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RESOLUTION OF WEYL MODULE IN THE CASE OF PARTITION (7, 6, 3)

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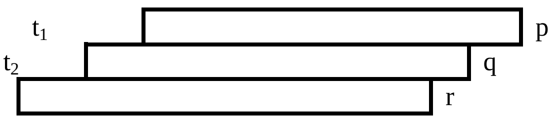
ABSTRACT

In this paper we study the relation between the resolution of weyl module $k_{(7,6,3)}F$ in characteristic-free mode and in the Lascoux mode (Characteristic zero), more precisely we obtain the Lascoux resolution of $k_{(7,6,3)}F$ in characteristic zero as an application of the resolution of $k_{(7,6,3)}F$ in characteristic - free.

Index Terms- *Resolution, weyl module, Lascoux module, divided power, characteristic free.*

I. INTRODUCTION

Let R be commutative ring with 1 and F be free R-module by D_nF we mean the divided power of degree n. The resolution Res (p. q, r, t_1, t_2) of weyl module $K_{\lambda/\mu}F$ associated to the three-roued skew-shaps (p+ t_1 + t_2, q + t_2, r)/ (t_1 +t_2, t_2, o) call the shape represented by the diagram



In general, the weyl module $k_{\lambda/\mu}\,F$ is presented hby the "box" map

$$\begin{matrix}\sum_{k>0} D_{p+t_1+k}F\otimes D_{q-t_1-k}\otimes D_rF\\ \oplus \\ \sum_{l>0} D_p\ F\otimes D_{q+t_2+l}\otimes D_{r-t_2-l}\ F\end{matrix} \xrightarrow{\ n\ } D_pF\otimes D_qF\otimes D_rF \xrightarrow{d'_{\lambda/\mu}} k_{\lambda/\mu}$$

Where the maps

$$\sum_{k>0} D_{p+t_1+k}F\otimes D_{q-t_1-k}\otimes D_rF\to D_pF\otimes D_qF\otimes D_rF$$

May be interpreted as k^{th} divided power of the place polarization from place one to place two $\left(i.e.\,\partial_{21}^{(k)}\right)$, the maps $\sum_{l>0} D_p\ F\otimes D_{q+t_2-l}\ F\ \otimes D_{r-t_2-l}\ F\to D_pF\otimes D_qF\otimes D_rF$ may be place two interpreted as l^{th} divided power of the place polarization from place two to place three $\left(i.e.\,\partial_{32}^{(l)}\right)$ [2].

We have to mention that we shall use D_n instead of D_nF to refer to divided power algebra of degree n.

II. CHARACTRISTIC-FREE RESOLUTION OF THE PARTITION (7, 6, 3)

We find the terms of the resolution of weyl module in the case of the partition (7, 6, 3), in general aterms of the resolution of weyl module in the case of a three-rowed partition (p, q, r) which appeared in [1] are

$$\mathbf{Res}([p,q;0])\otimes D_r\oplus\sum_{l\geq 0}\underline{Z}_{32}^{(l+1)}y\,\mathbf{Res}([p,q+l+1;l+1])\otimes D_{r-l-1}\oplus$$

$$\sum_{l_1\geq 0,l_2\geq l_1}\underline{Z}_{32}^{(l_2+1)}y\,\underline{Z}_{31}^{(l_1+1)}z\,\mathbf{Res}([p+l_1+1,q+l_2+1,l_2-l_1])\otimes D_{r-(l_1+l_2+2)}$$

where x, y and z stand for the separator variables, and the boundary map is $\partial_x+\partial_y+\partial_z$.

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Let again Bar$(M, A; S)$ be the free bar module on the set $S = \{x, y, z\}$ consisting of three separators x, y and z, where A is the free associative (non-commutative) algebra generated by Z_{21}, Z_{32} and Z_{31} and their divided powers with the following relation:

$$Z_{32}^{(a)} Z_{31}^{(b)} = Z_{31}^{(b)} Z_{32}^{(a)} \ \ and \ \ Z_{21}^{(a)} Z_{31}^{(b)} = Z_{31}^{(b)} Z_{21}^{(a)}$$

and the module M is the direct sum of tensor products of divided power module $D_{P1} \otimes D_{P2} \otimes D_{P3}$ for suitable p_1, p_2 and p_3 with the action of Z_{21}, Z_{32} and Z_{31} and their divided powers. we will consider the case when $p = 7, q = 6, and\ r = 3$.

We have:

$$\mathbf{Res}([7,6,0]) \otimes D_3 \oplus \sum_{l \geq 0} \underline{Z}_{32}^{(l+1)} y\, \mathbf{Res}([7,6+l+1; l+1]) \otimes D_{3-l-1} \oplus$$

$$\sum_{l_1 \geq 0, l_2 \geq l_1} \underline{Z}_{32}^{(l_2+1)}\, y \underline{Z}_{31}^{(l_1+1)}\, z\, \mathbf{Res}([7+l_1+1, 6+l_2+1; l_2-l_1]) \otimes D_{3-(l_1+l_2+2)}$$

So,

$$\sum_{l \geq 0} \underline{Z}_{32}^{(l+1)}\, y\, \mathbf{Res}([7,6+l+1; l+1]) \otimes D_{3-l-1} = \underline{Z}_{32}\, y\, \mathbf{Res}([7,7;1]) \otimes D_2 \oplus$$

$$\underline{Z}_{32}^{(2)}\, y\, \mathbf{Res}([7,8;2]) \otimes D_1 \oplus \underline{Z}_{32}^{(3)} y\, \mathbf{Res}([7,9;3]) \otimes D_0$$

and

$$\sum_{l_1 \geq 0, l_2 \geq l_1} \underline{Z}_{32}^{(l_2+1)}\, y \underline{Z}_{31}^{(l_1+1)}\, z\, \mathbf{Res}([7+l_1+1, 6+l_2+1; l_2-l_1]) \otimes D_{3-(l_1+l_2+2)}$$

$$= \underline{Z}_{32} y \underline{Z}_{31} z \mathbf{Res}([8,7;0]) \otimes D_1 \oplus \underline{Z}_{32}^{(2)}\, y\, \underline{Z}_{31}\, z\, \mathbf{Res}([8,8;1]) \otimes D_0$$

where $\underline{Z}_{32} y$ is the bar complex

$$0 \to Z_{32} y \xrightarrow{\partial_y} Z_{32} \to 0$$

$\underline{Z}_{32}^{(2)} y$ is the bar complex

$$0 \to Z_{32}\, y\, Z_{32}\, y \xrightarrow{\partial_y} Z_{32}^{(2)}\, y \xrightarrow{\partial_y} Z_{32}^{(2)} \to 0$$

$\underline{Z}_{32}^{(3)} y$ is the bar complex

$$0 \to Z_{32} y\, Z_{32} y\, Z_{32} y \xrightarrow{\partial_y} \begin{matrix} Z_{32}^{(2)} y\, Z_{32} y \\ \oplus \\ Z_{32} y\, Z_{32}^{(2)} y \end{matrix} \xrightarrow{\partial_y} Z_{32}^{(3)} y \longrightarrow Z_{32}^{(3)} \to 0$$ and $\underline{Z}_{31} z$ is the bar complex

$$0 \to Z_{31} z \xrightarrow{\partial_z} Z_{31} \to 0$$

Then in this case we have the following terms:

- In dimension Zero (M_0) we have $D_7 \otimes D_6 \otimes D_3$

- In dimension one (M_1) we have
  - $Z_{21}^{(b)} x\, D_{7+b} \otimes D_{6-b} \otimes D_3; with\ b = 1,2,3,4,5,6$
  - $Z_{32}^{(b)} x\, D_7 \otimes D_{6+b} \otimes D_{3-b}; with\ b = 1,2,3$

- In dimension two (M_2) we have the sum of the following terms:
  - $Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, D_{7+|b|} \otimes D_{6-|b|} \otimes D_3; with\ |b| = b_1 + b_2 = 2,3,4,5,6$
  - $Z_{32} y\, Z_{21}^{(b)} x\, D_{7+|b|} \otimes D_{7-b} \otimes D_2; with\ b = 2,3,4,5,6,7$
  - $Z_{32}^{(2)} y\, Z_{21}^{(b)} x\, D_{7+b} \otimes D_{8-b} \otimes D_1; with\ b = 3,4,5,6$
  - $Z_{32}^{(3)} y\, Z_{21}^{(b)} x\, D_{7+b} \otimes D_{9-b} \otimes D_0; with\ b = 4,5,6,7,8,9$
  - $Z_{32}^{(b_1)} y\, Z_{32}^{(b_2)} y\, D_7 \otimes D_{6+|b|} \otimes D_{3-|b|}; with\ |b| = b_1 + b_2 = 2,3$
  - $Z_{32}^{(b)} y\, Z_{31} z\, D_8 \otimes D_{6+b} \otimes D_{2-b}; with\ b = 1,2$

