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NEW APPROACH TO SOLVE QUADRATIC PROGRAMMING PROBLEMS FOR FUZZY WOLFE'S MODIFIED SIMPLEX METHOD

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ABSTRACT

In this paper, we proposed the method for solving fuzzy wolfe's modified simplex method. This method is easy to solve fuzzy nonlinear programming problem (NLPP). The fuzzy non-linearity of the functions makes the solution of the problem much more involved as compared to NLPPs and there is no single algorithm like the modified simplex method, which can be employed to solve efficiently all fuzzy NPPs.

Keywords: *New approach, Fuzzy nonlinear programming problem, Optimal solution, Trapezoidal fuzzy number.*

1. INTRODUCTION

The fuzzy set theory is being applied in many fields these days. One of these is nonlinear programming problems. Quadratic programming problems (QPP) deals with the nonlinear programming problem (NLPP) of maximizing (or minimizing) the quadratic objective function subjective to a set of linear inequality constraints. In general form of QPP [1, 3] be:

$$\text{Max } \tilde{z} = \text{f}(\tilde{x}) = \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j + \frac{1}{2}\sum_{j=1}^{n} \sum_{k=1}^{n} \tilde{c}_{jk} \tilde{x}_j \tilde{x}_k$$

Subject to the constraints

$$\sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j \preceq \tilde{b}_i, \quad \tilde{x}_j \geq 0 \quad (\text{I} = 1, 2\ldots\ldots \text{m}, \ \text{j} = 1, 2\ldots.\text{n})$$

Where $\tilde{c}_{jk} = \tilde{c}_{kj}$ for all j, k and $\tilde{b}_i \geq 0$, i = 1, 2…m. The quadratic form $\sum\sum \tilde{c}_{jk} \tilde{x}_j \tilde{x}_k$ be negative semi-definite.

The simplex method for fuzzy variable linear programming problem discussed for single objective by [7, 13, 14]. The concept of a fuzzy decision making was first proposed by Bellman and Zadeh [2]. Lalitha and Loganathan [11] proposed an objective fuzzy non linear programming problem with symmetric trapezoidal fuzzy numbers. Detail literature on fuzzy linear and non-linear programming with application is available in two well-known books of Lie and Hwang (1992, 1994). Mokhter, Hanif and Shetty [12] presented nonlinear programming theory and algorithms**.** Frank and Wolfe [6] proposed an algorithm for quadratic programming. Terlaky's algorithm [15] is active set method which starts from a primal feasible solution construct dual feasible solution which is complementary to the primal feasible solution. Hildreth [8] presented a Quadratic Programming Procedure. Wolfe Philip [16] has given algorithm which based on fairly simple modification of simplex method and converges in finite number of iterations. Dantzig [4] suggestion is to choose that entering vector corresponding to which is most negative. In this paper, we find the solution of fuzzy nonlinear programming problem by proposed method which is an alternative for wolfe's method. This method is different from Terlaky, wolfe's and earlier approaches.

2. PRELIMINARIES

Definition 2.1: Fuzzy Number: A fuzzy number is a normal and convex fuzzy set of real line R.

Definition 2.2: A fuzzy set Ã is called normal if its core is non-empty. In other words, there is at least one point $x \in$ X with $\mu_{\tilde{A}}(x) = 1$.

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Definition 2.3: Let X is a nonempty set A. Fuzzy set A in X is characterized by its membership function $\mu_A : x \rightarrow [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element X in fuzzy set A for each $x \in X$.

3. TRAPEZOIDAL FUZZY NUMBER

There are various types of fuzzy numbers, but the triangular and trapezoidal are the most important fuzzy memberships. In this research we use the trapezoidal fuzzy numbers.

$$\mu_{\tilde{A}}(x) = \begin{cases} L((A^L - x) \mid \alpha) & \text{if } x \le A^L, \alpha > 0 \\ R((x - A^U) \mid \beta) & \text{if } x \ge A^U, \beta > 0 \\ 1 & \text{otherwise,} \end{cases}$$

where $A^L < A^U$, $[A^L, A^U]$ is the core of $\tilde{A}$, $\mu_{\tilde{A}}(x) = 1\ \forall x + [A^L, A^U]$, A^L, A^U are the lower and upper model values of $\tilde{A}$ and $\alpha > 0$, $\beta > 0$ are the left hand and right hand spreads [13].

4. SOLVING A FUZZY NONLINEAR PROGRAMMING PROBLEM

$$\text{Max:} \quad \sum_{j=1}^{p} \tilde{C}_j x_j^{\alpha} \qquad j = 1, 2, 3,\ldots p$$

$$\text{S .t} \quad \sum_{j=1}^{p} \tilde{a}_j x_j \le \tilde{b}_i, \qquad i = 1, 2,\ldots m,$$

$$x_j \ge 0, \qquad j = 1, 2,\ldots p$$

Where $\bar{c}_j$, $\bar{a}_j$ and $\bar{b}_i$ are fuzzy numbers.

5. PROPOSED ALGORITHM

Step-1: Convert inequality constraints into equations by introducing slack variables $\tilde{x}_i^2$ (i = 1,2..p) in the i^th^ Constraints and the slack variables $\tilde{x}_j^2$ (j=1, 2, 3…..q) in the j^th^ constraints.

Step-2: Convert the Lagrangian function

$$L(\tilde{x}, \tilde{s}, \tilde{\lambda}, \tilde{\mu}) = f(\tilde{x}) - \sum_{i=1}^{m} \tilde{\lambda}_i \left[\sum_{j=1}^{n} \tilde{m}_{ij} \tilde{x}_j - \tilde{n}_i + \tilde{s}_i^2 \right] - \sum_{j=1}^{n} \tilde{\mu}_j \left[-\tilde{x}_j + \tilde{s}_j^2 \right]$$

Differentiating the Lagrangian function $L(\tilde{x}, \tilde{s}, \tilde{\lambda}, \tilde{\mu})$with respect to the components of $\tilde{x}, \tilde{s}, \tilde{\lambda}, \tilde{\mu}$ and equating the first order partial derivative to zero. Derive Kuhn tucker condition from the resulting equations.

