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EFFECT OF HALL CURRENT ON MHD FLOW PAST A MOVING VERTICAL PLATE WITH CONSTANT MASS DIFFUSION AND VARIABLE TEMPERATURE

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ABSTRACT

In the present paper, effect of Hall current on MHD flow past a moving vertical plate with constant mass diffusion and variable temperature is studied. The fluid considered is an electrically conducting. The solution of the model is obtained by Laplace transform method. The effect of parameters is shown with the help of graphs. The velocity profile and Skin friction have been studied for different parameters. The effect of parameters are shown graphically and the value of the skin-friction for different parameters has been tabulated.

Key Words: *MHD, Hall current, Skin friction, Heat and mass.*

INTRODUCTION

The problems related to Hall current, heat transfer and mass diffusion are of great interest due to their applications in many processes. The effect of Hall current on MHD flows have been carried out by many authors due to its applications in the problems of MHD generators and Hall accelerators. Some related research works are mentioned here. Beg and Ghosh [1] investigated magneto hydrodynamic flow with oscillatory surface temperature. Mukherjee [6] investigated effect of radiation and porosity parameter on hydro magnetic flow due to exponentially stretching sheet in a porous media. Datta *et al.* [3] have studied oscillatory MHD flow over plate. Sato [9] analyzed the Hall effect on ionized gas between parallel plates. Yamanishi [10] has studied MHD flow between two parallel plates. Hall effect on MHD flow past an accelerated plate was investigated by Deka [4]. Chambre *et al.* [2] have studied the diffusion of chemically reactive species in a laminar boundary layer flow. Ibrahim *et al.* [5] have studied chemically reacting magneto hydrodynamic boundary layer flow over a moving plate with suction. Rajput and Kumar [8] have studied magneto hydrodynamic flow over exponentially accelerated plate. Earlier we [7] have studied MHD flow past an impulsively started vertical plate with variable temperature in the presence of Hall current. In this model, we are considering effect of Hall current on MHD flow past a moving vertical plate with constant mass diffusion and variable temperature.

MATHEMATICAL FORMULATION

The flow is unsteady and incompressible. The x axis is considered along the fluid motion. Magnetic field B_0 of uniform strength is taken perpendicular to x axis. Initially it has been considered that the fluid and the plate are at the same temperature (T_∞). C_∞ is the species concentration within the fluid. The plate starts accelerating after $t = 0$. The species concentration and temperature of the plate are raised to C_w and T_w, respectively. The fluid model with usual approximations is as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0{}^2}{\rho(1+m^2)}(u + mw), \tag{1}$$

$$\frac{\partial w}{\partial t} = \upsilon \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0{}^2}{\rho(1+m^2)}(w - mu), \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

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$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

The following boundary conditions have been assumed:

$$\begin{aligned}
&t \le 0 : u = 0, w = 0, C = C_\infty, T = T_\infty, \text{ for all the values of y},\\
&t > 0 : u = e^{bt}, w = 0, C = C_w, T = T_\infty + (T_w - T_\infty)\frac{u_0{}^2 t}{\upsilon} \text{ at y} = 0,\\
&u \to 0, w \to 0, C \to C_\infty, \text{T} \to T_\infty \quad \text{as y} \to \infty.
\end{aligned} \tag{5}$$

Here u is the velocity of the fluid in x- direction, w - the velocity of the fluid in z- direction, m - Hall parameter, g – acceleration due to gravity, β - volumetric coefficient of thermal expansion, β^* - volumetric coefficient of concentration expansion, t- time, C_∞ - the concentration in the fluid far away from the plate, C - species concentration in the fluid , C_w - species concentration at the plate, D - mass diffusion, T_∞ - the temperature of the fluid near the plate, T_w - temperature of the plate, T - the temperature of the fluid , k - the thermal conductivity, υ – the kinematic viscosity, ρ - the fluid density, σ - electrical conductivity, μ - the magnetic permeability, and C_P - specific heat at constant pressure. Here $m = \omega_e \tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To write the equations (1) - (4) in dimensionless from, we introduce the following non - dimensional quantities:

$$\left.\begin{aligned}
&\bar{u} = \frac{u}{u_0}, \bar{w} = \frac{w}{u_0}, \bar{y} = \frac{y u_0}{\upsilon}, Sc = \frac{\upsilon}{D}, \text{Pr} = \frac{\mu C_P}{k},\\
&M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, \bar{t} = \frac{t u_0{}^2}{\upsilon}, Gr = \frac{g\beta\upsilon(T_w - T_\infty)}{u_0{}^3}, \bar{b} = \frac{b\upsilon}{u_0^2},\\
&Gm = \frac{g\beta^*\upsilon(C_w - C_\infty)}{u_0{}^3}, \bar{C} = \frac{C - C_\infty}{C_w - C_\infty}, \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}.
\end{aligned}\right\} \tag{6}$$