- In dimension three (M_3) we have the sum of the following terms:
  - $Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, D_{7+|b|} \otimes D_{6-|b|} \otimes D_3; with\ |b| = b_1 + b_2 + b_3 = 3,4,5,6\ \ and\ b_1 \geq 1$
  - $Z_{32} y\, Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, D_{7+|b|} \otimes D_{7-|b|} \otimes D_2; with\ |b| = b_1 + b_2 = 2,3,4,5,6,7\ \ and\ b_1 \geq 2$

- $Z_{21}^{(2)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, D_{7+|b|}\otimes D_{8-|b|}\otimes D_1; with\ |b| = b_1 + b_2 = 4,5,6,7,8\ \ and\ b_1 \geq 3$
- $Z_{32}y\, Z_{32}y\, Z_{21}^{(b)}x\, D_{7+b}\otimes D_{8-b}\otimes D_1; with\ b = 3,4,5,6,7,8$
- $Z_{32}^{(3)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, D_{7+|b|}\otimes D_{9-|b|}\otimes D_0; with\ |b| = b_1 + b_2 = 5,6,7,8,9\ \ \ and\ b_1 \geq 4$
- $Z_{32}^{(c_1)}y\, Z_{32}^{(c_2)}y\, Z_{21}^{(b)}x\ \ D_{7+b}\otimes D_{9-b}\otimes D_0; with\ c_1 + c_2 = 3\ \ and\ \ \ b = 4,5,6,7,8,9$
- $Z_{32}y\, Z_{32}y\, Z_{32}y\, D_7\otimes D_9\otimes D_0$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(b)}x\, D_{8+b}\otimes D_{7-b}\otimes D_1; with\ 1\ \ \ b = 1,2,3,4,5,6,7$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}^{(b)}x\, D_{8+b}\otimes D_{8-b}\otimes D_0; with\ \ \ b = 2,3,4,5,6,7,8$
- $Z_{32}y\, Z_{32}y\, Z_{31}z\, D_8\otimes D_8\otimes D_0$

o In dimension four (M_4) we have the sum of the following terms:

- $Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, D_{7+|b|}\otimes D_{6-|b|}\otimes D_3; with\ |b| = \sum_{i=1}^{4} b_i = 4,5,6\ and\ b_1 \geq 1$
- $Z_{32}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, D_{7+|b|}\otimes D_{7-|b|}\otimes D_2; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 + \mathrm{b}_3 = 4,5,6,7\ \text{ and }\ b_1 \geq 2$
- $Z_{32}^{(2)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, D_{7+|b|}\otimes D_{8-|b|}\otimes D_1; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 + \mathrm{b}_3 = 5,6,7,8\ \text{ and }\ b_1 \geq 3$
- $Z_{32}y\, Z_{32}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, D_{7+|b|}\otimes D_{8-|b|}\otimes D_1; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 = 4,5,6,7,8\ \text{ and }\ b_1 \geq 3$
- $Z_{32}^{(3)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, D_{7+|b|}\otimes D_{9-|b|}\otimes D_0; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 + \mathrm{b}_3 = 6,7,8,9\ \text{ and }\ b_1 \geq 4$
- $Z_{32}^{(c_1)}y\, Z_{32}^{(c_2)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, D_{7+|b|}\otimes D_{9-|b|}\otimes D_0; with\ b = \mathrm{c}_1 + \mathrm{c}_2 = 3\ \ \ |b| = \mathrm{b}_1 + \mathrm{b}_2 = 5,6,7,8,9\ \text{ and } b_1 \geq 4$
- $Z_{32}y\, Z_{32}y\, Z_{32}y\ \ Z_{21}^{(b)}x\, D_{7+b}\otimes D_{9-b}\otimes D_0\ \ ; with\ \ b = 4,5,6,7,8,9$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, D_{8+|b|}\otimes D_{7-|b|}\otimes D_1; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 = 2,3,4,5,6,7\ \text{ and }\ b_1 \geq 1$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, D_{8+|b|}\otimes D_{8-|b|}\otimes D_0; with|b| = \mathrm{b}_1 + \mathrm{b}_2 = 3,4,5,6,7,8\ \text{ and }\ b_1 \geq 2$
- $Z_{32}y\, Z_{32}y\, Z_{31}z\, Z_{21}^{(b)}x\, D_{8+b}\otimes D_{8-b}\otimes D_0; with\ b = 2,3,4,5,6,7,8$

o In dimension five (M_5) we have the sum of the following terms:

- $Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, Z_{21}^{(b_5)}x\, D_{7+|b|}\otimes D_{6-|b|}\otimes D_3; with\ |b| = \sum_{i=1}^{5} b_i = 5,6\ and\ b_1 \geq 1$
- $Z_{32}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, D_{7+|b|}\otimes D_{7-|b|}\otimes D_2; with\ |b| = \sum_{i=1}^{4} b_i = 5,6,7\ \text{ and }\ b_1 \geq 2$
- $Z_{32}^{(2)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, D_{7+|b|}\otimes D_{8-|b|}\otimes D_1; with\ |b| = \sum_{i=1}^{4} b_i = 6,7,8\ \text{ and }\ b_1 \geq 3$
- $Z_{32}y\, Z_{32}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\ \ D_{7+|b|}\otimes D_{8-|b|}\otimes D_1; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 + \mathrm{b}_3 = 5,6,7,8\ \text{ and }\ b_1 \geq 3$
- $Z_{32}^{(3)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, D_{7+|b|}\otimes D_{9-|b|}\otimes D_0; with\ |b| = \sum_{i=1}^{4} b_i = 7,8,9\ \text{ and }\ b_1 \geq 4$
- $Z_{32}^{(c_1)}y\, Z_{32}^{(c_2)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, D_{7+|b|}\otimes D_{9-|b|}\otimes D_0;$
  $with\ c_1 + c_2 = 3\ \text{ and } |b| = \mathrm{b}_1 + \mathrm{b}_2 + \mathrm{b}_3 = 6,7,8,9\ \text{ and }\ b_1 \geq 4$
- $Z_{32}y\, Z_{32}y\, Z_{32}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, D_{7+|b|}\otimes D_{9-|b|}\otimes D_0; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 = 5,6,7,8,9\ \text{ and }\ b_1 \geq 4$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, D_{8+|b|}\otimes D_{7-|b|}\otimes D_1; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 + \mathrm{b}_3 = 3,4,5,6,7\ \text{ and }\ b_1 \geq 1$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, D_{8+|b|}\otimes D_{8-|b|}\otimes D_0; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 + \mathrm{b}_3 = 4,5,6,7,8\ \text{ and }\ b_1 \geq 2$
- $Z_{32}y\, Z_{32}y\, Z_{31}z\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, D_{8+|b|}\otimes D_{8-|b|}\otimes D_0; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 = 3,4,5,6,7,8\ \text{ and }\ b_1 \geq 2$

o In dimension six (M_6) we have the sum of the following terms:

- $Z_{21}x\, Z_{21}x\, Z_{21}x\, Z_{21}x\, Z_{21}x\, Z_{21}x\, D_{13}\otimes D_0\otimes D_3$
- $Z_{32}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, Z_{21}^{(b_5)}x\, D_{7+|b|}\otimes D_{7-|b|}\otimes D_2; with\ |b| = \sum_{i=1}^{5} b_i = 6,7\ \text{ and }\ b_1 \geq 2$
- $Z_{32}^{(2)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, Z_{21}^{(b_5)}x\, D_{7+|b|}\otimes D_{8-|b|}\otimes D_1; with\ |b| = \sum_{i=1}^{5} b_i = 7,8\ \text{ and }\ b_1 \geq 3$
- $Z_{32}y\, Z_{32}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, D_{7+|b|}\otimes D_{8-|b|}\otimes D_1; with\ |b| = \sum_{i=1}^{4} b_i = 6,7,8\ \text{ and }\ b_1 \geq 3$
- $Z_{32}^{(3)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, Z_{21}^{(b_5)}x\, D_{7+|b|}\otimes D_{9-|b|}\otimes D_0; with\ |b| = \sum_{i=1}^{5} b_i = 8,9\ \ \text{ and }\ b_1 \geq 4$
- $Z_{32}^{(c_1)}y\, Z_{32}^{(c_2)}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, D_{7+|b|}\otimes D_{9-|b|}\otimes D_0; with\ \ c_1 + c_2 = 3$
  $\text{and } |b| = \sum_{i=1}^{4} b_i = 7,8,9\ \text{ and } b_1 \geq 4$
- $Z_{32}y\, Z_{32}y\, Z_{32}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, D_{7+|b|}\otimes D_{9-|b|}\otimes D_0; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 + \mathrm{b}_3 = 6,7,8,9\ \text{ and } b_1 \geq 4$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, D_{8+|b|}\otimes D_{7-|b|}\otimes D_1; with\ |b| = \sum_{i=1}^{4} b_i = 4,5,6,7\ \text{ and }\ b_1 \geq 1$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, D_{8+|b|}\otimes D_{8-|b|}\otimes D_0; with\ |b| = \sum_{i=1}^{4} b_i = 5,6,7,8\ \text{ and }\ b_1 \geq 2$
- $Z_{32}y\, Z_{32}y\, Z_{31}z\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, D_{8+|b|}\otimes D_{8-|b|}\otimes D_0; with\ |b| = \mathrm{b}_1 + \mathrm{b}_2 + \mathrm{b}_3 = 4,5,6,7,8\ \text{ and } b_1 \geq 2$

o In dimension seven (M_7) we have the sum of the following terms:

- $Z_{32}y\, Z_{21}x\, Z_{21}x\, Z_{21}x\, Z_{21}x\, Z_{21}x\, Z_{21}x\, D_{14}\otimes D_0\otimes D_2$
- $Z_{32}^{(2)}y\, Z_{21}^{(3)}x\, Z_{21}x\, Z_{21}x\, Z_{21}x\, Z_{21}x\, Z_{21}x\, D_{15}\otimes D_0\otimes D_1$
- $Z_{32}y\, Z_{32}y\, Z_{21}^{(b_1)}x\, Z_{21}^{(b_2)}x\, Z_{21}^{(b_3)}x\, Z_{21}^{(b_4)}x\, Z_{21}^{(b_5)}x\, D_{7+|b|}\otimes D_{8-|b|}\otimes D_1; with\ |b| = \sum_{i=1}^{5} b_i = 7,8\ \text{ and }\ b_1 \geq 3$

- $Z_{32}^{(3)} y\, Z_{21}^{(4)} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(c_1)} y\, Z_{32}^{(c_2)} y\, Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, Z_{21}^{(b_4)} x\, Z_{21}^{(b_5)} x\, D_{7+|b|} \otimes D_{9-|b|} \otimes D_0;$
  $with\ c_1 + c_2 = 3$ and $|b| = \sum_{i=1}^{5} b_i = 8{,}9$ and $b_1 \geq 4$
- $Z_{32} y\, Z_{32} y\, Z_{32} y\ Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, Z_{21}^{(b_4)} x\, D_{7+|b|} \otimes D_{9-|b|} \otimes D_0; with\ |b| = \sum_{i=1}^{4} b_i = 7{,}8{,}9$ and $b_1 \geq 4$
- $Z_{32} y\, Z_{32} y\, Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, Z_{21}^{(b_4)} x\, Z_{21}^{(b_5)} x\, D_{8+|b|} \otimes D_{7-|b|} \otimes D_1; with\ |b| = \sum_{i=1}^{5} b_i = 5{,}6{,}7{,}8$ and $b_1 \geq 1$
- $Z_{32}^{(2)} y\, Z_{31} z\, Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, Z_{21}^{(b_4)} x\, Z_{21}^{(b_5)} x\, D_{8+|b|} \otimes D_{8-|b|} \otimes D_1; with\ |b| = \sum_{i=1}^{5} b_i = 6{,}7{,}8$ and $b_1 \geq 2$
- $Z_{32} y\, Z_{32} y\, Z_{31} z\, Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, Z_{21}^{(b_4)} x\, D_{8+|b|} \otimes D_{8-|b|} \otimes D_0; with\ |b| = \sum_{i=1}^{4} b_i = 5{,}6{,}7{,}8$ and $b_1 \geq 2$

o In dimension eight (M_8) we have the sum of the following terms:

- $Z_{32} y\, Z_{32} y\, Z_{21}^{(3)} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)} y\, Z_{32} y\, Z_{21}^{(4)} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, D_{16} \otimes D_0 \otimes D_0$
- $Z_{32} y\, Z_{32}^{(2)} y\, Z_{21}^{(4)} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, D_{16} \otimes D_0 \otimes D_0$
- $Z_{32} y\, Z_{32} y\, Z_{32} y\, Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, Z_{21}^{(b_4)} x\, Z_{21}^{(b_5)} x\, D_{7+|b|} \otimes D_{9-|b|} \otimes D_0; with\ |b| = \sum_{i=1}^{5} b_i = 8{,}9\ and\ b_1 \geq 4$
- $Z_{32} y\, Z_{31} z\, Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, Z_{21}^{(b_4)} x\, Z_{21}^{(b_5)} x\, Z_{21}^{(b_6)} x\, D_{8+|b|} \otimes D_{7-|b|} \otimes D_1; with\ |b| = \sum_{i=1}^{6} b_i = 6{,}7$ and $b_1 \geq 1$
- $Z_{32}^{(2)} y\, Z_{31} z\, Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, Z_{21}^{(b_4)} x\, Z_{21}^{(b_5)} x\, Z_{21}^{(b_6)} x\, D_{8+|b|} \otimes D_{8-|b|} \otimes D_0; with\ |b| = \sum_{i=1}^{6} b_i = 7{,}8$ and $b_1 \geq 2$
- $Z_{32} y\, Z_{32} y\, Z_{31} z\, Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, Z_{21}^{(b_4)} x\, Z_{21}^{(b_5)} x\, D_{8+|b|} \otimes D_{8-|b|} \otimes D_0; with\ |b| = \sum_{i=1}^{5} b_i = 6{,}7{,}8$ and $b_1 \geq 2$

o In dimension nine (M_9) we have the sum of the following terms:

- $Z_{32} y\, Z_{32} y\, Z_{32} y\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, D_{16} \otimes D_0 \otimes D_0$
- $Z_{32} y\, Z_{31} z\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)} y\, Z_{31} z\, Z_{21}^{(2)} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, D_{16} \otimes D_0 \otimes D_0$
- $Z_{32} y\, Z_{32} y\, Z_{31} z\, Z_{21}^{(b_1)} x\, Z_{21}^{(b_2)} x\, Z_{21}^{(b_3)} x\, Z_{21}^{(b_4)} x\, Z_{21}^{(b_5)} x\, Z_{21}^{(b_6)} x\, D_{8+|b|} \otimes D_{8-|b|} \otimes D_0;$
  $with\ |b| = \sum_{i=1}^{6} b_i = 7{,}8$ and $b_1 \geq 2$

Finally in dimension ten (M_{10}) we have

- $Z_{32} y\, Z_{32} y\, Z_{31} z\, Z_{21}^{(2)}\ Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, Z_{21} x\, D_{16} \otimes D_0 \otimes D_0$

In [1], it is necessary to introduce a quotient of bar complex module the caplli identities relations; the proof these velation are compatible with the boundary map $\partial_x + \partial_y + \partial_z$ is complicated [1].

III. LASCOUX RESOLUTION OF THE PARTITION (7, 6, 3)

The Lascoux resolution of the weyl module associated to the partition (7, 6, 3) looks like this

$$0 \to D_9 F \otimes D_6 F \otimes D_1 F \to \begin{matrix} D_8 F \otimes D_7 F \otimes D_1 F \\ \oplus \\ D_9 F \otimes D_5 F \otimes D_2 F \end{matrix} \to \begin{matrix} D_7 F \otimes D_7 F \otimes D_2 F \\ \oplus \\ D_8 F \otimes D_5 F \otimes D_3 F \end{matrix} \to D_7 F \otimes D_6 F \otimes D_3 F \to 0$$

Were the positions of the terms of the complex are determined by the length of the permutation to which they correspond. The correspondence between the terms of the resolution above and permutations is as follows:

$D_7 F \otimes D_6 F \otimes D_3 F \leftrightarrow identity$

$D_8 F \otimes D_5 F \otimes D_3 F \leftrightarrow (12)$

$D_7 F \otimes D_7 F \otimes D_2 F \leftrightarrow (23)$

$D_9 F \otimes D_5 F \otimes D_2 F \leftrightarrow (123)$

$D_8 F \otimes D_7 F \otimes D_1 F \leftrightarrow (132)$

$D_9 F \otimes D_6 F \otimes D_1 F \leftrightarrow (13)$

Now, the terms can be presented as below, following Buchsbaum method [2].

