), M'(y_n, y_n, \frac{t}{c_2}) * M'(y_n, y_{n+1}, \frac{t}{c_2})\} \\
&= \min\{M(x_{n-1}, x_n, \frac{t}{c_2}), 1, 1 * M'(y_n, y_{n+1}, \frac{t}{c_2})\} \\
&= \min\{M(x_{n-1}, x_n, \frac{t}{c_2}), M'(y_n, y_{n+1}, \frac{t}{c_2})\} \\
&= M(x_{n-1}, x_n, \frac{t}{c_2}) \qquad (2.8)
\end{aligned}$$

From (2.7) and (2.8),

$$\begin{aligned}
M(x_n, x_{n+1}, t) &\geq M(x_{n-1}, x_n, \frac{t}{c_1 c_2}) \qquad \ldots\ldots\ldots \\
&\geq M(x_{n-1}, x_n, \frac{t}{(c_1 c_2)^n}) \to 0 \text{ as } n \to \infty \text{ (since } c_1 c_2 \in (0,1))
\end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence in (X, M. *). Since (X, M, *) is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also Cauchy sequence in (Y, M', *). Since (Y, M', *) is complete, it converges to a point w in Y.

Now, we prove that Tz = w

Consider,

$$\begin{aligned} M'(Tz, w, c_2t) &= \lim_{n\to\infty} M'(Tz, y_{n+1}, c_2t) = \lim_{n\to\infty} M'(Tz, TSy_n, c_2t) \\ &\geq \lim_{n\to\infty} \min\{M(z, Sy_n, t), M'(y_n, Tz, t), M'(y_n, Tz, t)* M'(y_n, TSy_n, t)\} \\ &= \lim_{n\to\infty} \min\{M(z, x_n, t), M'(y_n, Tz, t), M'(y_n, Tz, t)* M'(y_n, y_{n+1}, t)\} \\ &= \min\{M(z, z, t), M'(w, Tz, t), M'(w, Tz, t)* M'(w, w, t)\} \\ &= \min\{1, M'(w, Tz, t), M'(w, Tz, t)*1\} \\ &= M'(w, Tz, t), \end{aligned}$$

So, Tz = w

Now, we prove that Sw = z

Consider,

$$\begin{aligned} M(Sw, z, c_1t) &= \lim_{n\to\infty} M(Sw, x_{n+1}, c_1t) = \lim_{n\to\infty} M(Sw, STx_n, c_1t) \\ &\geq \lim_{n\to\infty} \min\{M(x_n, Sw, t), M'(w, Tx_n, t), M(x_n, Sw, t)* M(x_n, STx_n, t)\} \\ &= \lim_{n\to\infty} \min\{M(x_n, Sw, t), M'(w, y_n, t), M(x_n, Sw, t)* M(x_n, x_{n+1}, t)\} \\ &= \min\{M(z, Sw, t), M'(w, w, t), M(z, Sw, t)* M(z, z, t)\} \\ &= \min\{M(z, Sw, t), 1, M(z, Sw, t)* 1\} \\ &= M(z, Sw, t) \end{aligned}$$

So, Sw = z

We have STz = Sw = z. Thus the point z is a fixed point of ST in X and the point w is fixed point of Ts in Y.

Uniqueness: Let $z \neq z'$ be another fixed point of ST in X. We have

$$\begin{aligned} M(z, z', c_1 c_2 t) &= M(STz, STz', c_1 c_2 t) \\ &\geq \min\{M(z', STz, c_2 t), M'(Tz, Tz', c_2 t), M(z', STz, c_2 t)* M(z', STz', c_2 t)\} \\ &= \min\{M(z', z, c_2 t), M'(Tz, Tz', c_2 t), M(z', z, c_2 t)* M(z', z', c_2 t)\} \\ &= M'(Tz, Tz', c_2 t) \end{aligned} \tag{2.9}$$

Also,

$$\begin{aligned} M'(Tz, Tz', c_2 t) &= M'(Tz, TSTz', c_2t) \\ &\geq \min\{M(z, STz', t), M'(Tz', Tz, t), M'(Tz', Tz, t)* M'(Tz', TSTz', t)\} \\ &= \min\{M(z, z', t), M'(Tz', Tz, t), M'(Tz', Tz, t)* M'(Tz', Tz', t)\} \\ &= M(z, z', t) \end{aligned} \tag{2.10}$$

From (2.9) and (2.10) we have,

$$M(z, z', c_1 c_2 t) \geq M(z, z', t)$$

Thus z = z'.

So the point z is a unique fixed point of ST. Similarly, we prove that the point w is a unique fixed point of TS.