Here the symbols used are:

$\bar{u}$ -dimensionless velocity, $\bar{w}$ -dimensionless velocity, θ-the dimensionless temperature, $\bar{b}$ -dimensionless acceleration parameter $\bar{C}$ - the dimensionless concentration, Gr - thermal Grashof number, Gm - mass Grashof number, μ - the coefficient of viscosity, Pr - the Prandtl number, Sc - the Schmidt number, M - the magnetic parameter.

The dimensionless forms of Equations (1), (2), (3) and (4) are as follows

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + Gr\theta + Gm\bar{C} - \frac{M(\bar{u} + m\bar{w})}{(1+m^2)}, \tag{7}$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} = \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - \frac{M(\bar{w} - m\bar{u})}{(1+m^2)}, \tag{8}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{Sc}\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}, \tag{9}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{\text{Pr}}\frac{\partial^2 \theta}{\partial \bar{y}^2}. \tag{10}$$

The corresponding boundary conditions are:

$$\begin{aligned}
&\bar{t} \le 0, \bar{u} = 0, \bar{C} = 0, \theta = 0, \vec{w} = 0, \text{ for all value of } \bar{\text{y}},\\
&\bar{t} > 0, \bar{u} = e^{\bar{b}\bar{t}}, \bar{w} = 0, \bar{\theta} = \bar{t}, C = 1 \text{ at } \bar{\text{y}} = 0,\\
&\bar{u} \to 0, \bar{C} \to 0, \theta \to 0, \bar{w} \to 0 \text{ as } \bar{\text{y}} \to \infty.
\end{aligned} \tag{11}$$

Dropping the bars and combining the Equations (7) and (8), we get

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} + Gr\theta + GmC - \left(\frac{M}{1+m^2}(1-mi)\right)q, \tag{12}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}, \tag{13}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}. \tag{14}$$

Here $q = u + iw,$ with corresponding boundary conditions

$$\left.\begin{array}{l} t \le 0 : q = 0, \theta = 0, C = 0, \text{for all value of } y, \\ t > 0 : q = e^{bt},, w = 0, \theta = t, C = 1, \text{at } y = 0, \\ q \to 0, C \to 0, \theta \to 0, \text{as } y \to \infty. \end{array}\right\} \tag{15}$$

The solution obtained is as under:

$$\begin{aligned} q = {} & \frac{1}{2}e^{bt\sqrt{a+b}} + A_0 + \frac{1}{4a^2}\, y\; Gr\{A_{13}(1-\Pr-at)\} + \sqrt{a}\; e^{-\sqrt{a}y}(A_1 - e^{2\sqrt{a}y}) + B_{13}\{A_{14}(1-\Pr)\} \\ & + \frac{1}{2a}Gm(-e^{-\sqrt{a}y}A_1 + e^{\frac{at}{-1+Sc}\sqrt{\frac{aSc}{-1+Sc}}}(1 + B_{11} + e^{2\sqrt{\frac{aSc}{-1+Sc}}}B_{12})) - \\ & \frac{1}{2a^2\sqrt{\pi}}Gr\sqrt{\Pr}y(-B_{14}\{-1+\sqrt{\Pr}+at\} + a(2e^{\frac{-\Pr y^2}{4t}}\sqrt{t} - \sqrt{\pi}\sqrt{\Pr}y\{1 - Erf[\frac{\sqrt{\Pr}y}{2\sqrt{t}}]\}) \\ & + \frac{1}{y}e^{\frac{at}{-1+\Pr}\sqrt{\frac{a}{-1+\Pr}}\sqrt{\Pr}y}\sqrt{\pi}B_{17}\frac{1}{\sqrt{\Pr}}\Pr - 1) \\ & - \frac{1}{2a}Gm[-2Erfc[\frac{\sqrt{Sc}y}{2\sqrt{t}}] + e^{\frac{at}{-1+Sc}\sqrt{\frac{a}{-1+Sc}}\sqrt{Sc}y}(1 + B_{18} + e^{2\sqrt{\frac{aSc}{-1+Sc}}y}B_{19})] \end{aligned} \tag{16}$$

