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ON CONTRACTIBILITY OF A SIMPLE CLOSED ORIENTED SURFACE

Dr (Smt). S. N. BANASODE[1], SEEMA B PATIL*[2], JAYASHREE.S. RADDER[3]

[1]Associate Professor of Mathematics,
R. L Science Institute Belagavi, Karnataka State, India.

[2]Research Scholar,
Rani Channamma University Belagavi. Karnataka State, India.

[3]Assistant Professor of Mathematics,
Tontadarya College of Engg, Gadag, Karnataka State, India.

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ABSTRACT

Up to homeomorphism all compact simple oriented close surfaces are either a sphere S^2 or S^2 with g handles, g = 1, 2, 3, …..Called the genius of the surface. In this paper we observe that there is a nice characterization come from the topology at the surface and with the classification theme for contractibility. These are some hard results.

Keywords: *Contractibility, Orientable surface, S^2 with g handles.*

INTRODUCTION

Classification of spaces by topologies up to homeomorphism for successful in case of two dimensional manifolds. This gave an importance for them to try the problem of classifying their higher analogs (manifolds or spaces with n>2). But it proved to be futile. Certain probabical abstractions came in the way to resolve this problem. Poincare conjecture have the case of three sphere is a classic example. It remained for almost hundred years as an unsettled problem till in very recently; Perelman finally settling this problem posted on internet during year 2000, and finally it had the approval of mathematical community as a completely settled conjecture.

What does this conjecture state?

Following is the statement of Poincare conjecture: states that every simply connected closed three manifold is homeomorphic to the three sphere (in a topologist's sense) S^3, where a three sphere is simply a generalization of usual sphere to one dimension higher. More colloquially, the conjecture says that three sphere is the only type of bounded three dimensional space possible that contains no holes. This conjecture was first proposed in 1904 by H. Poincare and subsequently generalized to conjecture that every compact n-manifold is homotopy – equivalent to the n – sphere iff it is homeomorphic to the n-sphere. The generalized statement reduces to original conjecture for n=3.

The n=1 case of the generalized conjecture is trivial, the n=2 case is classical (and was known to 19th century mathematicians), n=3 (original conjecture) appear to have been proved by recent work by G. Perelman (although the proof has not yet been fully verified), n=4 was proved by Freedman (1982) (for which he was awarded the 1986 Fields medal), n=5 was demonstration by Stallings (1962), and n $\geq$ 7 was shown by Smale in 1961 (although Smale subsequently extended his proof to include all n $\geq$ 5).

The work of Perelman (2002, 2003, Robinson 2003) established a more general result known as the Thurston's geometrization conjecture, some background information is useful. Three-dimensional manifolds possess what is known as a standard two-level decomposition. First, there is the connected sum decomposition, which says that every compact three-manifold is the connected sum of a unique collection of prime three-manifolds. From which the Poincare conjecture immediately follows.

Corresponding Author: Seema B Patil[2]*
[2]Research Scholar, Rani Channamma University Belagavi. Karnataka State, India.

Poincare was led to make his conjecture during his pioneering studies in topology. The mathematical study of the properties of objects that stays unchanged when they are stretched or bent. In order to translate topological questions into algebra, Poincare invented so called “homotopy groups”–quantities which capture the essence of multi-dimensional spaces in algebraic terms, and have the power to reveal similarities between them. He thought the same would be true in three dimensions – but could not prove it.

Homotopy: A continuous transformation from one function to another. A homotopy between two functions f and g from a space X to a space Y is a continuous map G from X x [0,1] →Y is such that G(x, 0)=f(x) and G(x,1) = g(x), where X denotes set pairing. Another way of saying this is that a homotopy is path in the mapping space map (X, Y) from the first function to the second.

Two mathematical objects are set to be homotopic if one can be continuously deformed into the other. The concept of homotopy was first formulated by Poincare around 1900.

Orientable surfaces:

The genus of a connected, Orientable surface is an integer representing the maximum number of cuttings along non-intersecting closed simple curves without rendering the resultant manifold disconnected. It is equal to the number of handles on it. Alternatively, it can be defined in terms of Euler characteristic X, via the relationship X= 2-2g for closed surfaces, where g is the genus. For surfaces with b boundary components, the equation reads X=2-2g-b.
For instance:

i) The sphere S^2 and a disc both have genus zero.
ii) A torus has genus one, as does the surface of a coffee mug with a handle. This is the source of the joke that “a topologist is someone who can’t tell his donut from his coffee mug.”

An explicit construction of surfaces of genus g is given in the article on the fundamental polygon. In the simpler terms, the value of an Orientable surface’s genus is equal to the nu

Genus of a surface is the number of handles of the surface. An abstract and major way to construct surfaces is by connecting along some deleted disk of each surface. The connected sum of two surfaces S_1 and S_2 is denoted by $S_1\#S_2$. It is formed by deleting the interior of disks Di from each Si and attaching the resulting punctured surfaces Si−Di to each other by a one-to-one continuous map.

Non-Orientable Surfaces:

The non-orientable genus, demigenus, or Euler genus of a connected, non-orientable closed surface is a positive integer representing the number of cross-caps attached to a sphere. Alternatively, it can be defined for a closed surface in terms of the Euler characteristic X, via the relationship X=2-k, where k is the non-orientable genus.

For instance:

i) A projective plane has non-orientable genus one.
ii) A Klein bottle has non-orientable genus two.

Topology is concerned with those properties of geometric figures that are invariant under continuous transformations. A continuous transformation, also called a topological transformation or homeomorphism, is a one-to-one correspondence between the points of one figure and the points of another figure such that points that are arbitrarily close on one figure are transformed into points that are also arbitrarily close on the other figure. Figures that are related in this way are said to be topologically equivalent. If a figure is transformed into an equivalent figure by bending, stretching, etc., the change is a special type of topological transformation called a continuous deformation. Two figures (e.g, certain types of knots) may be topologically equivalent, however, without being changeable into one another by a continuous deformation.

It is intuitively evident that all simple closed curves in the plane and all polygons are topologically equivalent to a circle; similarly, all closed cylinders, cones, convex polyhedra, and other simple closed surfaces are equivalent to a sphere. On the other hand, a closed surface such as a torus (doughnut) is not equivalent to a sphere, since no amount of bending or stretching will make it into a sphere, nor is a surface with a boundary equivalent to a sphere, e.g., a cylinder with an open top, which may be stretched into a disk (a circle plus its interior).

Suppose we have a topological space X, and suppose this space is subject to deformation, then the space shall remain unchanged. For example if the space X is a 2 sphere i.e, S^2 defirm this object. We also observe following fact, consider a loop r (a closed simple curve) in S^2, then we can shrunk it to a point (in algebraic topology).

If in X, every simple closed curve is shrunk to a point then we say that the space X is contractable. Our sphere S^2 is contractable. Our classification theme in two sphere space X _□S^2 or S^2 in with a g handles, g = 1, 2,……

If g = n then X _□ S^2 with n handles. If g=1, X _□S^2 with 1 handle.S^2 with one handle is our torus.

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