Corollary 2.4: Let (X, M, $*$) be complete fuzzy metric spaces. If T and S are mapping from X into X satisfying the following conditions:

$$M(Sy, STx, t) \geq \min\{M(x, Sy, \frac{t}{c_1}), M(y, Tx, \frac{t}{c_1}), M(x, Sy, \frac{t}{c_1})* M(x, STx, \frac{t}{c_1})\}$$

$$M(Tx, TSy, t) \geq \min\{M(x, Sy, \frac{t}{c_2}), M(y, Tx, \frac{t}{c_2}), M(y, Tx, \frac{t}{c_2})* M'(y, TSy, \frac{t}{c_2})\}$$

for all x in X and y in Y where $0 < c_1 < 1$ and $0 < c_2 < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in X. Futher Tz = w and Sw = z and if z=w, then z is the unique common fixed point of S and T.

Proof: If (X, M, $*$) and (Y, M', $*$) are same fuzzy metric spaces, then by above theorem 2.3, we get the required result.

Theorem 2.5: Let (X, M, $*$) and (Y, M', $*$) be complete fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X satisfying the following conditions:

$$M(Sy, STx, t) \geq \min\{M(x, Sy, \tfrac{t}{c_1}), M'(y, Tx, \tfrac{t}{c_1}), M(x, STx, \tfrac{t}{c_1}), M'(Tx, TSy, \tfrac{t}{c_1})\}$$

$$M'(Tx, TSy, t) \geq \min\{M(x, Sy, \tfrac{t}{c_2}), M'(y, Tx, \tfrac{t}{c_2}), M'(y, TSy, \tfrac{t}{c_2}), M(x, STx, \tfrac{t}{c_2})\}$$

for all x in X and y in Y where $c_1, c_2 \in (0,1)$, then ST has a unique fixed point in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X and a sequence $\{y_n\}$ in Y, as follows:

$$x_n = (ST)^n x_0,\ y_n = T(x_{n-1}) \text{ for } n=1,2,\ldots\ldots$$

We have,

$$\begin{aligned} M(x_n, x_{n+1}, t) &= M((ST)^n x_0, (ST)^{n+1} x_0, t) \\ &= M(S(Tx_{n-1}), STx_n, t) = M(Sy_n, STx_n, t) \\ &\geq \min\{M(x_n, Sy_n, \tfrac{t}{c_1}), M'(y_n, Tx_n, \tfrac{t}{c_1}), M(x_n, STx_n, \tfrac{t}{c_1}), M'(Tx_n, TSy_n, \tfrac{t}{c_1})\} \\ &= \min\{M(x_n, x_n, \tfrac{t}{c_1}), M'(y_n, y_{n+1}, \tfrac{t}{c_1}), M(x_n, x_{n+1}, \tfrac{t}{c_1}), M'(y_{n+1}, y_{n+1}, \tfrac{t}{c_1})\} \\ &= \min\{1, M'(y_n, y_{n+1}, \tfrac{t}{c_1}), M(x_n, x_{n+1}, \tfrac{t}{c_1})\} \\ &= M'(y_n, y_{n+1}, \tfrac{t}{c_1}) \end{aligned} \tag{2.11}$$

Now,

$$\begin{aligned} M'(y_n, y_{n+1}, t) &= M'(Tx_{n-1}, Tx_n, t) = M'(Tx_{n-1}, TSy_n, t) \\ &\geq \min\{M(x_{n-1}, Sy_n, \tfrac{t}{c_2}), M'(y_n, Tx_{n-1}, \tfrac{t}{c_2}), M'(y_n, TSy_n, \tfrac{t}{c_2}), M(x_{n-1}, STx_{n-1}, \tfrac{t}{c_2})\} \\ &= \min\{M(x_{n-1}, x_n, \tfrac{t}{c_2}), M'(y_n, y_n, \tfrac{t}{c_2}), M'(y_n, y_{n+1}, \tfrac{t}{c_2}), M(x_{n-1}, x_n, \tfrac{t}{c_2})\} \\ &= \min\{M(x_{n-1}, x_n, \tfrac{t}{c_2}), 1, M'(y_n, y_{n+1}, \tfrac{t}{c_2})\} \\ &= \min\{M(x_{n-1}, x_n, \tfrac{t}{c_2}), M'(y_n, y_{n+1}, \tfrac{t}{c_2})\} \\ &= M(x_{n-1}, x_n, \tfrac{t}{c_2}) \end{aligned} \tag{2.12}$$

From (2.11) and (2.12),

$$M(x_n, x_{n+1}, t) \geq M(x_{n-1}, x_n, \frac{t}{c_1 c_2}) \quad \ldots\ldots\ldots$$

$$\geq M(x_{n-1}, x_n, \frac{t}{(c_1 c_2)^n}) \to 0 \text{ as } n \to \infty \text{ (since } c_1 c_2 \in (0,1))$$

Thus $\{x_n\}$ is a Cauchy sequence in (X, M. *) . Since (X,M, *) is complete, it converges to a point z in X. Similarly, we can prove that the sequence $\{y_n\}$ is also Cauchy sequence in (Y, M', *) . Since (Y, M', *) is complete, it converges to a point w in Y.