$$\theta = \left[\left(t + \frac{\Pr y^2}{2}\right)Erf\left(\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right) - e^{-\frac{y^2}{4t}\Pr}\,\frac{yt\sqrt{\Pr}}{\sqrt{\pi}}\right] \tag{17}$$

$$C = Erfc\left[\frac{\sqrt{Sc}y}{2\sqrt{t}}\right]. \tag{18}$$

The expressions for the constants involved in the above equations are given in the appendix.

Skin friction

The dimensionless skin friction at the plate y = 0 is computed by

$$\left(\frac{dq}{dy}\right)_{y=0} = \tau_x + i\tau_z.$$

Discussion and Results

The numerical values of velocity and skin friction are computed for different parameters like Hall parameter (*m*), mass Grashof number (*Gm*), Schmidt number (*Sc*), time (*t*), thermal Grashof number (*Gr*), magnetic field parameter (*M*), Prandtl number (*Pr*), and (*b*) acceleration parameter. The values of the main parameters considered are:

m = 1, 5; Gm = 10, 20, 30; Sc = 2.01, 3, 4; t = 0.15, 0.2, 0.25;
b = 1, 3, 5; Gr = 10, 20, 30; M = 1, 2, 5; Pr = 0.71, 7;

Figures *1, 2, 3, 7,* and *8* show that primary velocity (*u*) increases when *Gm, Gr, m, t* and *b* are increased. Figures *4, 5,* and *8* show that primary velocity (*u*) decreases when *M, Pr,* and *Sc* are increased. And figures *9, 10, 12, 15,* and *16* show that the secondary velocity (*w*) increases when *Gm, Gr, M, t* and *b* are increased. Figures *11, 13* and *14* show that secondary velocity (*w*) decreases when *m, Pr,* and *Sc* are increased.

Skin friction is given in table 1. The value of τ_x increases with the rise in *Gm, Gr, m,* and *t*. However, it decreases with *M, b, Pr* and *Sc*. Similar effects are observed with τ_z when *M, t Gm, Gr* and *b* are increased then τ_z increases. Again, τ_z decreases when *m, Pr,* and *Sc* increases.