Step-3: Introduce non- negative artificial variables $\tilde{y}_j$ (j=1, 2…q) in the Kuhn tucker condition

$$\tilde{c}_j + \sum_{k=1}^{n} \tilde{c}_{jk} \tilde{x}_k - \sum_{i=1}^{m} \tilde{\lambda}_i \tilde{m}_{ij} + \tilde{\mu}_j = 0$$

For j = 1, 2…n and construct an objective function

$$\tilde{z} = \tilde{y}_1 + \tilde{y}_2 \ldots\ldots\ldots \tilde{y}_n$$

Step-4: Obtain an initial basic feasible solution to the LPP

$$Min \tilde{z} = \tilde{y}_1 + \tilde{y}_2 \ldots\ldots\ldots \tilde{y}_n$$

Subject to the constraints:

$$\sum_{k=1}^{n} \tilde{c}_{jk} \tilde{x}_k - \sum_{i=1}^{m} \tilde{\lambda}_i \tilde{m}_{ij} + \tilde{y}_j = -\tilde{c}_j \ ;(j=1,2\ldots q)$$

$$\sum_{j=1}^{n} \tilde{m}_{ij} \tilde{x}_j + \tilde{s}_i^2 = \tilde{n}_i \ ; \ (i=1,2\ldots p)$$

$$\tilde{\lambda}_i, \ \tilde{\mu}_j, \tilde{x}_j, \tilde{y}_j \ge 0, \ (i = 1,2,\ldots.p)(j = 1,2,3\ldots..q)$$

and satisfying the slackness condition:

$$\tilde{\lambda}_i \tilde{s}_i = 0 \quad \text{and} \quad \tilde{\mu}_j \tilde{x}_j = 0$$

Step-5: solve this LPP by proposed method .Choose greatest coefficients. If greatest coefficient is unique, then variable corresponding to this column becomes incoming variable. If greatest coefficient is not unique, then use tie breaking technique.

Step-6: Compute the ratio with $\tilde{x}_B$. Choose minimum ratio, the variable corresponding to this row is Outgoing variable. If artificial variable is outgoing in the basis means corresponding artificial column also will be removed.

Step-7: Then proceed this table given by [10] and go to next step.

Step-8: Ignore corresponding row and column. Proceed to step5 for remaining elements and repeat the same procedure. Either an optimal solution is obtained or there is an indication of an unbounded solution.

Step-9: If all rows and columns are ignored, current solutions an optimal solution. Thus optimum solution is obtained and which is the given solution of given QPP.

6. SOLVED PROBLEMS

6.1. Problem 1: Solve the following quadratic programming problem:

$$MaxZ = \tilde{4}\tilde{x}_1 + \tilde{2}\tilde{x}_2 - \tilde{x}_1^2 - \tilde{x}_2^2 - \tilde{5}$$

Subject to: $\tilde{x}_1 + \tilde{x}_2 \leq \tilde{4}$, $\tilde{x}_1, \tilde{x}_2 \geq 0$

Solution: First, we convert the inequality constraint into equation by introducing slack variable $\tilde{s}_1^2$. Also the inequality constraints $\tilde{x}_1, \tilde{x}_2 \geq 0$. We convert them into equations by introducing slack variables $\tilde{s}_2^2$ and $\tilde{s}_3^2$. So the problem becomes

$$MaxZ = \tilde{4}\tilde{x}_1 + \tilde{2}\tilde{x}_2 - \tilde{x}_1^2 - \tilde{x}_2^2 - \tilde{5}$$

Subject to: $\tilde{x}_1 + \tilde{x}_2 + \tilde{s}_1^2 = \tilde{4}$

$$-\tilde{x}_1 + \tilde{s}_2^2 = 0$$

$$-\tilde{x}_2 + \tilde{s}_3^2 = 0$$

Where

$\tilde{4} = (3.2,3.3,4.3,4.8)$

$\tilde{2} = (1.9,2.1,2.2,2.6)$

$\tilde{1} = (0.9,0.6,0.7,0.5)$

$\tilde{1} = (0.8,0.6,0.9,1.7)$

$\tilde{5} = (4.3,4.6,5.2,5.8)$

$\tilde{1} = (1.8,0.7,0.9,1.5)$

$\tilde{1} = (1.4,0.8,0.9,0.6)$

$\tilde{1} = 1.2,0.6,0.5,0.4)$

$\tilde{4} = (4.4,4.1,5.2,5.8)$

$\tilde{1} = 0.3,0.2,1.2,0.8)$

$\tilde{1} = (2.1,2.2,1.1,1.7)$

$\tilde{1} = (0.5,0.8,0.9,1.4)$

$\tilde{1} = (2.4,2.8,0.3,0.9)$

$$Max\tilde{Z} = 3.2\tilde{x}_1 + 1.9\tilde{x}_2 - 0.9\tilde{x}_1^2 - 0.6\tilde{x}_2^2 - 4.3$$
$$0.7\tilde{x}_1 + 0.8\tilde{x}_2 + 1.2\tilde{s}_1^2 = 4.1$$
$$-0.3\tilde{x}_1 + 2.1\tilde{s}_2^2 = 0$$
$$-0.5\tilde{x}_2 + 2.4\tilde{s}_3^2 = 0$$

Construct the Lagrangian function

$$L(\tilde{x}_1, \tilde{x}_2, \tilde{s}_1, \tilde{s}_2, \tilde{s}_3, \tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3) = (3.2\tilde{x}_1 + 1.9\tilde{x}_2 - 0.9\tilde{x}_1^2 - 0.6\tilde{x}_2^2 - 4.2) - \tilde{\lambda}_1(0.7\tilde{x}_1 + 0.8\tilde{x}_2 + 1.2\tilde{s}_1^2 - 4.1)$$
$$- \tilde{\lambda}_2(-0.3\tilde{x}_1 + 2.1\tilde{s}_1^2) - \tilde{\lambda}_3(-0.5\tilde{x}_2 + 2.4\tilde{s}_3^2)$$

By Kuhn-Tucker conditions, we get

$$1.8\tilde{x}_1 + 0.7\tilde{\lambda}_1 - 0.3\tilde{\lambda}_2 = 3.2$$
$$1.2\tilde{x}_2 + 0.8\tilde{\lambda}_1 - 0.5\tilde{\lambda}_3 = 1.9$$
$$\tilde{\lambda}_2\tilde{s}_2 = \tilde{\lambda}_1\tilde{s}_1 = \tilde{\lambda}_3\tilde{s}_3 = 0$$
$$0.7\tilde{x}_1 + 0.8\tilde{x}_2 + 1.2\tilde{s}_1^2 = 4.1$$
$$-0.3\tilde{x}_1 + 2.1\tilde{s}_2^2 = 0$$
$$-0.5\tilde{x}_2 + 2.4\tilde{s}_3^2 = 0$$
$$\tilde{\lambda}_2\tilde{s}_2 = \tilde{\lambda}_1\tilde{s}_1 = \tilde{\lambda}_3\tilde{s}_3 = 0$$

