

## PRE\*GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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### ABSTRACT

*In this paper a new class of generalized closed sets, namely  $p^*g$ -closed sets is introduced in topological spaces. We find some basic properties and characterizations of  $p^*g$ -closed sets.*

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*Key Words:  $g$ -closed sets,  $p^*g$ -closed sets,  $g^*p$ -closed sets,  $\pi gp$ -closed sets.*

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### 1. INTRODUCTION

In 1970, N. Levine [8] introduced the concept of generalized closed sets (briefly  $g$ -closed). In 1982, Dunham [6] introduced the generalized closure (briefly  $g$ -closure). In 1996, H. Maki, J. Umehara and T. Noiri [4, 10] introduced the class of pre generalized closed sets and used them to obtain properties of pre- $T_{1/2}$  spaces. Selvi [20] introduced pre\*closed sets using the  $g$ -closure operator due to Dunham. Y. Gnanambal [7], H. Maki, R. Devi, K. Balachandran [9], J. Dontchev [4, 5], Veerakumar [23, 24, 25], N. Palaniappan and K. C. Rao [17], N. Nagaveni [14], J. H. Park [18], S. Muthuvel and R. Parimelazhagan, Milby Mathew [12, 13], Sarsak. M. S. and N. Rajesh [19], introduced and investigated  $gpr$ -closed,  $\alpha$ -closed,  $\alpha g$ -closed,  $gsp$ -closed,  $\pi g$ -closed,  $g^*$ -closed,  $g^*p$ -closed, pre semi closed,  $rg$ -closed,  $wg$ -closed,  $rwg$ -closed,  $\pi gp$ -closed,  $\alpha^m$ -closed,  $b^*$ -closed,  $\pi gsp$ -closed respectively.

In this paper we introduce a new class of sets called  $p^*g$ -closed sets. We give characterizations of  $p^*g$ -closed sets also investigate some fundamental properties of  $p^*g$ -closed set.

### 2. PRELIMINARIES

Throughout this paper  $(X, \tau)$  represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset  $A$  of a topological space  $X$ ,  $cl(A)$  and  $int(A)$  denote the closure of  $A$  and the interior of  $A$  respectively.  $(X, \tau)$  will be replaced by  $X$  if there is no changes of confusion. We recall the following definitions and results.

**Definition 2.1:** Let  $(X, \tau)$  be a topological space. A subset  $A$  of  $X$  is said to be generalized closed [8] (briefly  $g$ -closed) if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an open in  $(X, \tau)$ .

**Definition 2.2:** Let  $(X, \tau)$  be a topological space and  $A \subseteq X$ . The generalized closure of  $A$  [6], denoted by  $cl^*(A)$  and is defined by the intersection of all  $g$ -closed sets containing  $A$  and generalized interior of  $A$  [6], denoted by  $int^*(A)$  and is defined by union of all  $g$ -open sets contained in  $A$ .

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**Definition 2.3:** Let  $(X, \tau)$  be a topological space and  $A \subseteq X$ . Then

- (i)  $A$  is  $\alpha$ -open if  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$  and  $\alpha$ -closed if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$  [15].
- (ii)  $A$  is pre open if  $A \subseteq \text{int}(\text{cl}(A))$  and pre closed if  $\text{cl}(\text{int}(A)) \subseteq A$  [11].
- (iii)  $A$  is pre\*open if  $A \subseteq \text{int}^*(\text{cl}(A))$  and pre\*closed if  $\text{cl}^*(\text{int}(A)) \subseteq A$  [20].
- (iv)  $A$  is regular open if  $A = \text{int}(\text{cl}(A))$  and regular closed if  $A = \text{cl}(\text{int}(A))$  [21].
- (v)  $A$  is semi pre open if  $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$  and semi pre closed if  $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$  [1].
- (vi)  $\pi$ -closed set [26] if  $A$  is a finite intersection of regular closed sets. The complement of a  $\pi$ -closed set is called a  $\pi$ -open set.
- (vii) a regular  $\alpha$ -open set (briefly  $r\alpha$ -open) [22] if there is a regular open set  $U$  such that  $U \subseteq A \subseteq \alpha\text{cl}(U)$ .