$M_0 = A_0$

$M_1 = A_1 \oplus B_1$

$M_2 = A_2 \oplus B_2$

$M_3 = A_3 \oplus B_3$

$M_j = B_j$; for $j = 4,5,6,,7,8,9,10.$

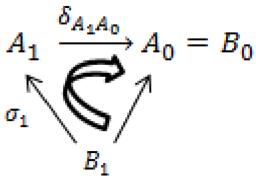
Where the A_S are the sums of the lascoux terms, and the B_S are the sums of the others.

Now, we define the map σ_1 $from\ B_1\ to\ A_1$ as follows:

- $Z_{21}^{(2)}x(v) \longmapsto \frac{1}{2}Z_{21}\ x\ \partial_{21}\ (v);\quad where\ v \in D_9 \otimes D_4 \otimes D_3$
- $Z_{21}^{(3)}x(v) \longmapsto \frac{1}{3}Z_{21}\ x\ \partial_{21}^{(2)}\ (v);\quad where\ v \in D_{10} \otimes D_3 \otimes D_3$
- $Z_{21}^{(4)}x(v) \longmapsto \frac{1}{4}Z_{21}\ x\ \partial_{21}^{(3)}\ (v);\quad where\ v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}^{(5)}x(v) \longmapsto \frac{1}{5}Z_{21}\ x\ \partial_{21}^{(4)}\ (v);\quad where\ v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}^{(6)}x(v) \longmapsto \frac{1}{6}Z_{21}\ x\ \partial_{21}^{(5)}\ (v);\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{32}^{(2)}y(v) \longmapsto \frac{1}{2}Z_{32}\ y\ \partial_{32}\ (v);\quad where\ v \in D_7 \otimes D_8 \otimes D_1$
- $Z_{32}^{(3)}y(v) \longmapsto \frac{1}{3}Z_{32}\ y\ \partial_{32}^{(2)}\ (v);\quad where\ v \in D_7 \otimes D_9 \otimes D_0$

We should point out that the map σ_1 satisfies the identity:

$$\delta_{A_1A_0}\sigma_1 = \delta_{B_1B_0}. \tag{3.1}$$



Where by $\delta_{A_1A_0}$ we mean the component of the boundary of the fat complex which conveys $A_1\ to\ A_0$. We will use notation $\delta_{A_{i+1}A_i}$, $\delta_{A_{i+1}B_i}$ etc.

Then we can define $\partial_1 : A_1 \to A_0$ as $\partial_1 = \delta_{A_1A_0}$ .It is easy to show that ∂_1 which we defined above satisfies (3.1) for example:

$$\left(\delta_{A_1A_0} o\ \sigma_1\right)\left(Z_{21}^{(4)}x\ (v)\right) = \delta_{A_1A_0}\left(\frac{1}{4}Z_{21}x\partial_{21}^{(3)}(v)\right)$$
$$= \frac{1}{4}\left(\partial_{21}\partial_{21}^{(3)}(v)\right) = \partial_{21}^{(4)}(v) = \delta_{B_1B_0}\left(Z_{21}^{(4)}x\ (v)\right)$$

At this point we are in position to define

$$\partial_2 : A_2 \to A_1 \quad as\ \partial_2 = \delta_{A_2A_1} + \sigma_1\ \delta_{A_2B_1}$$

Proposition (3.1)

The composition $\partial_1\ o\ \partial_2 = 0$

Proof: [2], [3]

$$\partial_1 \text{o}\ \partial_2(b) = \delta_{A_1A_0} o\ \left(\delta_{A_2A_1}(b) + \sigma_1 o\ \delta_{A_2B_1}(b)\right)$$
$$= \delta_{A_1A_0}\ o\ \delta_{A_2A_1}(b) + \delta_{A_1A_0} o\ \sigma_1\ \delta_{A_2B_1}(b)$$

but $\delta_{A_1A_0} o\ \sigma_1 = \delta_{B_1B_0}$ we have

$$\partial_1 \text{o}\ \partial_2(b) = \delta_{A_1A_0} o\ \left(\delta_{A_2A_1}(b) + \delta_{B_1B_2} o\ \delta_{A_2B_1}(b)\right)$$

Which equal to zero because the properties of the boundary map δ [2], so we get $\partial_1 \text{o}\ \partial_2 = 0$

Now, we define map $\sigma_2 : B_2 \to A_2$ such that

$$\delta_{A_2A_1} + \sigma_1\ o\ \delta_{B_2B_1} = \left(\delta_{A_2A_1} + \sigma_1\ o\ \delta_{A_2B_1}\right) o\ \sigma_2 \tag{3.2}$$

We define this map as follows:

- $Z_{21}x\ Z_{21}x(v) \longmapsto 0;\quad where\ v \in D_9 \otimes D_4 \otimes D_3$
- $Z_{21}^{(2)}x\ Z_{21}x(v) \longmapsto 0;\quad where\ v \in D_{10} \otimes D_3 \otimes D_3$
- $Z_{21}x\ Z_{21}^{(2)}x(v) \longmapsto 0;\quad where\ v \in D_{10} \otimes D_3 \otimes D_3$
- $Z_{21}^{(3)}x\ Z_{21}x(v) \longmapsto 0;\quad where\ v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}^{(2)}x\ Z_{21}^{(2)}x(v) \longmapsto 0;\quad where\ v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}x\ Z_{21}^{(3)}x(v) \longmapsto 0;\quad where\ v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}^{(4)}x\ Z_{21}x(v) \longmapsto 0;\quad where\ v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}^{(3)}x\ Z_{21}^{(2)}x(v) \longmapsto 0;\quad where\ v \in D_{12} \otimes D_1 \otimes D_3$