Where $\tilde{x}_1,\ \tilde{x}_2,\ \tilde{s}_i^2,\ \tilde{\lambda}_i \geq 0$, i = 1, 2, 3 satisfying the complementary slackness conditions

$$\tilde{\lambda}_2\tilde{x}_1 + \tilde{\lambda}_1\tilde{s}_1^2 + \tilde{\lambda}_3\tilde{x}_2 = 0$$

Now, introducing the artificial variables $\tilde{a}_1, \tilde{a}_2 \geq 0$ the given QPP is equivalent to :

$$Min\tilde{Z} = 0.8\tilde{a}_1 + 0.6\tilde{a}_2$$
$$1.8\tilde{x}_1 + 0.7\tilde{\lambda}_1 - 0.3\tilde{\lambda}_2 = 3.2$$
$$1.2\tilde{x}_2 + 0.8\tilde{\lambda}_1 - 0.5\tilde{\lambda}_3 = 1.9$$
$$0.7\tilde{x}_1 + 0.8\tilde{x}_2 + 1.2\tilde{s}_1^2 = 4.1$$

c_B	BVS	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{s}_1^2$	Ratio
0.8	$\tilde{a}_1$	3.2	1.8	0	0.7	0.3	0	0.8	0	0	1.7
0.6	$\tilde{a}_2$	1.9	0	1.2	0.8	0	0.5	0	0.6	0	-
0	s_1^2	4.1	0.7	0.8	0	0	0	0	0	1.2	5.8
0	$\tilde{x}_1$	1.7	1	0	0.4	-0.2	0	0.4	0	0	0
0.6	$\tilde{a}_2$	1.9	0	1.2	0.8	0	-0.5	0	0.6	0	1.6
0	s_1^2	2.9	0	0.8	-0.3	1.4	0	-0.3	0	1.2	3.6
0	$\tilde{x}_1$	1.7	1	0	0.4	-0.2	0	0.4	0	0	
0	$\tilde{x}_2$	1.6	0	1	0.7	0	-0.4	0	0.5	0	
0	s_1^2	1.6	0	0	-1.3	1.4	0.3	-0.3	-0.4	1.2	

Table-1: Comparison with Kirtiwant and et al., method

NLPP	x_1	x_2	Max Z
Kirtiwant and et al.,	2	1	0
FNLPP	$\tilde{x}_1$	$\tilde{x}_2$	$Max\,\tilde{Z}$
Our proposed method	1.7	1.6	0.2

From the Table 1, Objective (maximum) value obtained by our method is better than Kirtiwant and *et al.*, [10] Method.

Current solution is an optimal solution $\tilde{x}_1 = 1.7$, $\tilde{x}_2 = 1.6$, $Max\tilde{Z} = 0.2$

6.2. Problem 2: Solve the following quadratic programming problem:

$$Min\tilde{Z} = \tilde{5} - \tilde{6}\tilde{x}_1 + \tilde{2}\tilde{x}_1^2 - \tilde{2}\tilde{x}_1\tilde{x}_2 + \tilde{2}\tilde{x}_2^2$$

Subject to : $\tilde{x}_1 + \tilde{x}_2 \le 2$, $\tilde{x}_1, \tilde{x}_2 \ge 0$

Solution: First, we convert the inequality constraint into equation by introducing slack variable $\tilde{s}_1^2$.

Also the inequality constraints $\tilde{x}_1, \tilde{x}_2 \ge 0$. We convert them into equations by introducing slack variables $\tilde{s}_2^2$ and $\tilde{s}_3^2$. So the problem becomes

$$MinZ = \tilde{5} - \tilde{6}\tilde{x}_1 + \tilde{2}\tilde{x}_1^2 - \tilde{2}\tilde{x}_1\tilde{x}_2 + \tilde{2}\tilde{x}_2^2$$

Subject to: $\tilde{x}_1 + \tilde{x}_2 + \tilde{s}_1^2 = \tilde{2}$,

$$-\tilde{x}_1 + \tilde{s}_2^2 = 0$$

$$-\tilde{x}_2 + \tilde{s}_3^2 = 0$$

Where

$\tilde{5} = (5.8,4.4,5.2,4.2)$

$\tilde{2} = (1.3,2.1,2.4,1.5)$

$\tilde{6} = (7.3,7.5,6.6,6.8)$

$\tilde{2} = (2.4,2.6,3.2,3.3)$

$\tilde{2} = (2.2,2.3,1.6,1.8)$

$\tilde{1} = (1.3,0.8,0.9,1.5)$

$\tilde{1} = (2.4,1.8,2.2,1.6)$

$\tilde{1} = 1.4,1.6,2.8,2.4)$

$\tilde{2} = (2.3,2.2,1.8,1.5)$

$\tilde{1} = (1.3,0.2,1.1,0.4)$

$\tilde{1} = (2.3,2.4,1.4,1.6)$

$\tilde{1} = (2.5,2.7,1.9,1.8)$

$\tilde{1} = (1.4,1.5,0.3,0.2)$

$$Max\tilde{Z} = 5.2 - 5.2\tilde{x}_1 + 3.2\tilde{x}_1\tilde{x}_2 - 2.1\tilde{x}_1^2 - 1.6\tilde{x}_2^2$$

$$0.8\tilde{x}_1 + 1.8\tilde{x}_2 + 1.4\tilde{s}_1^2 = 1.5$$

$$-1.1\tilde{x}_1 + 1.4\tilde{s}_2^2 = 0,$$

$$-2.5\tilde{x}_2 + 0.3\tilde{s}_3^2 = 0$$

Construct the Lagrangian function

$$L(\tilde{x}_1,\tilde{x}_2,\tilde{s}_1,\tilde{s}_2,\tilde{s}_3,\tilde{\lambda}_1,\tilde{\lambda}_2,\tilde{\lambda}_3) = (5.2-5.2\tilde{x}_1+3.2\tilde{x}_1\tilde{x}_2-2.1\tilde{x}_1^2-1.6\tilde{x}_2^2)-\tilde{\lambda}_1(0.8\tilde{x}_1+1.8\tilde{x}_2+1.4\tilde{s}_1^2-1.5)$$
$$-\tilde{\lambda}_2(-1.1\tilde{x}_1+1.4\tilde{s}_2^2)-\tilde{\lambda}_3(-2.5\tilde{x}_2+0.3\tilde{s}_3^2)$$