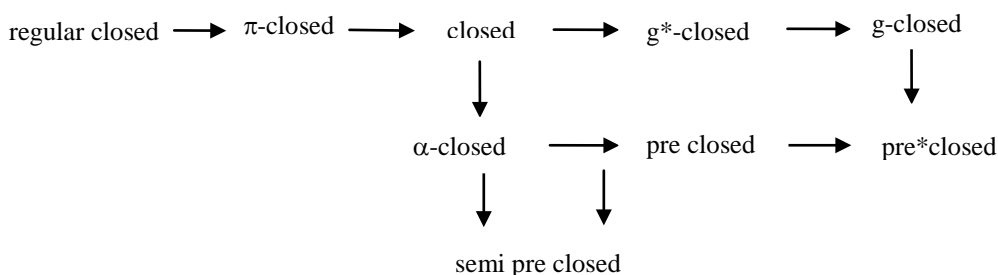
**Definition 2.4:** Let  $(X, \tau)$  be a topological space and  $A \subseteq X$ . The pre closure of  $A$  [11], denoted by  $\text{pcl}(A)$  and is defined by the intersection of all pre closed sets containing  $A$ .

**Definition 2.5:** Let  $(X, \tau)$  be a topological space. A subset  $A$  of  $X$  is said to be

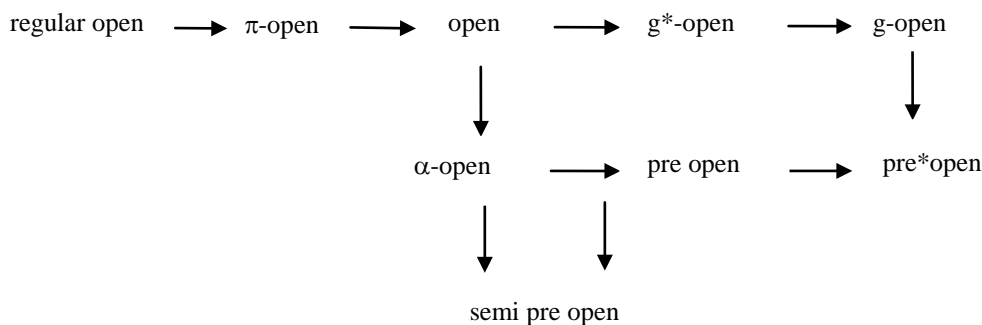
1.  $\alpha$ -generalized closed set (briefly  $\alpha g$ -closed) [9] if  $\alpha\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
2. generalized pre closed set (briefly  $gp$ -closed) [10] if  $\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
3. strongly generalized closed set (briefly  $g^*$ -closed) [23] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$ .
4. generalized\*pre closed set (briefly  $g^*p$ -closed) [24] if  $\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$ .
5. regular generalized closed set (briefly  $rg$ -closed) [17] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $(X, \tau)$ .
6. weakly generalized closed set (briefly  $wg$ -closed) [14] if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
7. generalized pre regular closed set (briefly  $gpr$ -closed) [7] if  $\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $(X, \tau)$ .
8. generalized semi preclosed set (briefly  $gsp$ -closed) [5] if  $\text{spcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
9. pre semi closed set [25] if  $\text{spcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$ .
10.  $\pi gp$ -closed set [18] if  $\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\pi$ -open in  $(X, \tau)$ .
11. regular weakly generalized closed set (briefly  $rwg$ -closed) [14] if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $(X, \tau)$ .
12.  $b$ -closed set [13] if  $\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subseteq A$ .
13.  $b^*$ -closed set [13] if  $\text{int}(\text{cl}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $b$ -open in  $(X, \tau)$ .
14.  $\alpha^m$ -closed set [12] if  $\text{int}(\text{cl}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $(X, \tau)$ .
15.  $\pi$ -generalized semi pre closed set [19] (briefly  $\pi gsp$ -closed) if  $\text{spcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\pi$ -open in  $(X, \tau)$ .