- $Z_{21}^{(2)} x\, Z_{21}^{(3)} x(v) \longmapsto 0;$ *where* $v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21} x\, Z_{21}^{(4)} x(v) \longmapsto 0;$ *where* $v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}^{(5)} x\, Z_{21} x(v) \longmapsto 0;$ *where* $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(4)} x\, Z_{21}^{(2)} x(v) \longmapsto 0;$ *where* $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)} x\, Z_{21}^{(3)} x(v) \longmapsto 0;$ *where* $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)} x\, Z_{21}^{(4)} x(v) \longmapsto 0;$ *where* $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21} x\, Z_{21}^{(5)} x(v) \longmapsto 0;$ *where* $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{32} y\, Z_{21}^{(3)} x(v) \longmapsto \frac{1}{3} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}(v);$ where $v \in D_{10} \otimes D_4 \otimes D_2$
- $Z_{32} y\, Z_{21}^{(4)} x(v) \longmapsto \frac{1}{6} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(2)}(v);$ where $v \in D_{11} \otimes D_3 \otimes D_2$
- $Z_{32} y\, Z_{21}^{(5)} x(v) \longmapsto \frac{1}{10} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(3)}(v);$ where $v \in D_{12} \otimes D_2 \otimes D_2$
- $Z_{32} y\, Z_{21}^{(6)} x(v) \longmapsto \frac{1}{15} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(4)}(v);$ where $v \in D_{13} \otimes D_1 \otimes D_2$
- $Z_{32} y\, Z_{21}^{(7)} x(v) \longmapsto \frac{1}{21} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(5)}(v);$ where $v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}^{(2)} y\, Z_{21}^{(3)} x(v) \longmapsto \frac{1}{3} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{31}(v) - \frac{1}{3} Z_{32} y\, Z_{31} z\, \partial_{21}^{(2)}(v);$ where $v \in D_{10} \otimes D_5 \otimes D_1$
- $Z_{32}^{(2)} y\, Z_{21}^{(4)} x(v) \longmapsto -\frac{1}{12} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(2)} \partial_{32}(v) - \frac{1}{2} Z_{32} y\, Z_{31} z\, \partial_{21}^{(3)}(v);$ where $v \in D_{11} \otimes D_4 \otimes D_1$
- $Z_{32}^{(2)} y\, Z_{21}^{(5)} x(v) \longmapsto -\frac{1}{30} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(3)} \partial_{32}(v) - \frac{1}{3} Z_{32} y\, Z_{31} z\, \partial_{21}^{(4)}(v);$ *where* $v \in D_{12} \otimes D_3 \otimes D_1$
- $Z_{32}^{(2)} y\, Z_{21}^{(6)} x(v) \longmapsto -\frac{1}{60} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(4)} \partial_{32}(v) - \frac{1}{4} Z_{32} y\, Z_{31} z\, \partial_{21}^{(5)}(v);$ *where* $v \in D_{13} \otimes D_2 \otimes D_1$
- $Z_{32}^{(2)} y\, Z_{21}^{(7)} x(v) \longmapsto -\frac{1}{105} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(5)} \partial_{32}(v) - \frac{1}{5} Z_{32} y\, Z_{31} z\, \partial_{21}^{(6)}(v);$ *where* $v \in D_{14} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)} y\, Z_{21}^{(8)} x(v) \longmapsto \frac{1}{42} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(5)} \partial_{31}(v);$ *where* $v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32} y\, Z_{32} y(v) \longmapsto 0;$ *where* $v \in D_7 \otimes D_8 \otimes D_1$
- $Z_{32}^{(3)} y\, Z_{21}^{(4)} x(v) \longmapsto -\frac{1}{6} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(2)} \partial_{32}^{(2)}(v) - \frac{1}{3} Z_{32} y\, Z_{31} z\, \partial_{21}^{(3)} \partial_{32}(v) + \frac{1}{3} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{31}^{(2)}(v);$
  where $v \in D_{11} \otimes D_5 \otimes D_0$
- $Z_{32}^{(3)} y\, Z_{21}^{(5)} x(v) \longmapsto \frac{1}{30} Z_{21}^{(2)} x\, Z_{31} z\, \partial_{32}\, \partial_{21}^{(2)} \partial_{31}(v) + \frac{1}{30} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(3)} \partial_{32}^{(2)}(v) - \frac{1}{15} Z_{32} y\, Z_{31} z\, \partial_{21}^{(3)} \partial_{31}(v);$
  where $v \in D_{12} \otimes D_4 \otimes D_0$
- $Z_{32}^{(3)} y\, Z_{21}^{(6)} x(v) \longmapsto \frac{1}{36} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{32}\, \partial_{21}^{(3)} \partial_{31}(v) - \frac{1}{36} Z_{32} y\, Z_{31} z\, \partial_{21}^{(5)} \partial_{32}(v) + \frac{1}{90} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(4)} \partial_{32}^{(2)}(v);$
  where $v \in D_{13} \otimes D_3 \otimes D_0$
- $Z_{32}^{(3)} y\, Z_{21}^{(7)} x(v) \longmapsto \frac{1}{210} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(3)} \partial_{31}^{(2)}(v) + \frac{1}{42} Z_{32} y\, Z_{31} z\, \partial_{21}^{(6)} \partial_{32}(v) - \frac{1}{14} Z_{32} y Z_{31} z \partial_{32} \partial_{21}^{(6)}(v);$
  where $v \in D_{14} \otimes D_2 \otimes D_0$
- $Z_{32}^{(3)} y\, Z_{21}^{(8)} x(v) \longmapsto \frac{1}{420} Z_{32} y\, Z_{21}^{(2)} x\, \partial_{21}^{(6)} \partial_{32}^{(2)}(v) - \frac{1}{3} Z_{32} y\, Z_{31} z\, \partial_{32} \partial_{21}^{(7)}(v)\ \frac{1}{30} Z_{32} y\, Z_{31} z\, \partial_{21}^{(6)} \partial_{31}(v);$
  where $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)} y\, Z_{21}^{(9)} x(v) \longmapsto -\frac{1}{18} Z_{32} y\, Z_{31} z\, \partial_{21}^{(7)} \partial_{31}(v);$ *where* $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(2)} y\, Z_{32} y(v) \longmapsto 0;$ *where* $v \in D_7 \otimes D_9 \otimes D_0$
- $Z_{32} y\, Z_{32}^{(2)} y(v) \longmapsto 0;$ *where* $v \in D_7 \otimes D_9 \otimes D_0$
- $Z_{32}^{(2)} y\, Z_{31} z(v) \longmapsto \frac{1}{3} Z_{32} y\, Z_{31} z\, \partial_{32}(v);$ *where* $v \in D_8 \otimes D_8 \otimes D_0$

It is easy to show that σ_2 which is defined above satisfies the condition (3.2), for example we chose one of them

- $\left(\delta_{B_2A_1} + \sigma_1 \delta_{B_2B_1}\right)\left(Z_{32} y\, Z_{21}^{(5)} x\,(v)\right)$; where $v \in D_{12} \otimes D_2 \otimes D_2$

$$= \sigma_1\left(Z_{21}^{(5)} x\, \partial_{32}(v)\right) + \sigma_1\left(Z_{21}^{(4)} x\, \partial_{31}(v)\right) - Z_{32} y\, \partial_{21}^{(5)}(v)$$
$$= \frac{1}{5} Z_{21} x\, \partial_{21}^{(4)} \partial_{32}(v) + \frac{1}{4} Z_{21} x\, \partial_{21}^{(3)} \partial_{31}(v) - Z_{32} y\, \partial_{21}^{(5)}(v)$$

and

$$\left(\delta_{A_2A_1} + \sigma_2 \delta_{A_2B_1}\right)\left(\frac{1}{10} Z_{32} y\ Z_{21}^{(2)} x\, \partial_{21}^{(3)}(v)\right)$$
$$= \sigma_1\left(\frac{1}{10} Z_{21}^{(2)} x\, \partial_{32}\, \partial_{32}^{(3)}(v)\right) + \frac{1}{10} Z_{21}\, x\, \partial_{31}\, \partial_{21}^{(3)}(v) - \frac{1}{10} Z_{32}\, y\, \partial_{21}^{(2)}\, \partial_{21}^{(3)}(v)$$
$$= \frac{1}{5} Z_{21} x\, \partial_{21}^{(4)}\, \partial_{32}\,(v) + \frac{3}{20} Z_{21} x\, \partial_{21}^{(3)}\, \partial_{31}\,(v) + \frac{1}{10} Z_{21} x\, \partial_{21}^{(3)}\, \partial_{31}\,(v) - Z_{32}\, y \partial_{21}^{(5)}(v)$$
$$= \frac{1}{5} Z_{21} x\, \partial_{21}^{(4)}\, \partial_{32}\,(v) + \frac{1}{4} Z_{21} x\, \partial_{21}^{(3)}\, \partial_{31}\,(v) - Z_{32} y\, \partial_{21}^{(5)}(v)$$

Proposition (3.2): we have exactness at A_i

Proof: see [2] and [3]

Now by using σ_2 we can also define

$$\partial_3 : A_3 \to A_2 \ by \ \delta_{A_3A_2} + \sigma_2 \ o \ \delta_{A_3B_2}$$

Proposition (3.3):

The composition $\partial_2 \ o \ \partial_3 = 0$

Proof: the same way used in proposition (3.1)

We need to definition of map $\sigma_3 : B_3 \to A_3$ such that

$$\delta_{B_3A_3} + \sigma_2\delta_{B_3B_2} = (\delta_{A_3A_2} + \sigma_2\delta_{A_3B_2})\sigma_3 \quad (3.3)$$

As follows:

- $Z_{21}x\, Z_{21}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{10} \otimes D_3 \otimes D_3$
- $Z_{21}^{(2)}x\, Z_{21}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}x\, Z_{21}^{(2)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}x\, Z_{21}x\, Z_{21}^{(2)}x(v) \mapsto 0;$ $\quad where\ v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}^{(3)}x\, Z_{21}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}^{(2)}x\, Z_{21}^{(2)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}^{(2)}x\, Z_{21}x\, Z_{21}^{(2)}x(v) \mapsto 0;$ $\quad where\ v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}x\, Z_{21}^{(3)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}x\, Z_{21}^{(2)}x\, Z_{21}^{(2)}x(v) \mapsto 0;$ $\quad where\ v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}x\, Z_{21}x\, Z_{21}^{(3)}x(v) \mapsto 0;$ $\quad where\ v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}^{(4)}x\, Z_{21}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}x\, Z_{21}^{(2)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}x\ \ Z_{21}x\, Z_{21}^{(2)}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}x\, Z_{21}^{(3)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}x\, Z_{21}^{(2)}x\, Z_{21}^{(2)}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}x\, Z_{21}x\, Z_{21}^{(3)}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}x\, Z_{21}^{(4)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}x\, Z_{21}^{(3)}x\, Z_{21}^{(2)}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}x\, Z_{21}^{(2)}x\, Z_{21}^{(3)}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}x\, Z_{21}x\, Z_{21}^{(4)}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{32}y\, Z_{21}^{(2)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{10} \otimes D_4 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(3)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{11} \otimes D_3 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(2)}x\, Z_{21}^{(2)}x(v) \mapsto 0;$ $\quad where\ v \in D_{11} \otimes D_3 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(4)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{12} \otimes D_2 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(3)}x\, Z_{21}^{(2)}x(v) \mapsto 0;$ $\quad where\ v \in D_{12} \otimes D_2 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(2)}x\, Z_{21}^{(3)}x(v) \mapsto 0;$ $\quad where\ v \in D_{12} \otimes D_2 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(5)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_1 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(4)}x\, Z_{21}^{(2)}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_1 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(3)}x\, Z_{21}^{(3)}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_1 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(2)}x\, Z_{21}^{(4)}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_1 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(6)}x\, Z_{21}x(v) \mapsto 0;$ $\quad where\ v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(5)}x\, Z_{21}^{(2)}x(v) \mapsto 0;$ $\quad where\ v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(4)}x\, Z_{21}^{(3)}x(v) \mapsto 0;$ $\quad where\ v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(3)}x\, Z_{21}^{(4)}x(v) \mapsto 0;$ $\quad where\ v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}y\, Z_{21}^{(2)}x\, Z_{21}^{(5)}x(v) \mapsto 0;$ $\quad where\ v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}^{(2)}y\, Z_{21}^{(3)}x\, Z_{21}x(v) \mapsto \frac{1}{3} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(2)}\,(v);$ $\quad where\ v \in D_{11} \otimes D_4 \otimes D_1$