By Kuhn-Tucker conditions, we get

$$-3.2x_2+4.2\tilde{x}_1+0.8\tilde{\lambda}_1-1.1\tilde{\lambda}_2=5.2$$
$$-3.2\tilde{x}_1+3.2\tilde{x}_2+1.8\tilde{\lambda}_1-2.5\tilde{\lambda}_3=0$$
$$0.8x_1+1.8x_2+1.4s_1^2=1.5$$
$$-1.1\tilde{x}_1+1.4\tilde{s}_2^2=0,$$
$$-2.5\tilde{x}_2+0.3\tilde{s}_3^2=0$$
$$\tilde{\lambda}_2\tilde{s}_2=\tilde{\lambda}_1\tilde{s}_1=\tilde{\lambda}_3\tilde{s}_3=0$$

Where $\tilde{x}_1,\tilde{x}_2,\tilde{s}_i^2,\tilde{\lambda}_i \geq 0$,i = 1,2,3 satisfying the complementary slackness conditions

$$\tilde{\lambda}_2\tilde{x}_1+\tilde{\lambda}_1\tilde{s}_1^2+\tilde{\lambda}_3\tilde{x}_2=0$$

Now, introducing the artificial variables $\tilde{a}_1,\ \tilde{a}_2 \geq 0$ the given QPP is equivalent to: $Min\tilde{Z}=0.4\tilde{a}_1+0.6\tilde{a}_2$

$$-3.2x_2+4.2\tilde{x}_1+0.8\tilde{\lambda}_1-1.1\tilde{\lambda}_2=5.2$$
$$-3.2\tilde{x}_1+3.2\tilde{x}_2+1.8\tilde{\lambda}_1-2.5\tilde{\lambda}_3=0$$
$$0.8x_1+1.8x_2+1.4s_1^2=1.5$$

c_B	BVS	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{s}_1^2$	Ratio
0.4	$\tilde{a}_1$	5.2	4.2	-3.2	0.8	-1.1	0	0.4	0	0	1.6
0.6	$\tilde{a}_2$	0	-3.2	3.2	1.8	0	-2.5	0	1.2	0	0
0	s_1^2	1.5	0.8	1.8	0	0	0	0	0	1.4	2.9
0	$\tilde{x}_1$	1.2	1	-0.8	0.2	-0.3	0	0.1	0	0	-ve
0.6	$\tilde{a}_2$	5.1	0	0.6	2.4	-1	-2.5	0.3	1.2	0	8.5
0	s_1^2	0.5	0	2.4	-0.2	0.2	0	-0.1	0	1.4	0.2
0	$\tilde{x}_1$	1.4	1	0	0.1	-0.2	0	0.1	0	0.5	14
0.6	$\tilde{a}_2$	5	0	0	2.5	-1.1	-2.5	0.3	1.2	-0.4	2
0	$\tilde{x}_2$	0.5	0	1	-0.1	0.1	0	-0.04	0	0.6	-ve
0	$\tilde{x}_1$	1.6	1	0	1	-0.2	0.1	0.1	-0.1	0.5	
0	$\tilde{\lambda}_1$	2	0	0	1	-0.4	-1	0.1	0.5	-0.2	
0	$\tilde{x}_2$	0.4	0	1	0	0.1	-0.1	-0.03	0.1	0.6	

Table-2: Comparison with Kirtiwant and et al., method

NLPP	x_1	x_2	Min Z
Kirtiwant and et al.,	1.5	0.5	0.5
FNLPP	$\tilde{x}_1$	$\tilde{x}_2$	$Min\,\tilde{Z}$
Our proposed method	1.2	0.4	0.3

From the Table 2, Objective (minimum) value obtained by our method is better than Kirtiwant and *et al.*, [10] method.

Current solution is an optimal solution $x_1 = 1.2 \;\; x_2 = 0.4$, $MaxZ = 0.3$

6.3. Problem 3: Solve the following quadratic programming problem:

$$Max\tilde{Z} = \tilde{2}\tilde{x}_1 + \tilde{3}\tilde{x}_2 - \tilde{2}\tilde{x}_1^2$$

Subject to: $\tilde{x}_1 + \tilde{x}_2 \le \tilde{2}$, $\tilde{x}_1, \tilde{x}_2 \ge 0$

Solution: First, we convert the inequality constraint into equation by introducing slack variable $\tilde{s}_1^2$. Also the inequality constraints $\tilde{x}_1, \tilde{x}_2 \ge 0$. We convert them into equations by introducing slack variables $\tilde{s}_2^2$ and $\tilde{s}_3^2$. So the problem becomes

$$Max\tilde{Z} = \tilde{2}\tilde{x}_1 + \tilde{3}\tilde{x}_2 - \tilde{2}\tilde{x}_1^2$$

Subject to: $\tilde{x}_1 + \tilde{x}_2 + \tilde{s}_1^2 = \tilde{2}$,

$$-\tilde{x}_1 + \tilde{s}_2^2 = 0$$

$$-\tilde{x}_2 + \tilde{s}_3^2 = 0$$

Where

$\tilde{2} = (2.3, 2.4, 0.8, 0.5)$

$\tilde{2} = (3.3, 3.8, 2.5, 2.7)$

$\tilde{1} = (1.3, 1.1, 2.2, 2.1)$

$\tilde{2} = (2.1, 2.4, 3.3, 3.8)$

$\tilde{1} = (1.8, 1.9, 0.4, 0.5)$

$\tilde{1} = (1.7, 1.9, 0.3, 0.4)$

$\tilde{3} = (2.4, 2.8, 3.2., 3.3)$

$\tilde{1} = (0.3, 0.4, 1.7, 1.8)$

$\tilde{1} = (1.8, 1.9, 0.9, 0.3)$

$\tilde{1} = (0.1, 0.8, 1.7, 1.4)$

$\tilde{1} = (1.9, 1.7, 2.2, 2.1)$

$$Max\tilde{Z} = 2.2\tilde{x}_1 + 2.4\tilde{x}_2 - 2.5\tilde{x}_1^2$$

$$0.3\tilde{x}_1 + 1.3\tilde{x}_2 + 1.9\tilde{s}_1^2 = 3.3$$

$$-0.1\tilde{x}_1 + 0.4\tilde{s}_2^2 = 0$$

$$-1.9\tilde{x}_2 + 0.3\tilde{s}_3^2 = 0$$

Construct the Lagrangian function

$$L(\tilde{x}_1, \tilde{x}_2, \tilde{s}_1, s_2, \tilde{s}_3, \tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3) = (2.2\tilde{x}_1 + 2.4\tilde{x}_2 - 2.5\tilde{x}_1^2) - \tilde{\lambda}_1(0.3\tilde{x}_1 + 1.3\tilde{x}_2 + 1.9\tilde{s}_1^2 - 3.3) - \tilde{\lambda}_2(-0.1\tilde{x}_1 + 0.4\tilde{s}_2^2) - \tilde{\lambda}_3(-1.9\tilde{x}_2 + 0.3\tilde{s}_3^2)$$