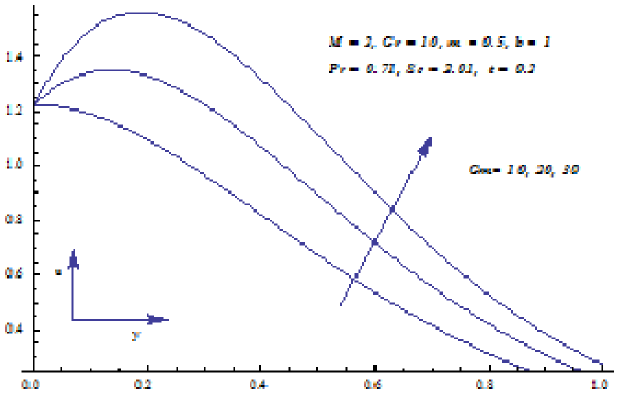

Figure 1: velocity u for different values of *Gm*

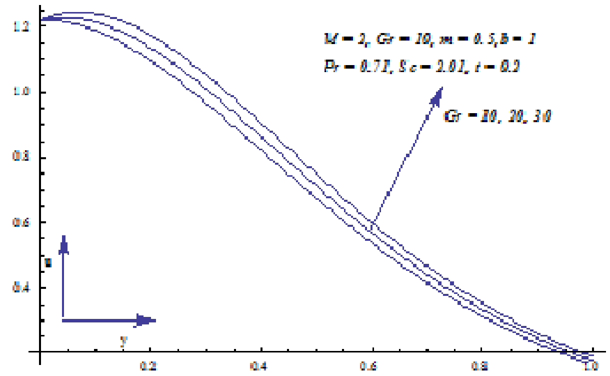

Figure 2: velocity u for different values of *Gr*

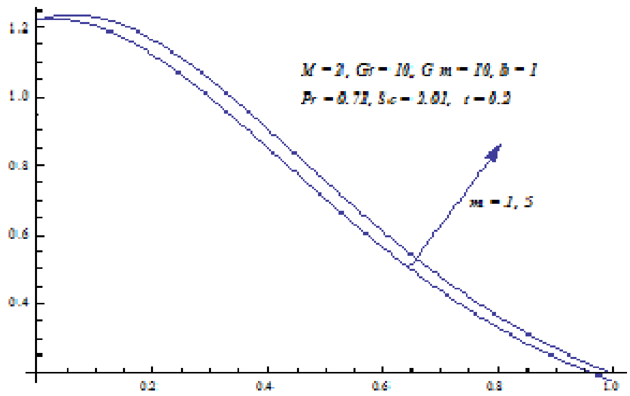

Figure 3: velocity u for different values of *m*

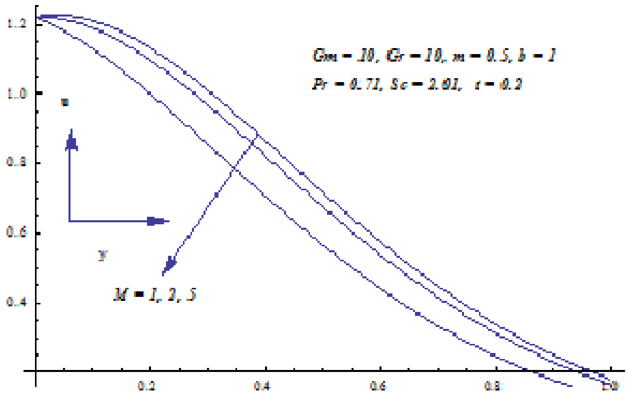

Figure 4: velocity u for different values of *M*

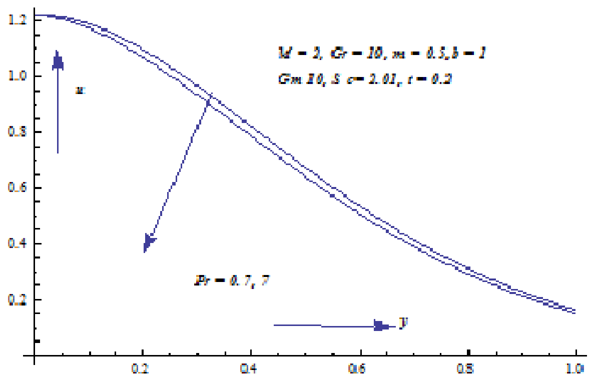

Figure 5: velocity u for different values of *Pr*

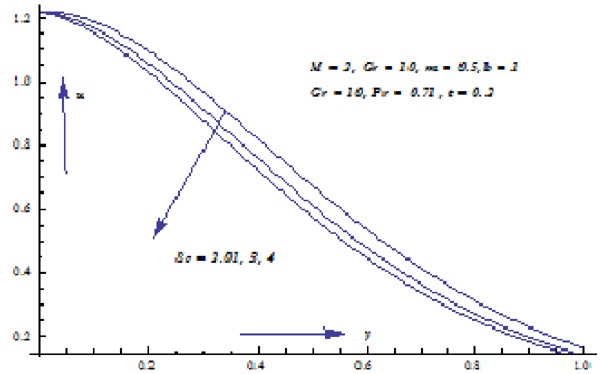

Figure 6: velocity u for different values of *Sc*

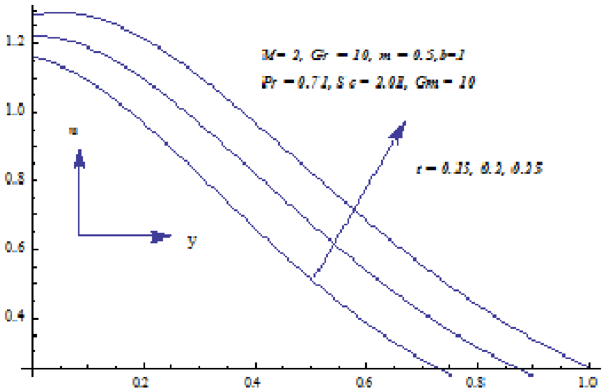

Figure 7: velocity u for different values of *t*

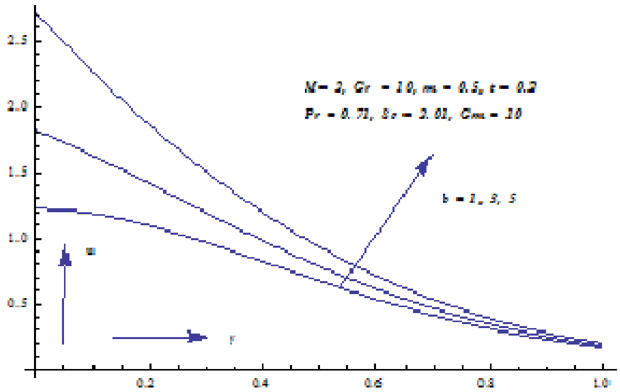

Figure 8: velocity u for different values of *b*

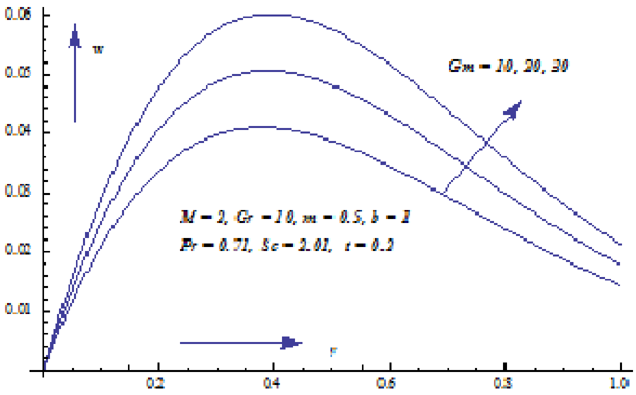