The complements of the above mentioned closed sets are their respective open sets.

**Remark 2.6:**



**Remark 2.7:**



**Theorem 2.8:** [3] Let  $(X, \tau)$  be a topological space. Then  $\text{pcl}(A \cap B) \subseteq \text{pcl}(A) \cap \text{pcl}(B)$ .

**Lemma 2.9:** [1] For any subset A of X,  $pcl(A) = A \cup cl(int(A))$ .

**Lemma 2.10:** [2] If A is semi closed in X, then  $pcl(A \cup B) = pcl(A) \cup pcl(B)$ .

**Theorem 2.11:** [20] Arbitrary union of pre\*open sets is pre\*open.

### 3. PRE\*GENERALIZED CLOSED SETS

**Definition 3.1:** A subset A of a topological space  $(X, \tau)$  is called pre\*generalized closed (briefly p\*g-closed) if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is pre\*open in  $(X, \tau)$ .

**Theorem 3.2:** Let  $(X, \tau)$  be a topological space. Then every closed set is p\*g-closed.

**Proof:** Let A be a closed set. Let  $A \subseteq U$ , U is pre\*open. Since A is closed,  $cl(A) = A \subseteq U$ . But  $pcl(A) \subseteq cl(A)$ . Thus we have  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is pre\*open. Therefore, A is p\*g-closed.

**Remark 3.3:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.4:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{c\}, X\}$ . Then {a} and {b} are p\*g-closed but not closed.

**Theorem 3.5:** Let  $(X, \tau)$  be a topological space. Then every regular closed set is p\*g-closed.

**Proof:** Let A be a regular closed set. Let  $A \subseteq U$ , U is pre\*open. By Remark 2.6,  $pcl(A) \subseteq rcl(A) = A \subseteq U$ . Thus we have  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is pre\*open. Therefore, A is p\*g-closed.

**Remark 3.6:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.7:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a, b\}, \{c\}, X\}$ . Then {b, c} and {a} are p\*g-closed but not regular closed.

**Theorem 3.8:** Let  $(X, \tau)$  be a topological space. Then every  $\alpha$ -closed set is p\*g-closed.

**Proof:** Let A be a  $\alpha$ -closed set. Let  $A \subseteq U$ , U is pre\*open. By Remark 2.6,  $pcl(A) \subseteq \alpha cl(A) = A \subseteq U$ . Thus we have  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is pre\*open. Therefore, A is p\*g-closed.

**Remark 3.9:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.10:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a, b\}, X\}$ . Then {a}, {b} are p\*g-closed but not  $\alpha$ -closed.

**Theorem 3.11:** Let  $(X, \tau)$  be a topological space. Then every p\*g-closed set is gp-closed.

**Proof:** Let A be a p\*g-closed set. Let  $A \subseteq U$ , U is open. Then by Remark 2.7, U is pre\*open. Since A is p\*g-closed,  $pcl(A) \subseteq U$ . Thus we have  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open. Therefore, A is gp-closed.

**Remark 3.12:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.13:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{c\}, X\}$ . Then {a, c} and {b, c} are gp-closed but not p\*g-closed.

**Theorem 3.14:** Let  $(X, \tau)$  be a topological space. Then every p\*g-closed set is gpr-closed.

**Proof:** Let A be a p\*g-closed set. Let  $A \subseteq U$ , U is regular open. Then by Remark 2.7, U is pre\*open. Since A is p\*g-closed,  $pcl(A) \subseteq U$ . Thus we have  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is regular open. Therefore, A is gpr-closed.