By Kuhn-Tucker conditions, we get

$$0.3\tilde{x}_1 + 1.3\tilde{x}_2 + 1.9\tilde{s}_1^2 = 3.3$$
$$5\tilde{x}_1 + 0.3\tilde{\lambda}_1 - 0.1\tilde{\lambda}_2 = 2.2$$
$$1.3\tilde{\lambda}_1 - 1.9\tilde{\lambda}_3 = 2.4$$
$$-0.1\tilde{x}_1 + 0.4\tilde{s}_2^2 = 0$$
$$-1.9\tilde{x}_2 + 0.3\tilde{s}_3^2 = 0$$
$$\tilde{\lambda}_2\tilde{s}_2 = \tilde{\lambda}_1\tilde{s}_1 = \tilde{\lambda}_3\tilde{s}_3 = 0$$

Where $\tilde{x}_1, \tilde{x}_2, \tilde{s}_i^2, \tilde{\lambda}_i \geq 0$, i = 1,2,3 satisfying the complementary slackness conditions

$$\tilde{\lambda}_2\tilde{x}_1 + \tilde{\lambda}_1\tilde{s}_1^2 + \tilde{\lambda}_3\tilde{x}_2 = 0$$

Now, introducing the artificial variables $\tilde{a}_1, \tilde{a}_2 \geq 0$ the given QPP is equivalent to:

$$Min\tilde{Z} = 0.7\tilde{a}_1 + 0.5\tilde{a}_2$$
$$0.3\tilde{x}_1 + 1.3\tilde{x}_2 + 1.9\tilde{s}_1^2 = 3.3$$
$$5x_1 + 0.3\lambda_1 - 0.1\lambda_2 + 0.3a_1 = 2.2$$
$$1.3\lambda_1 - 1.9\lambda_3 + 0.5a_2 = 2.4$$

c_B	BVS	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{s}_1^2$	Ratio
1	$\tilde{a}_1$	2.2	5	0	0.3	-0.1	0	0.3	0	0	
1	$\tilde{a}_2$	2.4	0	0	`1.3	0	-1.9	0	0.5	0	
0	s_1^2	3.3	0.3	1.3	0	0	0	0	0	1.9	
0	$\tilde{x}_1$	0.4	1	0	0.1	-0.02	0	0.1	0	0	
1	$\tilde{a}_2$	2.4	0	0	1.3	0	-1.9	0	0.5	0	
0	s_1^2	3.2	0	1.3	-0.03	0.01	0	-0.03	0	1.9	
0	$\tilde{x}_1$	0.4	1	0	0.1	-0.02	0	0.1	0	0	
1	$\tilde{a}_2$	2.4	0	0	1.3	0	-1.9	0	0.5	0	
0	$\tilde{x}_2$	2.5	0	1	-0.02	0.01	0	0.02	0	1.5	
0	$\tilde{x}_1$	0.2	1	0	0	-0.02	0.2	0.1	0.04	0	
0	$\tilde{\lambda}_1$	1.9	0	0	1	0	-1.5	2	0.4	0	
0	$\tilde{x}_2$	2.6	0	1	0	0.01	-0.03	0.02	0.01	1.5	

Table-3: Comparison with Kirtiwant and *et al.*, method

NLPP	x_1	x_2	Max Z
Kirtiwant and et al.,	0.5	1.5	1.5
FNLPP	$\tilde{x}_1$	$\tilde{x}_2$	$Max\,\tilde{Z}$
Our proposed method	0.2	2.6	6.58

From the Table 3, Objective (maximum) value obtained by our method is better than Kirtiwant and *et al.*, [10] Method.

Current solution is an optimal solution $\tilde{x}_1 = 0.2$ $\tilde{x}_2 = 2.6$, $Max\tilde{Z} = 6.58$

6.4. Problem 4: Solve the following quadratic programming problem:

$$Max\tilde{Z} = \tilde{2}\tilde{x}_1 + \tilde{x}_2 - \tilde{x}_1^2$$

Subject to: $2\tilde{x}_1 + 3\tilde{x}_2 \le 6$

$$2\tilde{x}_1 + \tilde{x}_2 \le 4 \;, \quad \tilde{x}_1, \tilde{x}_2 \ge 0$$

Solution: First, we convert the inequality constraint into equation by introducing slack variable $\tilde{s}_1^{\,2}$. Also the inequality constraints $\tilde{x}_1,\ \tilde{x}_2 \ge 0$. We convert them into equations by introducing slack variables $\tilde{s}_2^{\,2}$ and $\tilde{s}_3^{\,2}$. So the problem becomes

$$Max\tilde{Z} = \tilde{2}\tilde{x}_1 + \tilde{3}\tilde{x}_2 - \tilde{2}\tilde{x}_1^2$$