Figure 9: velocity w for different values of *Gm*

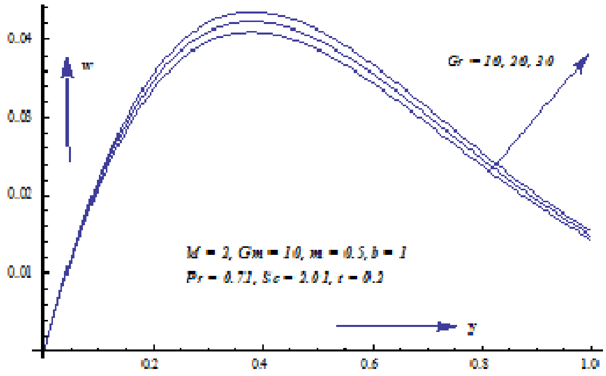

Figure 10: velocity w for different values of *Gr*

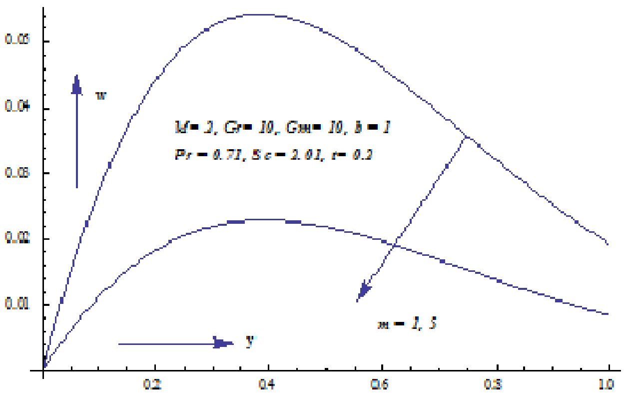

Figure 11: velocity w for different values of *m*

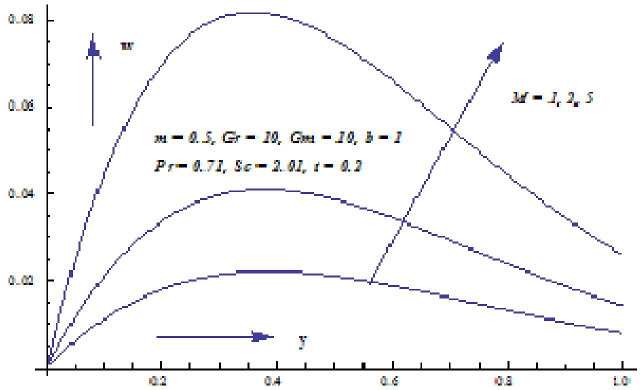

Figure 12: velocity w for different values of *M*

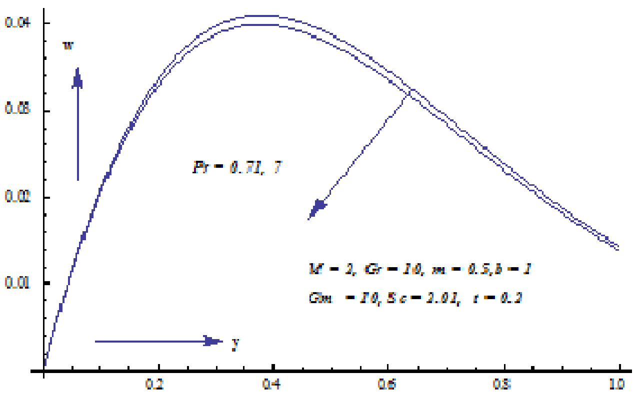

Figure 13: velocity w for different values of *Pr*

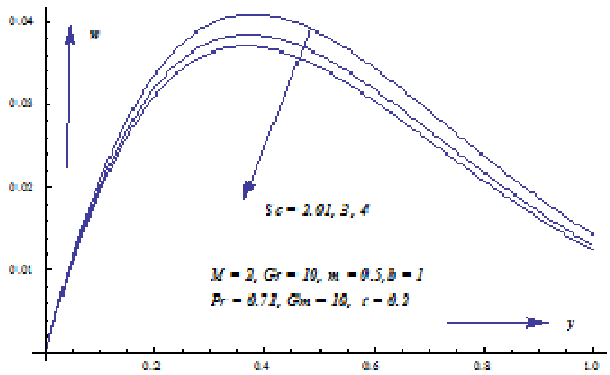

Figure 14: velocity w for different values of *Sc*

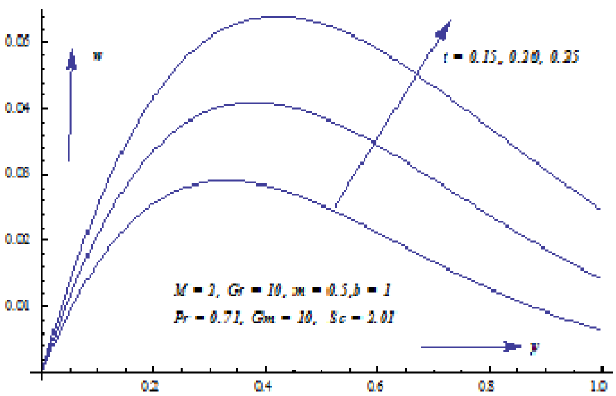

Figure 15: velocity w for different values of *t*

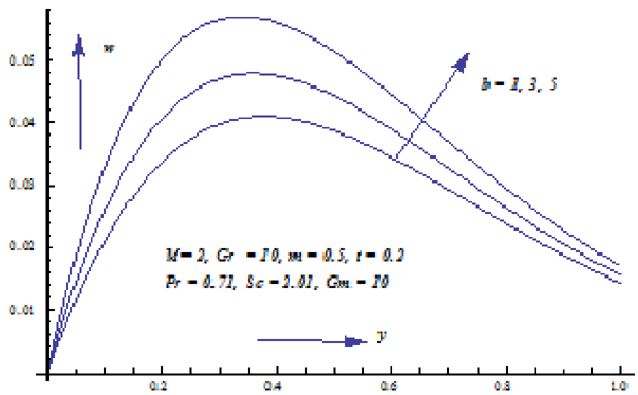

Figure 16: velocity w for different values of *b*

Table – 1: For skin friction

M	*m*	*Pr*	*Sc*	*Gm*	*Gr*	*b*	*t*	τ_x	τ_z