**Remark 3.15:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.16:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a, b\}, X\}$ . Then {a, b} is gpr-closed but not p\*g-closed.

**Theorem 3.17:** Let  $(X, \tau)$  be a topological space. Then every  $p^*g$ -closed set is  $wg$ -closed.

**Proof:** Let  $A$  be a  $p^*g$ -closed set. Let  $A \subseteq U$ ,  $U$  is open. Then by Remark 2.7,  $U$  is pre\*open. Since  $A$  is  $p^*g$ -closed,  $pcl(A) \subseteq U$ . By Lemma 2.9,  $A \cup cl(int(A)) \subseteq U$ . Thus we have  $cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open. Therefore,  $A$  is  $wg$ -closed.

**Remark 3.18:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.19:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{c\}, X\}$ . Then  $\{a, c\}$  and  $\{b, c\}$  are  $wg$ -closed but not  $p^*g$ -closed.

**Theorem 3.20:** Let  $(X, \tau)$  be a topological space. Then every  $p^*g$ -closed set is  $rwg$ -closed.

**Proof:** Let  $A$  be a  $p^*g$ -closed set. Let  $A \subseteq U$ ,  $U$  is regular open. Then by Remark 2.7,  $U$  is pre\*open. Since  $A$  is  $p^*g$ -closed,  $pcl(A) \subseteq U$ . By Lemma 2.9,  $A \cup cl(int(A)) \subseteq U$ . Thus we have  $cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open. Therefore,  $A$  is  $rwg$ -closed.

**Remark 3.21:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.22:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ . Then  $\{a, b\}$  is  $rwg$ -closed but not  $p^*g$ -closed.

**Theorem 3.23:** Let  $(X, \tau)$  be a topological space. Then every  $p^*g$ -closed set is  $\pi gp$ -closed.

**Proof:** Let  $A$  be a  $p^*g$ -closed set. Let  $A \subseteq U$ ,  $U$  is  $\pi$ -open. Then by Remark 2.7,  $U$  is pre\*open. Since  $A$  is  $p^*g$ -closed,  $pcl(A) \subseteq U$ . Thus we have  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\pi$ -open. Therefore,  $A$  is  $\pi gp$ -closed.

**Remark 3.24:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.25:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ . Then  $\{a, b\}$  is  $\pi gp$ -closed but not  $p^*g$ -closed.

**Theorem 3.26:** Let  $(X, \tau)$  be a topological space. Then every  $p^*g$ -closed set is  $gsp$ -closed.

**Proof:** Let  $A$  be a  $p^*g$ -closed set. Let  $A \subseteq U$ ,  $U$  is open. Then by Remark 2.7,  $U$  is pre\*open. Since  $A$  is  $p^*g$ -closed,  $pcl(A) \subseteq U$ . But  $spcl(A) \subseteq pcl(A) \subseteq U$ . Thus we have  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open. Therefore,  $A$  is  $gsp$ -closed.

**Remark 3.27:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.28:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ . Then  $\{a, c\}$  is  $gsp$ -closed but not  $p^*g$ -closed.

**Theorem 3.29:** Let  $(X, \tau)$  be a topological space. Then every  $p^*g$ -closed set is  $\pi gsp$ -closed.

**Proof:** Let  $A$  be a  $p^*g$ -closed set. Let  $A \subseteq U$ ,  $U$  is  $\pi$ -open. Then by Remark 2.7,  $U$  is pre\*open. Since  $A$  is  $p^*g$ -closed,  $pcl(A) \subseteq U$ . But  $spcl(A) \subseteq pcl(A) \subseteq U$ . Thus we have  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\pi$ -open. Therefore,  $A$  is  $\pi gsp$ -closed.

**Remark 3.30:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.31:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a, b\}, X\}$ . Then  $\{a, b\}$  is  $\pi gsp$ -closed but not  $p^*g$ -closed.