Subject to: $\tilde{2}\tilde{x}_1 + \tilde{3}\tilde{x}_2 + \tilde{s}_1^2 = \tilde{6}$,

$$\tilde{2}\tilde{x}_1 + \tilde{x}_2 + \tilde{s}_2^2 = \tilde{4}$$

$$-\tilde{x}_1 + \tilde{s}_3^2 = 0$$

$$-\tilde{x}_2 + \tilde{s}_4^2 = 0$$

Where

$\tilde{2} = (2.2,2.9,1.6,1.8)$

$\tilde{2} = (2.4,2.5,1.4,1.6)$

$\tilde{1} = (1.1,1.3,2.2,2.4)$

$\tilde{1} = (0.3,0.4,1.2,1.4)$

$\tilde{3} = (2.2,2.8,3.3,3.4)$

$\tilde{6} = (6.1,6.3,5.2,5.5)$

$\tilde{1} = (1.4,1.5,0.1,0.2)$

$\tilde{1} = (2.6,2.7,1.3,1.4)$

$\tilde{4} = (4.4,4.3,3.3,3.4)$

$\tilde{1} = (1.3,1.5,2.5,2.6)$

$\tilde{1} = (1.2,1.4,0.4,0.5))$

$\tilde{1} = (1.8,0.8,0.7,1.7)$

$\tilde{1} = (2.2,2.4,0.5,0.4)$

$\tilde{1} = (0.3,0.4,1.6,1.7)$

$$Max\tilde{Z} = 2.9\tilde{x}_1 + 0.4\tilde{x}_2 - 1.1\tilde{x}_1^2$$

$$2.4\tilde{x}_1 + 3.3\tilde{x}_2 + 0.4\tilde{s}_1^2 = 5.2$$

$$3.2\tilde{x}_1 + 0.1\tilde{x}_2 + 0.2\tilde{s}_2^2 = 4.3$$

$$-1.6\tilde{x}_1 + 2.7\tilde{s}_3^2 = 0$$

$$-0.5\tilde{x}_2 + 1.3\tilde{s}_4^2 = 0$$

Construct the Lagrangian function

$$L(\tilde{x}_1, \tilde{x}_2, \tilde{s}_1, s_2, \tilde{s}_3, \tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3) = (2.9\tilde{x}_1 + 0.4\tilde{x}_2 - 1.1\tilde{x}_1^2) - \tilde{\lambda}_1(3.2\tilde{x}_1 + 0.1\tilde{x}_2 + 0.2\tilde{s}_2^2 - 4.3)$$
$$-\tilde{\lambda}_2(-1.6\tilde{x}_1 + 2.7\tilde{s}_3^2) - \tilde{\lambda}_3(-0.5\tilde{x}_2 + 1.3\tilde{s}_4^2)$$

By Kuhn-Tucker conditions, we get

$$2.2\tilde{x}_1 + 2.4\tilde{\lambda}_1 + 3.2\tilde{\lambda}_2 - 1.6\tilde{\lambda}_3 = 2.9$$

$$3.3\tilde{\lambda}_1 + 0.1\tilde{\lambda}_2 + 0.5\tilde{\lambda}_4 = 0.4$$

$$2.4\tilde{x}_1 + 3.3\tilde{x}_2 + 0.4\tilde{s}_1^2 = 5.2$$
$$3.2\tilde{x}_1 + 0.1\tilde{x}_2 + 0.2\tilde{s}_2^2 = 4.3$$
$$-1.6\tilde{x}_1 + 2.7\tilde{s}_3^2 = 0$$
$$-0.5\tilde{x}_2 + 1.3\tilde{s}_4^2 = 0$$
$$\tilde{\lambda}_2\tilde{s}_2 = \tilde{\lambda}_1\tilde{s}_1 = \tilde{\lambda}_3\tilde{s}_3 = 0$$

Where $\tilde{x}_1, \tilde{x}_2, \tilde{s}_i^{\,2}, \tilde{\lambda}_i \geq 0$, i = 1, 2, 3 satisfying the complementary slackness conditions

$$\tilde{\lambda}_2\tilde{x}_1 + \tilde{\lambda}_1\tilde{s}_1^{\,2} + \tilde{\lambda}_3\tilde{x}_2 = 0$$

Now, introducing the artificial variables $\tilde{a}_1, \tilde{a}_2 \geq 0$ the given QPP is equivalent to: $Min\tilde{Z} = 0.3\tilde{a}_1 + 0.6\tilde{a}_2$

$$2.2\tilde{x}_1 + 2.4\tilde{\lambda}_1 + 3.2\tilde{\lambda}_2 - 1.6\tilde{\lambda}_3 + 0.3\tilde{a}_1 = 2.9$$
$$3.3\tilde{\lambda}_1 + 0.1\tilde{\lambda}_2 + 0.5\tilde{\lambda}_4 + 0.6\tilde{a}_2 = 0.4$$
$$2.4\tilde{x}_1 + 3.3\tilde{x}_2 + 0.4\tilde{s}_1^2 = 5.2$$
$$3.2\tilde{x}_1 + 0.1\tilde{x}_2 + 0.2\tilde{s}_2^2 = 4.3$$

c_B	BVS	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{s}_1^2$	$\tilde{s}_2^2$
0.3	$\tilde{a}_1$	2.9	2.2	0	2.4	3.2	-1.6	0.3	0	0	0
0.6	$\tilde{a}_2$	0.4	0	0	3.3	0.1	0	0	0.6	0	0
0	$\tilde{s}_1^2$	5.2	2.4	3.3	0	0	0	0	0	0.4	0
0	$\tilde{s}_2^2$	4.3	3.2	0.1	0	0	0	0	0	0	0.2
0.3	$\tilde{a}_1$	2.9	2.2	0	0	2.5	-1.6	0.3	0	0	0
0	$\tilde{\lambda}_1$	0.1	0	0	1	0.3	0	0	0.6	0	0
0	$\tilde{s}_1^2$	5.2	2.4	3.3	0	0	0	0	0	0.4	0
0	$\tilde{s}_2^2$	4.3	3.2	0.1	0	0	0	0	0	0	0.2
0.3	$\tilde{a}_1$	2.9	2.2	0	0	2.5	-1.6	0.3	0	0	0
0	$\tilde{\lambda}_1$	0.1	0	0	1	0.3	0	0	0.6	0	0
0	$\tilde{x}_2$	1.6	0.7	1	0	0	0	0	0	0.1	0
0	$\tilde{s}_2^2$	4.1	3.1	0	0	0	0	0	0	-0.01	0.2
0.3	$\tilde{a}_1$	0.04	0	0	0	2.5	-1.6	0.3	0	0.01	-0.2
0	$\tilde{\lambda}_1$	0.1	0	0	1	0.3	0	0	0	0.6	0
0	$\tilde{x}_2$	0.7	0	1	0	0	0	0	0	0.1	-0.1
0	$\tilde{x}_1$	1.3	1	0	0	0	0	0	0	-0.003	0.1
0.3	$\tilde{\lambda}_2$	0.02	0	0	0	1	-0.6	0.1	0	0.004	-0.1
0	$\tilde{\lambda}_1$	0.1	0	0	1	0	0.2	-0.03	0	-.001	.03
0	$\tilde{x}_2$	0.7	0	1	0	0	0	0	0	0.1	-0.1
0	$\tilde{x}_1$	1.3	1	0	0	0	0	0	0	-0.003	0.1

Table-4: Comparison with Kirtiwant and et al., method

NLPP	x_1	x_2	Max Z
Kirtiwant and et al.,	0.6	1.5	2.01
FNLPP	$\tilde{x}_1$	$\tilde{x}_2$	$Max\,\tilde{Z}$
Our proposed method	1.3	o.7	2.19

From the Table 4, Objective (maximum) value obtained by our method is better than Kirtiwant and *et al.*, [10] Method.