2	0.5	**0.71**	2.01	10	10	1	0.2	0.06781	0.25639
2	0.5	**7.00**	2.01	10	10	1	0.2	-0.10370	0.25212
2	0.5	0.71	**3.00**	10	10	1	0.2	-0.15335	0.24768
2	0.5	0.71	**4.00**	10	10	1	0.2	-0.30661	0.24228
2	0.5	0.71	2.01	**20**	10	1	0.2	2.06686	0.29742
2	0.5	0.71	2.01	**30**	10	1	0.2	4.06590	0.33846
2	0.5	0.71	2.01	10	**20**	1	0.2	0.42072	0.26216
2	0.5	0.71	2.01	10	**30**	1	0.2	0.77363	0.26790
2	0.5	0.71	2.01	10	10	**3**	0.2	-1.79420	0.32701
2	0.5	0.71	2.01	10	10	**5**	0.2	-4.82680	0.42573
2	0.5	0.71	2.01	10	10	1	**.015**	-0.33517	0.209744
2	0.5	0.71	2.01	10	10	1	**0.25**	0.39374	0.30292
1	0.5	0.71	2.01	10	50	1	0.2	0.33515	0.13467
5	0.5	0.71	2.01	10	10	1	0.2	-0.67460	0.55820
2	**1**	0.71	2.01	10	10	1	0.2	0.25966	0.33222
2	**5**	0.71	2.01	10	10	1	0.2	0.58427	0.13556