- $Z_{32}^{(2)}y\,Z_{21}^{(4)}x\,Z_{21}x(v) \mapsto -\frac{1}{12}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\,(v);$ $\quad where\ v \in D_{12}\otimes D_3\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(3)}x\,Z_{21}^{(2)}x(v) \mapsto \frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\,(v);$ $\quad where\ v \in D_{12}\otimes D_3\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(5)}x\,Z_{21}x(v) \mapsto -\frac{1}{30}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\,(v);$ $\quad where\ v \in D_{13}\otimes D_2\otimes D_1$
- $Z_{32}^{(2)}\,yZ_{21}^{(4)}xZ_{21}^{(2)}x(v) \mapsto -\frac{1}{4}Z_{32}y\,Z_{31}zZ_{21}x\partial_{21}^{(4)}\,(v);$ $\quad where\ v \in D_{13}\otimes D_2\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(3)}x\,Z_{21}^{(3)}x(v) \mapsto \frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\,(v);$ $\quad where\ v \in D_{13}\otimes D_2\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(6)}x\,Z_{21}x(v) \mapsto -\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\,(v);$ $\quad where\ v \in D_{14}\otimes D_1\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(5)}x\,Z_{21}^{(2)}x(v) \mapsto -\frac{2}{15}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\,(v);$ $\quad where\ v \in D_{14}\otimes D_1\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(4)}x\,Z_{21}^{(3)}x(v) \mapsto -\frac{1}{2}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\,(v);$ $\quad where\ v \in D_{14}\otimes D_1\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(3)}x\,Z_{21}^{(4)}x(v) \mapsto \frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\,(v);$ $\quad where\ v \in D_{14}\otimes D_1\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(7)}x\,Z_{21}x(v) \mapsto -\frac{1}{5}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\,(v);$ $\quad where\ v \in D_{15}\otimes D_0\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(6)}x\,Z_{21}^{(2)}x(v) \mapsto -\frac{3}{4}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\,(v);$ $\quad where\ v \in D_{15}\otimes D_0\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(5)}x\,Z_{21}^{(3)}x(v) \mapsto -\frac{5}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\,(v);$ $\quad where\ v \in D_{15}\otimes D_0\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(4)}x\,Z_{21}^{(4)}x(v) \mapsto -\frac{5}{2}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\,(v);$ $\quad where\ v \in D_{15}\otimes D_0\otimes D_1$
- $Z_{32}^{(2)}y\,Z_{21}^{(3)}x\,Z_{21}^{(5)}x(v) \mapsto -Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\,(v);$ $\quad where\ v \in D_{15}\otimes D_0\otimes D_1$
- $Z_{32}y\,Z_{32}y\,Z_{21}^{(3)}x(v) \mapsto -\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}\,(v);$ $\quad where\ v \in D_{10}\otimes D_5\otimes D_1$
- $Z_{32}y\,Z_{32}y\,Z_{21}^{(4)}x(v) \mapsto -\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(2)}\,(v);$ $\quad where\ v \in D_{11}\otimes D_4\otimes D_1$
- $Z_{32}y\,Z_{32}y\,Z_{21}^{(5)}x(v) \mapsto -\frac{1}{6}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\,(v);$ $\quad where\ v \in D_{12}\otimes D_3\otimes D_1$
- $Z_{32}y\,Z_{32}y\,Z_{21}^{(6)}x(v) \mapsto -\frac{1}{10}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\,(v);$ $\quad where\ v \in D_{13}\otimes D_2\otimes D_1$
- $Z_{32}y\,Z_{32}y\,Z_{21}^{(7)}x(v) \mapsto -\frac{1}{15}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\,(v);$ $\quad where\ v \in D_{14}\otimes D_1\otimes D_1$
- $Z_{32}y\,Z_{32}y\,Z_{21}^{(8)}x(v) \mapsto 0;$ $\quad ;$ $\quad where\ v \in D_{15}\otimes D_0\otimes D_1$
- $Z_{32}^{(3)}y\,Z_{21}^{(4)}x\,Z_{21}x(v) \mapsto -\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\ \partial_{32}(v);$ $\quad where\ v \in D_{12}\otimes D_4\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(5)}x\,Z_{21}x(v) \mapsto \frac{2}{15}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\ \partial_{32}(v)$
  $+\frac{1}{30}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{10}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\ \partial_{21}^{(4)}(v);$ $\quad where\ v \in D_{13}\otimes D_3\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(4)}x\,Z_{21}^{(2)}x(v) \mapsto -\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\ \partial_{32}(v)$
  $-\frac{1}{4}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\ \partial_{21}^{(4)}(v) - \frac{1}{12}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\ \partial_{31}(v);$ $\quad where\ v \in D_{13}\otimes D_3\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(6)}x\,Z_{21}x(v) \mapsto \frac{2}{45}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\ \partial_{21}^{(5)}(v)$
  $+\frac{1}{20}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\ \partial_{31}(v) - \frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\ \partial_{32}(v);$ $\quad where\ v \in D_{14}\otimes D_2\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(5)}x\,Z_{21}^{(2)}x(v) \mapsto \frac{1}{6}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\ \partial_{21}^{(5)}(v);$ $\quad where\ v \in D_{14}\otimes D_2\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(7)}x\,Z_{21}x(v) \mapsto -\frac{1}{105}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\ \partial_{21}^{(6)}(v) + \frac{2}{105}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\ \partial_{31}(v);$
  $where\ v \in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(6)}x\,Z_{21}^{(2)}x(v) \mapsto \frac{17}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\ \partial_{21}^{(6)}(v)$
  $-\frac{7}{30}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\ \partial_{32}(v);$ $\quad where\ v \in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(5)}x\,Z_{21}^{(3)}x(v) \mapsto \frac{2}{5}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\ \partial_{21}^{(6)}(v)$
  $-\frac{2}{15}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\ \partial_{32}(v);$ $\quad where\ v \in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(4)}x\,Z_{21}^{(4)}x(v) \mapsto -\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\ \partial_{21}^{(6)}(v)$
  $-Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\ \partial_{32}(v);$ $\quad where\ v \in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(8)}x\,Z_{21}x(v) \mapsto 0;\ where\ v \in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(7)}x\,Z_{21}^{(2)}x(v) \mapsto \frac{1}{42}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v \in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(6)}x\,Z_{21}^{(3)}x(v) \mapsto \frac{7}{12}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v \in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(5)}x\,Z_{21}^{(4)}x(v) \mapsto \frac{2}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v \in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(4)}x\,Z_{21}^{(5)}x(v) \mapsto -\frac{2}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v \in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(4)}x(v) \mapsto -\frac{1}{6}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(2)}\partial_{32}(v) + \frac{1}{6}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}\ \partial_{31}(v);$ $\quad where\ v \in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(5)}x(v) \mapsto \frac{1}{20}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\ \partial_{21}^{(3)}(v)$
  $+\frac{1}{20}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(2)}\partial_{31}(v) + \frac{1}{30}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\partial_{32}(v);$ $\quad where\ v \in D_{12}\otimes D_4\otimes D_0$

- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(6)}x(v)\longmapsto\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\partial_{32}(v)$
  $+\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(4)}(v)+\frac{1}{15}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\partial_{31}(v);$ $\quad where\ v\in D_{13}\otimes D_3\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(7)}x(v)\longmapsto\frac{1}{105}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(5)}(v)-\frac{1}{420}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\partial_{31}(v);$
  $where\ v\in D_{14}\otimes D_2\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(8)}x(v)\longmapsto-\frac{1}{70}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{32}(v);$ $\quad where\ v\in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(9)}x(v)\longmapsto-\frac{1}{42}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v\in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(4)}x(v)\longmapsto-\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(2)}\partial_{32}(v);$ $\quad where\ v\in D_{11}\otimes D_5\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(5)}x(v)\longmapsto\frac{1}{15}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\partial_{32}(v)-\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(3)}(v);$ $\quad where\ v\in D_{12}\otimes D_4\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(6)}x(v)\longmapsto-\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\partial_{32}(v)\quad where\ v\in D_{13}\otimes D_3\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(7)}x(v)\longmapsto\frac{1}{42}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\partial_{32}(v)$
  $+\frac{1}{210}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\partial_{31}(v)-\frac{1}{21}\,Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(5)}(v);$ $\quad where\ v\in D_{14}\otimes D_2\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(8)}x(v)\longmapsto-\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(6)}(v)$
  $-\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\partial_{31}(v)+\frac{1}{420}\,Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{32}(v);$ $\quad where\ v\in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(9)}x(v)\longmapsto-\frac{1}{42}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v\in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}y\,Z_{32}y\,Z_{32}y(v)\longmapsto 0;$ $\quad where\ v\in D_7\otimes D_9\otimes D_0$
- $Z_{32}y\,Z_{31}z\,Z_{21}^{(2)}x(v)\longmapsto\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}(v);$ $\quad where\ v\in D_{10}\otimes D_5\otimes D_0$
- $Z_{32}y\,Z_{31}z\,Z_{21}^{(3)}x(v)\longmapsto 0;\quad where\ v\in D_{11}\otimes D_4\otimes D_1$
- $Z_{32}^{(3)}y\,Z_{21}^{(6)}x\,Z_{21}^{(2)}x(v)\longmapsto\frac{17}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(6)}(v)-\frac{7}{30}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\,\partial_{32}(v);$
  $where\ v\in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(5)}x\,Z_{21}^{(3)}x(v)\longmapsto\frac{2}{5}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(6)}(v)-\frac{2}{15}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\,\partial_{32}(v);$
  $where\ v\in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(4)}x\,Z_{21}^{(4)}x(v)\longmapsto-\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(6)}(v)-Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\,\partial_{32}(v);$
  $where\ v\in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(8)}x\,Z_{21}x(v)\longmapsto 0;\ where\ v\in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(7)}x\,Z_{21}^{(2)}x(v)\longmapsto\frac{1}{42}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v\in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(6)}x\,Z_{21}^{(3)}x(v)\longmapsto\frac{7}{12}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v\in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(5)}x\,Z_{21}^{(4)}x(v)\longmapsto\frac{2}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v\in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(3)}y\,Z_{21}^{(4)}x\,Z_{21}^{(5)}x(v)\longmapsto-\frac{2}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v\in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(4)}x(v)\longmapsto-\frac{1}{6}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(2)}\partial_{32}(v)\ +\frac{1}{6}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}\,\partial_{31}(v);$ $where\ v\in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(5)}x(v)\longmapsto\frac{1}{20}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(3)}(v)$
  $+\frac{1}{20}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(2)}\partial_{31}(v)+\frac{1}{30}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\partial_{32}(v);$ $\quad where\ v\in D_{12}\otimes D_4\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(6)}x(v)\longmapsto\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\partial_{32}(v)$
  $+\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(4)}(v)+\frac{1}{15}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\partial_{31}(v);$ $\quad where\ v\in D_{13}\otimes D_3\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(7)}x(v)\longmapsto\frac{1}{105}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(5)}(v)-\frac{1}{420}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\partial_{31}(v);$
  $where\ v\in D_{14}\otimes D_2\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(8)}x(v)\longmapsto-\frac{1}{70}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{32}(v);$ $\quad where\ v\in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}^{(2)}y\,Z_{32}y\,Z_{21}^{(9)}x(v)\longmapsto-\frac{1}{42}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v\in D_{16}\otimes D_0\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(4)}x(v)\longmapsto-\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(2)}\partial_{32}(v);$ $\quad where\ v\in D_{11}\otimes D_5\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(5)}x(v)\longmapsto\frac{1}{15}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(3)}\partial_{32}(v)-\frac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(3)}(v);$ $\quad where\ v\in D_{12}\otimes D_4\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(6)}x(v)\longmapsto-\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\partial_{32}(v);$ $\quad where\ v\in D_{13}\otimes D_3\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(7)}x(v)\longmapsto\frac{1}{42}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\partial_{32}(v)$
  $+\frac{1}{210}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(4)}\partial_{31}(v)-\frac{1}{21}\,Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(5)}(v);$ $\quad where\ v\in D_{14}\otimes D_2\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(8)}x(v)\longmapsto-\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{32}\,\partial_{21}^{(6)}(v)$
  $-\frac{1}{60}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(5)}\partial_{31}(v)+\frac{1}{420}\,Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{32}(v);$ $\quad where\ v\in D_{15}\otimes D_1\otimes D_0$
- $Z_{32}y\,Z_{32}^{(2)}y\,Z_{21}^{(9)}x(v)\longmapsto-\frac{1}{42}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v\in D_{16}\otimes D_0\otimes D_0$

- $Z_{32}y\, Z_{32}y\, Z_{32}y(v) \mapsto 0;$ $\quad where\ v \in D_7 \otimes D_9 \otimes D_0$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(2)}x(v) \mapsto \frac{1}{3} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}(v);$ $\quad where\ v \in D_{10} \otimes D_5 \otimes D_0$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(3)}x(v) \mapsto 0;$ $\quad where\ v \in D_{11} \otimes D_4 \otimes D_1$
- $Z_{32}^{(3)}y\, Z_{21}^{(6)}x\, Z_{21}^{(2)}x(v) \mapsto \frac{17}{60} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{32}\, \partial_{21}^{(6)}(v) - \frac{7}{30} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}\, \partial_{32}(v);$ $\quad where\ v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}y\, Z_{21}^{(5)}x\, Z_{21}^{(3)}x(v) \mapsto \frac{2}{5} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{32}\, \partial_{21}^{(6)}(v) - \frac{2}{15} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}\, \partial_{32}(v);$ $\quad where\ v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}y\, Z_{21}^{(4)}x\, Z_{21}^{(4)}x(v) \mapsto -\frac{1}{3} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{32}\, \partial_{21}^{(6)}(v) - Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}\, \partial_{32}(v);$ $\quad where\ v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}y\, Z_{21}^{(8)}x\, Z_{21}x(v) \mapsto 0;\ where\ v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y\, Z_{21}^{(7)}x\, Z_{21}^{(2)}x(v) \mapsto \frac{1}{42} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y\, Z_{21}^{(6)}x\, Z_{21}^{(3)}x(v) \mapsto \frac{7}{12} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y\, Z_{21}^{(5)}x\, Z_{21}^{(4)}x(v) \mapsto \frac{2}{3} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y\, Z_{21}^{(4)}x\, Z_{21}^{(5)}x(v) \mapsto -\frac{2}{3} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{32}y\, Z_{21}^{(4)}x(v) \mapsto -\frac{1}{6} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(2)}\partial_{32}(v) + \frac{1}{6} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}\, \partial_{31}(v);$ $\quad where\ v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{32}y\, Z_{21}^{(5)}x(v) \mapsto \frac{1}{20} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{32}\, \partial_{21}^{(3)}(v)$
  $+ \frac{1}{20} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(2)}\partial_{31}(v) + \frac{1}{30} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(3)}\partial_{32}(v);$ $\quad where\ v \in D_{12} \otimes D_4 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{32}y\, Z_{21}^{(6)}x(v) \mapsto \frac{1}{60} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(4)}\partial_{32}(v)$
  $+ \frac{1}{60} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{32}\, \partial_{21}^{(4)}(v) + \frac{1}{15} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(3)}\partial_{31}(v);$ $\quad where\ v \in D_{13} \otimes D_3 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{32}y\, Z_{21}^{(7)}x(v) \mapsto \frac{1}{105} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{32}\, \partial_{21}^{(5)}(v)$
  $- \frac{1}{420} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(4)}\partial_{31}(v);$ $\quad where\ v \in D_{14} \otimes D_2 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{32}y\, Z_{21}^{(8)}x(v) \mapsto -\frac{1}{70} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}\partial_{32}(v);$ $\quad where\ v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{32}y\, Z_{21}^{(9)}x(v) \mapsto -\frac{1}{42} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}\partial_{31}(v);$ $\quad where\ v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}y\, Z_{32}^{(2)}y\, Z_{21}^{(4)}x(v) \mapsto -\frac{1}{3} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(2)}\partial_{32}(v);$ $\quad where\ v \in D_{11} \otimes D_5 \otimes D_0$
- $Z_{32}y\, Z_{32}^{(2)}y\, Z_{21}^{(5)}x(v) \mapsto \frac{1}{15} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(3)}\partial_{32}(v) - \frac{1}{3} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{32}\, \partial_{21}^{(3)}(v);$ $\quad where\ v \in D_{12} \otimes D_4 \otimes D_0$
- $Z_{32}y\, Z_{32}^{(2)}y\, Z_{21}^{(6)}x(v) \mapsto -\frac{1}{60} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(4)}\partial_{32}(v);$ $\quad where\ v \in D_{13} \otimes D_3 \otimes D_0$
- $Z_{32}y\, Z_{32}^{(2)}y\, Z_{21}^{(7)}x(v) \mapsto \frac{1}{42} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(5)}\partial_{32}(v)$
  $+ \frac{1}{210} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{21} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{32}\, \partial_{21}^{(5)}(v);$ $\quad where\ v \in D_{14} \otimes D_2 \otimes D_0$
- $Z_{32}y\, Z_{32}^{(2)}y\, Z_{21}^{(8)}x(v) \mapsto -\frac{1}{60} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{32}\, \partial_{21}^{(6)}(v)$
  $- \frac{1}{60} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(5)}\partial_{31}(v) + \frac{1}{420} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}\partial_{32}(v);$ $\quad where\ v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}y\, Z_{32}y\, Z_{32}y(v) \mapsto 0;$ $\quad where\ v \in D_7 \otimes D_9 \otimes D_0$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(2)}x(v) \mapsto \frac{1}{3} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}(v);$ $\quad where\ v \in D_{10} \otimes D_5 \otimes D_0$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(3)}x(v) \mapsto 0;$ $\quad where\ v \in D_{11} \otimes D_4 \otimes D_1$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(5)}x(v) \mapsto 0;$ $\quad where\ v \in D_{13} \otimes D_2 \otimes D_1$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(6)}x(v) \mapsto 0;$ $\quad where\ v \in D_{14} \otimes D_1 \otimes D_1$
- $Z_{32}y\, Z_{31}z\, Z_{21}^{(7)}x(v) \mapsto \frac{1}{7} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(6)}(v);$ $\quad where\ v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}^{(2)}x(v) \mapsto \frac{1}{3} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{31}(v);$ $\quad where\ v \in D_{10} \otimes D_6 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}^{(3)}x(v) \mapsto -\frac{1}{6} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{32}(v) + \frac{1}{6} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}\, \partial_{31}(v);$ $\quad where\ v \in D_{11} \otimes D_5 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}^{(4)}x(v) \mapsto \frac{1}{15} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}\, \partial_{31}(v);$ $\quad where\ v \in D_{12} \otimes D_4 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}^{(5)}x(v) \mapsto \frac{1}{12} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(3)}\partial_{31}(v);$ $\quad where\ v \in D_{13} \otimes D_3 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}^{(6)}x(v) \mapsto \frac{1}{105} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{105} Z_{32}y\, Z_{31}z\, Z_{21}x\, \partial_{21}^{(5)}\partial_{32}(v);$ $\quad where\ v \in D_{14} \otimes D_2 \otimes D_0$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}^{(7)}x(v) \mapsto 0;$ $\quad where\ v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}y\, Z_{31}z\, Z_{21}^{(8)}x(v) \mapsto 0;$ $\quad where\ v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}y\, Z_{32}y\, Z_{31}z\,(v) \mapsto 0;$ $\quad where\ v \in D_8 \otimes D_8 \otimes D_0$