Current solution is an optimal solution $\tilde{x}_1 = 1.3\ \tilde{x}_2 = 0.7$, $Max\tilde{Z} = 2.19$

6.5. Problem 5: Solve the following quadratic programming problem:

$$Max\tilde{Z} = 4\tilde{x}_1 + 2\tilde{x}_1\tilde{x}_2 - \tilde{x}_1^2 - 2\tilde{x}_2^2$$

Subject to: $2\tilde{x}_1 + \tilde{x}_2 \le 6$,

$$\tilde{x}_1 - 4\tilde{x}_2 \le 0,\ \tilde{x}_1, \tilde{x}_2 \ge 0$$

Solution: First, we convert the inequality constraint into equation by introducing slack variable $\tilde{s}_1^2$. Also the inequality constraints $\tilde{x}_1$, $\tilde{x}_2 \ge 0$. We convert them into equations by introducing slack variables $\tilde{s}_2^2$ and $\tilde{s}_3^2$. So the problem becomes

$$Max\tilde{Z} = 4\tilde{x}_1 + 2\tilde{x}_1\tilde{x}_2 - \tilde{x}_1^2 - 2\tilde{x}_2^2$$

Subject to: $2\tilde{x}_1 + \tilde{x}_2 + s_1^2 = 6$,

$$\tilde{x}_1 - 4\tilde{x}_2 + s_2^2 = 0$$

$$-x_1 + s_3^2 = 0$$

$$-x_2 + s_4^2 = 0$$

Where

$\tilde{4} = (5.2, 5.4, 4.3, 4.4)$

$\tilde{2} = (1.8, 2.1, 2.2, 1.7)$

$\tilde{1} = (0.8, 0.9, 1.4, 1.6)$

$\tilde{1} = (1.3, 1.4, 0.9, 1.7)$

$\tilde{6} = (6.7, 6.3, 5.4, 5.5)$

$\tilde{4} = (5.8, 5.7, 4.3, 4.7)$

$\tilde{1} = (0.4, 1.7, 0.6, 1.6)$

$\tilde{1} = (1.8, 1.9, 0.8, 0.9)$

$\tilde{4} = (4.4, 4.1, 5.2, 5.8)$

$\tilde{2} = (2.4, 2.6, 3.1, 3.2)$

$\tilde{1} = (1.8, 1.9, 2.7, 2.9)$

$\tilde{1} = (1.1, 1.4, 2.3, 2.5)$

$\tilde{1} = (0.7, 0.8, 1.3, 1.7)$

$\tilde{2} = (2.7, 2.8, 1.3, 1.4)$

$\tilde{1} = (0.8, 0.9, 1.3, 1.5)$

$\tilde{1} = (0.3, 0.2, 1.7, 1.8)$

$$Max\tilde{Z} = 5.4\tilde{x}_1 + 2.4\tilde{x}_1\tilde{x}_2 - 1.7\tilde{x}_2^2 - 1.4\tilde{x}_1^2$$
$$2.4\tilde{x}_1 + 0.9\tilde{x}_2 + 1.8\tilde{s}_1^2 = 5.4$$
$$1.1\tilde{x}_1 - 4.3\tilde{x}_2 + 0.7\tilde{s}_2^2 = 0$$
$$-0.4\tilde{x}_1 + 1.3\tilde{s}_3^2 = 0$$
$$-1.8\tilde{x}_2 + 0.3\tilde{s}_4^2 = 0$$

Construct the Lagrangian function

$$L(\tilde{x}_1, \tilde{x}_2, \tilde{s}_1, s_2, \tilde{s}_3, \tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3) = (5.4\tilde{x}_1 + 2.4\tilde{x}_1\tilde{x}_2 - 1.4\tilde{x}_1^2 - 1.7\tilde{x}_2^2) - \tilde{\lambda}_1(2.4\tilde{x}_1 + 0.9\tilde{x}_2 + 1.8\tilde{s}_1^2 - 5.4)$$
$$- \tilde{\lambda}_2(1.1\tilde{x}_1 - 4.3\tilde{x}_2 + 0.7\tilde{s}_2^2) - \tilde{\lambda}_3(-0.4\tilde{x}_1 + 1.3\tilde{s}_3^2) - \tilde{\lambda}_4(-1.8\tilde{x}_2 + 0.3\tilde{s}_4^2)$$

By Kuhn-Tucker conditions, we get

$$2.8\tilde{x}_1 - 2.4\tilde{x}_2 + 2.4\tilde{\lambda}_1 + 1.1\tilde{\lambda}_2 - 0.4\tilde{\lambda}_3 = 5.4$$
$$-2.4\tilde{x}_1 - 3.4\tilde{x}_2 + 0.9\tilde{\lambda}_1 - 4.3\tilde{\lambda}_2 - 1.8\tilde{\lambda}_4 = 0$$
$$2.4\tilde{x}_1 + 0.9\tilde{x}_2 + 1.8\tilde{s}_1^2 = 5.4$$
$$1.1\tilde{x}_1 - 4.3\tilde{x}_2 + 0.7\tilde{s}_2^2 = 0$$
$$1.8\tilde{x}_2 + 0.3\tilde{s}_4^2 = 0$$
$$-0.4\tilde{x}_1 + 1.3\tilde{s}_3^2 = 0$$
$$\tilde{\lambda}_2\tilde{s}_2 = \tilde{\lambda}_1\tilde{s}_1 = \tilde{\lambda}_3\tilde{s}_3 = 0$$

Where $\tilde{x}_1, \tilde{x}_2, \tilde{s}_i^2, \tilde{\lambda}_i \geq 0$, i =1,2,3 satisfying the complementary slackness condition $\tilde{\lambda}_2\tilde{x}_1 + \tilde{\lambda}_1\tilde{s}_1^2 + \tilde{\lambda}_3\tilde{x}_2 = 0$