**Theorem 3.32:** Let  $(X, \tau)$  be a topological space. Then every  $p^*g$ -closed set is pre semi closed.

**Proof:** Let  $A$  be a  $p^*g$ -closed set. Let  $A \subseteq U$ ,  $U$  is  $g$ -open. Then by Remark 2.7,  $U$  is pre\*open. Since  $A$  is  $p^*g$ -closed,  $pcl(A) \subseteq U$ . But  $spcl(A) \subseteq pcl(A) \subseteq U$ . Thus we have  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open. Therefore,  $A$  is pre semi closed.

**Remark 3.33:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.34:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{c\}, \{b, c\}, X\}$ . Then  $\{a, c\}$  is pre semi closed but not  $p^*g$ -closed.

**Theorem 3.35:** Let  $(X, \tau)$  be a topological space. Then every  $p^*g$ -closed set is  $g^*p$ -closed.

**Proof:** Let  $A$  be a  $p^*g$ -closed set. Let  $A \subseteq U$ ,  $U$  is  $g$ -open. Then by Remark 2.7,  $U$  is pre\*open. Since  $A$  is  $p^*g$ -closed,  $pcl(A) \subseteq U$ . Thus we have  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open. Therefore,  $A$  is  $g^*p$ -closed.

**Remark 3.36:** The converse of the above theorem need not be true, as seen from the following example.

**Example 3.37:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{c\}, \{b, c\}, X\}$ . Then  $\{a, c\}$  is  $g^*p$ -closed but not  $p^*g$ -closed.

**Theorem 3.38:** Let  $(X, \tau)$  be a topological space. If  $A$  and  $B$  are two  $p^*g$ -closed in  $X$ , then  $A \cap B$  is  $p^*g$ -closed.

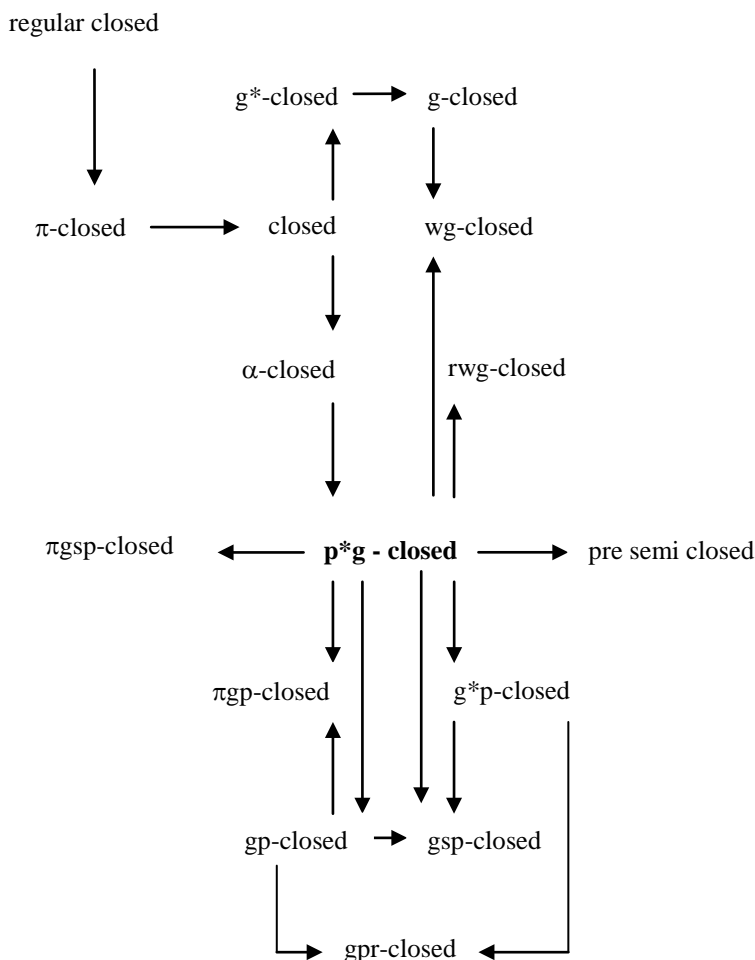
**Proof:** Let  $U$  be pre\*open such that  $A \cap B \subseteq U$ . Then by Theorem 2.11,  $U \cup (X-B)$  is pre\*open containing  $A$ . Since  $A$  is  $p^*g$ -closed,  $pcl(A) \subseteq U \cup (X-B)$ .