CONCLUSION

The conclusions of the investigation are as given:

- Primary velocity increases with the increase in *Gr*, *Gm*, *b*, *m* and *t*.
- Primary velocity decreases with *M*, *Pr* and *Sc*.
- Secondary velocity increases with the increase in *Gr*, *Gm*, *M*, *b* and *t*.
- Secondary velocity decreases with *m*, *Pr* and *Sc*.
- τ_x increases with the rise in *Gr*, *Gm*, *m*, and *t*.
- τ_x decreases with *α, M, b, Pr* and *Sc.*
- τ_z increases with the rise in *Gr*, *Gm*, *M*, *b* and *t*.
- τ_z decreases with *m*, *Pr* and *Sc*.

Appendix:

$$A_0 = (1 + A_{01} + e^{2\sqrt{a}y} A_{02}),\ A_{01} = 1 + Erf\left[\frac{2\sqrt{a+bt} - y}{2\sqrt{t}}\right],\ A_{02} = 1 + Erf\left[\frac{2\sqrt{a+bt} + y}{2\sqrt{t}}\right],\ A_1 = 1 + A_{11} + A_{12},$$

$$A_{11} = 1 + Erf\left[\frac{2\sqrt{at} - y}{2\sqrt{t}}\right],\ A_{12} = e^{2\sqrt{a}y} Erfc\left[\frac{2\sqrt{at} + y}{2\sqrt{t}}\right],\quad A_{13} = \frac{2e^{-\sqrt{a}y}\left(1 + e^{2\sqrt{a}y} + A_{11} - A_{12}\right)}{y},\quad A_{14} = \frac{2e^{-\sqrt{a}y}\left(-1 - e^{2\sqrt{a}y} - A_{11} + A_{12}\right)}{y},$$

$$B_1 = \frac{Efr\left[2\sqrt{\frac{a\Pr}{-1+\Pr}}t - y\right]}{2\sqrt{t}},\ B_2 = \frac{Efr\left[2\sqrt{\frac{a\Pr}{-1+\Pr}}t + y\right]}{2\sqrt{t}},\ B_{11} = \frac{Efr\left[2\sqrt{\frac{aSc}{-1+Sc}}t - y\right]}{2\sqrt{t}},\ B_{12} = \frac{Efr\left[2\sqrt{\frac{aSc}{-1+Sc}}t + y\right]}{2\sqrt{t}},$$

$$B_{13} = \left(-1 - e^{\frac{2\sqrt{a\Pr}}{-1+\Pr}y} - B_1 + e^{\frac{2\sqrt{a\Pr}}{-1+\Pr}y} B_2\right),\ B_{14} = \frac{2\sqrt{\pi}\left(-1 + Erf\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right]\right)}{\sqrt{\Pr}y},\ B_{15} = \frac{Efr\left[2\sqrt{\frac{a}{-1+\Pr}}t - \sqrt{\Pr}y\right]}{2\sqrt{t}},\ B_{16} = \frac{Efr\left[2\sqrt{\frac{a}{-1+\Pr}}t + \sqrt{\Pr}y\right]}{2\sqrt{t}},$$

$$B_{17} = \left(1 + e^{2\sqrt{\frac{a}{-1+\Pr}}\sqrt{\Pr}y} + B_{15} - e^{2\sqrt{\frac{a}{-1+\Pr}}\sqrt{\Pr}} B_{16}\right),\ B_{18} = Erf\frac{\left[2\sqrt{\frac{a}{1+Sc}}t - \sqrt{Sc}y\right]}{2\sqrt{t}},\quad B_{19} = Erfc\frac{\left[2\sqrt{\frac{a}{1+Sc}}t + \sqrt{Sc}y\right]}{2\sqrt{t}},\ a = \frac{M}{1+m^2}(1 - im),$$

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