Now, introducing the artificial variables $\tilde{a}_1,\ \tilde{a}_2 \geq 0$ the given QPP is equivalent to: $Min\tilde{Z} = 1.2\tilde{a}_1 + 1.8\tilde{a}_2$

$$2.8\tilde{x}_1 - 2.4\tilde{x}_2 + 2.4\tilde{\lambda}_1 + 1.1\tilde{\lambda}_2 - 0.4\tilde{\lambda}_3 + 1.2a_1 = 5.4$$
$$-2.4\tilde{x}_1 - 3.4\tilde{x}_2 + 0.9\tilde{\lambda}_1 - 4.3\tilde{\lambda}_2 - 1.8\tilde{\lambda}_4 + 0.8a_2 = 0$$
$$2.4\tilde{x}_1 + 0.9\tilde{x}_2 + 1.8\tilde{s}_1^2 = 5.4$$
$$1.1\tilde{x}_1 - 4.3\tilde{x}_2 + 0.7\tilde{s}_2^2 = 0$$

c_B	BVS	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	$\tilde{a}_1$	$\tilde{a}_2$	s_1^2	s_2^2
1.2	$\tilde{a}_1$	5.4	2.8	2.4	2.4	1.1	-0.4	0	1.2	0	0	0
1.8	$\tilde{a}_2$	0	-2.4	3.4	0.9	-4.3	0	1.8	0	0.8	0	0
0	s_1^2	5.4	2.4	0.9	0	0	0	0	0	0	1.8	0
0	s_2^2	0	1.1	-4.3	0	0	0	0	0	0	0	0.7
1.2	$\tilde{a}_1$	5.4	4.5	0	1.7	4.2	-0.4	-1.2	0	-0.5	0	0
0	$\tilde{x}_2$	0	-0.7	1	0.3	-1.3	0	0.5	0	0.2	0	0
0	s_1^2	5.4	3	0	-0.3	1.2	0	-0.5	0	-0.2	1.8	0
0	s_2^2	0	-1.9	0	1.3	-5.6	0	-2.1	0	-0.9	0	0.7
0	$\tilde{x}_1$	1.2	1	0	0.4	0.9	-0.1	-0.3	0	0.1	0	0
0	$\tilde{x}_2$	0.8	0	1	0.6	-0.7	-0.1	0.3	0	0.3	0	0
0	s_1^2	1.8	0	0	-1.5	-1.5	0.3	0.4	0	-0.5	1.8	0
0	s_2^2	2.1	0	0	2.1	-3.9	-0.2	-1.5	0	-0.7	0	0.7.

Table-5: Comparison with Kirtiwant and *et al.*, method

NLPP	x_1	x_2	Min Z
Kirtiwant and et al.,	2.4	1	-6.7
FNLPP	$\tilde{x}_1$	$\tilde{x}_2$	$Min\,\tilde{Z}$
Our proposed method	1.2	0.8	-5.68

From the Table 5, Objective (maximum) value obtained by our method is better than Kirtiwant and *et al.*, [10] Method.

Current solution is an optimal solution $x_1 = 1.2$, $x_2 = 0.8$, $Min\,Z = -5.68$

7. CONCLUSION

A proposed method to obtain the solution of fuzzy nonlinear programming problem has been derived. It gives better solution than the solution of nonlinear programming problem. A number of algorithms have been developed, each applicable to specific type of FNLPP only. The number of application of fuzzy nonlinear programming is very large and it is not possible to give a comprehensive survey of all of them. An algorithm that performs well on one type of the problem may perform poorly on problem with a different structure. However, an efficient method for the solution of general FNLPP is still. This technique is useful to apply on numerical problems, reduces the labour work and save valuable time.

8. REFERENCE

1. Beale E.M.L., On quadratic programming, Naval Research Logistics Quarterly 6, (1969), pp: 227-244.
2. Bellman R.E., and Zadeh L. A., Decision-Making in Fuzzy Environment, Management Science, Vol.17, Issue 4, (1970), pp: B141 - B164.
3. Boot J.C.G., Quadratic Programming, North-Holland, Amsterdam, (1964).
4. Dantzig G. B., Linear Programming and Extensions, Princeton University Press, Princeton, (1963).
5. S.C. Fang and C.F. Hu., Linear programming with fuzzy coefficients inconstraint, Comput. Math. Appl. 37 (1999), pp: 63-76.
6. Frank M. and Wolfe P., An Algorithm for Quadratic Programming, Naval Research Logistics Quarterly 3, (1956), pp: 95-110.
7. Gupta P. K., Man Mohan., Problems in Operation Research methods and solutions, Sultan Chand and Sons, Educational Publications, New Delhi.
8. Hildreth C., A Quadratic Programming Procedure, Naval Research Logistics Quarterly 4, pp: 79-85.
9. Khobragade N. W, Lamba N. K. and Khot P. G., Alternative Approach to Wolfe's Modified Simplex Method for Quadratic Programming Problems, Int. J. Latest Trend Math, Vol.2, No.1, (2012).
10. Kirtiwant P.Ghadle and Tanaji S. Pawar, New Approach for Wolfe's Modified Simplex Method to Solve Quadratic Programming Problems, International Journal of Research in Engineering and Technology, Vol.4, Issue: 01, (2015), pp: 371-376.
11. Lalitha M and Loganathan C, An Objective Fuzzy Nonlinear Programming Problem With Symmetric Trapezoidal Fuzzy Numbers, International Journal of Mathematics and Technology, Vol.4, Issue: 01, (2015), pp: 371-376.
12. Mokhter S Bazaraa, Hanif D. Sherali and C. N. Shetty., Nonlinear Programming Theory and Algorithms, John Wiley and Sons Third edition.
13. H. Rommelfanger, R. Hanuscheck and J. Wolf, Linear programming with fuzzy objective, Fuzzy Sets and Systems 29, (1989), pp:31-48.
14. Sharma S. D.: Operation Research, Kedar Nath Ram Nath, 132, R. G. Road, Meerut-250001 (U.P.), India.
15. Terlaky T: A New Algorithm for Quadratic Programming, European Journal of Operation Research, North-Holland, Vol.32, (1987), pp: 294- 301.
16. Wolfe Philip: The Simplex Method for Quadratic Programming, The Econometric Society, Econometrica, Vol. 27, No. 3, (1959), pp: 382-398.

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