Now  $pcl(A \cap B) \subseteq pcl(A) \cap pcl(B) \subseteq pcl(A) \cap cl(B) = pcl(A) \cap B \subseteq (U \cup (X-B)) \cap B = U \cap B \subseteq U$ . Thus we have  $pcl(A \cap B) \subseteq U$ ,  $U$  is pre\*open and  $A \cap B \subseteq U$ . Therefore  $A \cap B$  is  $p^*g$ -closed.

**Remark 3.39:** In general, union of any two  $p^*g$ -closed sets in  $(X, \tau)$  need not be a  $p^*g$ -closed set, as seen from the following example.

**Example 3.40:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a, b\}, X\}$ . Here,  $\{a\}$  and  $\{b\}$  are  $p^*g$ -closed. But their union  $\{a, b\}$  is not  $p^*g$ -closed.

**Remark 3.41:** The above discussions are summarized in the following implications.



**Remark 3.42:**  $p^*g$ -closedness and  $rg$ -closedness are independent concepts as we illustrate by means of the following example.

**Example 3.43:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ . Then the set  $\{c\}$  is  $p^*g$ -closed but not  $rg$ -closed and also  $\{a, b\}$  is  $rg$ -closed but not  $p^*g$ -closed.

**Remark 3.44:**  $p^*g$ -closedness and  $g$ -closedness are independent concepts as we illustrate by means of the following example.

**Example 3.45:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ . Then the set  $\{b\}$  is  $p^*g$ -closed but not  $g$ -closed and also  $\{a, c\}$  is  $g$ -closed but not  $p^*g$ -closed.

**Remark 3.46:**  $p^*g$ -closedness and  $g^*$ -closedness are independent concepts as we illustrate by means of the following example.

**Example 3.47:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ . Then the set  $\{b\}$  is  $p^*g$ -closed but not  $g^*$ -closed and also  $\{a, c\}$  is  $g^*$ -closed but not  $p^*g$ -closed.

**Remark 3.48:**  $p^*g$ -closedness and  $\alpha g$ -closedness are independent concepts as we illustrate by means of the following examples.

**Example 3.49:**

- i. Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a, b\}, X\}$ . Then the set  $\{a\}$  and  $\{b\}$  are  $p^*g$ -closed but not  $\alpha g$ -closed.
- ii. Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{c\}, X\}$ . Then the set  $\{a, c\}$  and  $\{b, c\}$  are  $\alpha g$ -closed but not  $p^*g$ -closed.

**Remark 3.50:**  $p^*g$ -closedness and regular  $\alpha$ -closedness are independent concepts as we illustrate by means of the following example.

**Example 3.51:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ . Then  $\{c\}$  is  $p^*g$ -closed but not regular  $\alpha$ -closed and also  $\{a\}$  and  $\{b\}$  are regular  $\alpha$ -closed but not  $p^*g$ -closed.

**Remark 3.52:**  $p^*g$ -closedness and  $b^*$ -closedness are independent concepts as we illustrate by means of the following examples.