A gain we can show that σ_3 which defined above satisfies the condition (3.3), and here we chose one of them as an example.

- $\left(\delta_{B_3A_2}+\sigma_2\delta_{B_3B_2}\right)\left(Z_{32}^{(2)}y\,Z_{21}^{(3)}Z_{21}x(v)\right);\ where\ \ v\in D_{11}\otimes D_4\otimes D_1$

$$=\sigma_2\left(Z_{21}^{(3)}x\,Z_{21}x\ \partial_{32}^{(2)}(v)\right)+\sigma_2\left(Z_{21}^{(2)}x\,Z_{21}x\,\partial_{32}\partial_{31}(v)\right)$$
$$+\sigma_2\left(Z_{21}\,xZ_{21}\,x\,\partial_{31}^{(2)}x(v)\right)-\sigma_2\left(4\,Z_{32}^{(2)}y\,Z_{21}^{(4)}x\,(v)\right)+\sigma_2\left(Z_{32}^{(2)}y\,Z_{21}^{(3)}x\partial_{21}\right)$$
$$=\frac{1}{3}\,Z_{32}y\,Z_{21}^{(2)}x\,\partial_{21}^{(2)}\partial_{32}(v)+2Z_{32}y\,Z_{31}z\,\partial_{21}^{(3)}(v)+\frac{1}{3}\,Z_{32}y\,Z_{21}^{(2)}x\,\partial_{21}\partial_{31}(v)-Z_{32}y\,Z_{31}z\,\partial_{21}^{(3)}(v)$$
$$=\frac{1}{3}\,Z_{32}y\,Z_{21}^{(2)}x\,\partial_{21}^{(2)}\partial_{32}(v)+Z_{32}y\,Z_{31}z\,\partial_{21}^{(3)}(v)+\frac{1}{3}\,Z_{32}y\,Z_{21}^{(2)}x\,\partial_{21}\partial_{32}(v).$$

And

$$\left(\delta_{A_3A_2}+\sigma_2\delta_{A_3B_2}\right)\left(\tfrac{1}{3}Z_{32}y\,Z_{31}z\,Z_{21}x\,\partial_{21}^{(2)}(v)\right)$$
$$=-\sigma_2\left(\frac{1}{3}Z_{21}\,x\,Z_{21}x\,\partial_{32}^{(2)}\,\partial_{21}^{(2)}(v)\right)+\frac{1}{3}Z_{32}\,y\,Z_{21}^{(2)}x\,\partial_{32}\,\partial_{21}^{(2)}(v)$$
$$-\sigma_2\left(\tfrac{1}{3}\,Z_{32}y\,Z_{32}y\ \ \partial_{21}^{(2)}\,\partial_{21}^{(2)}(v)\right)+Z_{32}\,y\,Z_{31}z\,\partial_{21}^{(3)}\,(v)$$
$$=\frac{1}{3}\,Z_{32}y\,Z_{21}^{(2)}x\,\partial_{32}\,\partial_{21}^{(2)}(v)+Z_{32}\,y\,Z_{31}z\,\partial_{21}^{(3)}(v)$$
$$=\tfrac{1}{3}\,Z_{32}y\,Z_{21}^{(2)}x\ \ \partial_{21}^{(2)}\partial_{32}(v)+Z_{32}\,y\,Z_{31}z\,\partial_{21}^{(3)}(v)+\tfrac{1}{3}\,Z_{32}y\,Z_{21}^{(2)}x\,\partial_{21}\partial_{31}(v)\ .$$

So from all we have done above we the complex.

$$0\ \rightarrow\ A_3\xrightarrow{\partial_3}A_2\xrightarrow{\partial_2}A_1\xrightarrow{\partial_1}A_0 \qquad (3.4)$$

Where ∂_i defied as follows:

- $\partial_1\left(Z_{21}x(v)\right)=\partial_{21}(v)$
- $\partial_1\left(Z_{32}y(v)\right)=\partial_{32}(v)$
- $\partial_2(Z_{32}\,y\,Z_{21}^{(2)}\,xv)=\frac{1}{2}Z_{21}\,x\,\partial_{21}\,\partial_{32}(v)+Z_{21}\,x\,\partial_{31}(v)-Z_{32}\,y\,\partial_{21}^{(2)}(v)$
- $\partial_2(Z_{32}\,y\,Z_{31}\,zv)=\frac{1}{2}Z_{32}\,y\,\partial_{32}\,\partial_{21}(v)+Z_{21}\,x\,\partial_{32}^{(2)}(v)-Z_{32}\,y\,\partial_{32}^{(2)}(v)$ finally, we defined the map ∂_3 by :
- $\partial_3(Z_{32}\,y\,Z_{31}\,z\,Z_{21}\,xv)=Z_{32}\,y\,Z_{21}^{(2)}\,x\,\partial_{32}(v)+Z_{32}\,y\,Z_{31}\,z\,\partial_{21}(v)$ Proposition (3.4).

The complex $0\rightarrow A_3\xrightarrow{\partial_3}A_2\xrightarrow{\partial_2}A_1\xrightarrow{\partial_1}A_0\rightarrow k_{(7,6,3)}$ is exact

Prove: see [2] and [3]

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