Now, we prove that $Tz = w$,

Suppose $Tz \neq w$

Consider,

$$\begin{aligned}
M'(Tz, w, c_1c_2t) &= \lim_{n\to\infty} M'(Tz, y_{n+1}, c_1 c_2t) = \lim_{n\to\infty} M'(Tz, TSy_n, c_1c_2t) \\
&\geq \lim_{n\to\infty} \min\{M(z, Sy_n, c_1t), M'(y_n, Tz, c_1t), M'(y_n, TSy_n, c_1t), M(z, STz, c_1t)\} \\
&= \lim_{n\to\infty} \min\{M(z, x_n, c_1t), M'(y_n, Tz, c_1t), M'(y_n, y_{n+1}, c_1t), M(z, STz, c_1t)\} \\
&= \min\{M(z, z, c_1t), M'(w, Tz, c_1t), M'(w, w, c_1t), M(z, STz, c_1t)\} \\
&= \min\{1, M'(w, Tz, c_1t), M(z, STz, c_1t)\} \\
&= M(z, STz, c_1t), \qquad (2.13)
\end{aligned}$$

$$\begin{aligned}
M(z, STz, c_1t) &= \lim_{n\to\infty} M(x_n, STz, c_1t) = \lim_{n\to\infty} M(Sy_n, STz, c_1t) \\
&\geq \lim_{n\to\infty} \min\{M(z, Sy_n, t), M'(y_n, Tz, t), M(z, STz, t), M'(Tz, TSy_n, t)\} \\
&= \lim_{n\to\infty} \min\{M(z, x_n, t), M'(y_n, Tz, t), M(z, STz, t), M'(Tz, y_{n+1}, t)\} \\
&= \min\{M(z, z, t), M'(w, Tz, t), M(z, STz, t), M'(Tz, w, t)\} \\
&= M'(Tz, w, t) \qquad (2.14)
\end{aligned}$$

From, (2.13) and (2.14),

$$M'(Tz, w, c_1c_2t) \geq M'(Tz, w, t)$$

So, $Tz = w$, since $c_1c_2 \in (0,1)$

Similarly, we prove that $Sw = z$

We have $STz = Sw = z$. Thus the point z is a fixed point of ST in X and the point w is fixed point of Ts in Y.

Uniqueness: Let $z \neq z'$ be another fixed point of ST in X. We have

$$\begin{aligned}
M(z, z', c_1 c_2 t) &= M(STz, STz', c_1 c_2 t) \\
&\geq \min\{M(z', STz, c_2 t), M'(Tz, Tz', c_2 t), M(z', STz, c_2 t), M'(Tz', Tz, c_2 t)\} \\
&= \min\{M(z', z, c_2 t), M'(Tz, Tz', c_2 t), M(z', z, c_2 t), M'(Tz', Tz, c_2 t)\} \\
&= M'(Tz, Tz', c_2 t) \qquad (2.15)
\end{aligned}$$

Also,

$$\begin{aligned}
M'(Tz, Tz', c_2 t) &= M'(Tz, TSTz', c_2t) \\
&\geq \min\{M(z, STz', t), M'(Tz', Tz, t), M'(Tz', TSTz', t), M(z, STz, t)\} \\
&= \min\{M(z, z', t), M'(Tz', Tz, t), M'(Tz', Tz', t), M(z, z, t)\} \\
&= M(z, z', t) \qquad (2.16)
\end{aligned}$$

From (2.15) and (2.16) we have,

$$M(z, z', c_1 c_2 t) \geq M(z, z', t)$$

Thus $z = z'$.

So the point z is a unique fixed point of ST. Similarly, we prove that the point w is a unique fixed point of TS.

Corollary 2.6: Let $(X, M, *)$ be complete fuzzy metric spaces. If T and S are mapping from X into X satisfying the following conditions:

$$M(Sy, STx, t) \geq \min\left\{M\left(x, Sy, \frac{t}{c_1}\right), M\left(y, Tx, \frac{t}{c_1}\right), M\left(Tx, TSy, \frac{t}{c_1}\right), M\left(x, STx, \frac{t}{c_1}\right)\right\}$$

$$M(Tx, TSy, t) \geq \min\left\{M\left(x, Sy, \frac{t}{c_2}\right), M\left(y, Tx, \frac{t}{c_2}\right), M\left(x, STx, \frac{t}{c_2}\right), M\left(y, TSy, \frac{t}{c_2}\right)\right\}$$

for all x in X and y in Y where $0 < c_1 < 1$ and $0 < c_2 < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in X. Futher $Tz = w$ and $Sw = z$ and if $z = w$, then z is the unique common fixed point of S and T.

Proof: If (X, M, $*$) and (Y, M', $*$) are same fuzzy metric spaces, then by above theorem 2.5, we get the required result.

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