**Example 3.53:**

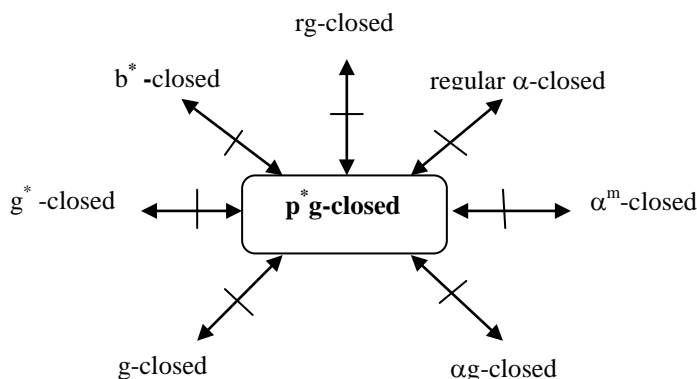
- i. Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a, b\}, X\}$ . Then the set  $\{a\}$  and  $\{b\}$  are  $p^*g$ -closed but not  $b^*$ -closed.
- ii. Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ . Then  $\{a\}$  and  $\{b\}$  are  $b^*$ -closed but not  $p^*g$ -closed.

**Remark 3.54:**  $p^*g$ -closedness and  $\alpha^m$ -closedness are independent concepts as we illustrate by means of the following examples.

**Example 3.55:**

- i. Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a, b\}, X\}$ . Then the set  $\{a\}$  and  $\{b\}$  are  $p^*g$ -closed but not  $\alpha^m$ -closed.
- ii. Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ . Then  $\{a\}$  and  $\{b\}$  are  $\alpha^m$ -closed but not  $p^*g$ -closed.

**Remark 3.56:**



#### 4. CHARACTERIZATION

In this section, we investigate some basic characterization of  $p^*g$ -closed set in topological spaces.

**Theorem 4.1:** If A is g-closed and p\*g-closed, then A is wg-closed.

**Proof:** Suppose A is g-closed and p\*g-closed. By Remark 2.6,  $pcl(A) \subseteq cl(A)$  which implies  $pcl(A) \subseteq cl(A) \subseteq U$ . By Lemma 2.9,  $A \cup cl(int(A)) \subseteq U$ . Thus we have  $cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and U is open. Therefore, A is wg-closed.

**Remark 4.2:** The converse of the above theorem need not be true, as seen from the following example.

**Example 4.3:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{c\}, X\}$ . Here,  $\{a, c\}$  and  $\{b, c\}$  are both g-closed and wg-closed but not p\*g-closed.

**Theorem 4.4:** If A is g-closed and p\*g-closed, then A is g\*p-closed.

**Proof:** Suppose A is g-closed and p\*g-closed. By Remark 2.6,  $pcl(A) \subseteq cl(A)$  which implies  $pcl(A) \subseteq cl(A) \subseteq U$  and by Remark 2.7, U is g-open. Thus we have  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is g-open. Therefore, A is g\*p-closed.

**Remark 4.5:** The converse of the above theorem need not be true, as seen from the following example.

**Example 4.6:** Consider the space  $(X, \tau)$  where  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{c\}, \{b, c\}, X\}$ . Here,  $\{a, c\}$  is both g-closed and g\*p-closed but not p\*g-closed.

**Theorem 4.7:** Let A be any p\*g-closed set in  $(X, \tau)$ . If  $A \subseteq B \subseteq pcl(A)$ , then B is also a p\*g-closed set.

**Proof:** Let  $B \subseteq U$  where U is pre\*open in  $(X, \tau)$ . Then  $A \subseteq U$ . Also since A is p\*g-closed,  $pcl(A) \subseteq U$ . Since  $B \subseteq pcl(A)$ ,  $pcl(B) \subseteq pcl(pcl(A)) = pcl(A) \subseteq U$ . This implies,  $pcl(B) \subseteq U$ . Thus B is a p\*g-closed set.

**Theorem 4.8:** If a set A is p\*g-closed in X, then  $pcl(A) - A$  contains no non empty pre\*open set in X.

**Proof:** Let  $U \subseteq pcl(A) - A$  be a non empty pre\*open set. Then  $U \subseteq pcl(A)$  and  $A \subseteq X - U$ , we have  $pcl(A) \subseteq X - U$ . So  $U \subseteq X - pcl(A)$ . Therefore  $U \subseteq pcl(A) \cap (X - pcl(A)) = \{\emptyset\}$ . Hence  $pcl(A) - A$  contains no non empty pre\*open set in X.

**Remark 4.9:** The converse of the above theorem need not be true, as seen from the following example.

**Example 4.10:** If  $pcl(A) - A$  contains no non empty pre\*open set in X, then A is not a p\*g-closed set. Consider  $X = \{a, b, c\}$  with the topology  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$  and  $A = \{a, b\}$ . Then  $pcl(A) - A = X - \{a, b\} = \{c\}$  contains no non empty pre\*open set in X, but A is not a p\*g-closed set in X.

**Theorem 4.11:** For every element x in a space X, the set  $X - \{x\}$  is p\*g-closed or pre\*open.

**Proof:** Suppose  $X - \{x\}$  is not pre\*open. Then X is the only pre\*open set containing  $X - \{x\}$ . This implies  $pcl(X - \{x\}) \subseteq X$ . Hence  $X - \{x\}$  is p\*g-closed.

**Theorem 4.12:** Let A and B be p\*g-closed sets in  $(X, \tau)$  such that  $cl(A) = pcl(A)$  and  $cl(B) = pcl(B)$ . Then  $A \cup B$  is p\*g-closed.

**Proof:** Let  $A \cup B \subseteq U$ , where U is pre\*open. Then  $A \subseteq U$  and  $B \subseteq U$ . Since A and B are p\*g-closed,  $pcl(A) \subseteq U$  and  $pcl(B) \subseteq U$ . Now  $cl(A \cup B) = cl(A) \cup cl(B) = pcl(A) \cup pcl(B) \subseteq U$ . But  $pcl(A \cup B) \subseteq cl(A \cup B)$ . So,  $pcl(A \cup B) \subseteq cl(A \cup B) \subseteq U$ . Therefore  $pcl(A \cup B) \subseteq U$  whenever  $A \cup B \subseteq U$ , U is pre\*open. Hence  $A \cup B$  is p\*g-closed.

**Theorem 4.13:** The union of two p\*g-closed sets is p\*g-closed if at least one of them is semi closed.

**Proof:** Let A and B be two p\*g-closed sets in X. Suppose A is semi closed. To prove that  $A \cup B$  is p\*g-closed. Let  $A \cup B \subseteq U$  and U is pre\*open. Then  $A \subseteq U$  and  $B \subseteq U$ . Since A and B are p\*g-closed,  $pcl(A) \subseteq U$  and  $pcl(B) \subseteq U$ . Therefore,  $pcl(A) \cup pcl(B) \subseteq U$ . Since by Lemma 2.10,  $pcl(A \cup B) \subseteq U$ . Thus we have  $pcl(A \cup B) \subseteq U$  whenever  $A \cup B \subseteq U$  and U is pre\*open. Therefore  $A \cup B$  is p\*g-closed.

**Theorem 4.14:** If  $A \subseteq Y \subseteq X$  and A is p\*g-closed in X then A is p\*g-closed relative to Y.

**Proof:** Given that  $A \subseteq Y \subseteq X$  and A is a p\*g-closed set in X. To prove that A is p\*g-closed set relative to Y. Let us assume that  $A \subseteq Y \cap U$ , where U is pre\*open in X. Since A is p\*g-closed,  $A \subseteq U$ . This implies that  $pcl(A) \subseteq U$ . It follows that  $Y \cap pcl(A) \subseteq Y \cap U$ . That is, A is p\*g-closed relative to Y.

## 5. CONCLUSION

The present paper has introduced a new concept of  $p^*g$ -closed set in topological spaces. It also analyzed some of the properties. The implication shows the relationship between  $p^*g$ -closed sets and the other existing sets.

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