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FIXED POINT THEOREM IN $\mathcal{L}$ -FM SPACE SATISFYING A CONTRACTIVE CONDITION OF INTEGRAL FORM

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ABSTRACT

In this paper, we prove a fixed point theorem in L- FM space satisfying a Contrractive condition of integral form and also it is a generalization of results.

***Mathematical classification**: 47A62, 47A63.*

***Keywords:** fixed point, fixed point theorem, L-Fm space, contraction, integral form.*

1. INTRODUCTION

In 1965, the concept of fuzzy sets was introduced initially by Zadeh [186].Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. In particular, George and Veeramani [61] have introduced and studied a notion of fuzzy metric space with the help of continuous t-norms, which constitutes a slight but appealing modification of the one due to Kramosil and Michalek [89]. Deschrijver and Kerre [44] have shown that intuitionistic fuzzy sets can also be seen as L- fuzzy sets in the sense of Goguen [6]. Using the idea of L-fuzzy sets [6], Saadati *et al.* introduced the notion of L-fuzzy metric spaces with the help of continuous t-norms as a generalization of fuzzy metric space due to George and Veeramani and intuitionistic fuzzy metric space due to Park and Saadati and prove a common fixed point theorem for a pair of commuting mappings. Later on in he introduce the notion of uniform continuity and equicontinuity in an L-fuzzy metric space and prove Uniform continuity theorem for L-fuzzy metric space and prove Ascoli–Arzela theorem for L-fuzzy metric space.

Branciari [30] initiated the study of contractive conditions of integral type in 2002 and give integral version of Banach contraction principle which was further generalized by Rhoades Several common fixed point theorems for a family of four mappings satisfying some contractive conditions of integral type were established in [14, 47] and [12].

In metric fixed point theory, various mathematicians weakened the notion of commutativity by introducing the notions of weak commutativity, compatibility and weak compatibility and produced a number of fixed point theorems using these notions. It is worth to mention that every pair of commuting self-maps is weakly commuting, each pair of weakly commuting self-maps is compatible and each pair of compatible self-maps is weak compatible but the reverse is not always true.

The main object of this paper is to prove common fixed point theorem in $\mathcal{L}$ -fuzzy metric space for weakly compatible mappings satisfying integral type contractive condition and property (C). Which is a generalization of some results Adibi *et al.* [5] for this first, we recall some definitions and known results that will be used in the sequel.

Definition 1.1: Let $\mathcal{L} = (\mathrm{L}, \leq_{\mathrm{L}})$ be a complete lattice, and U a nonempty set called a universe. An $\mathcal{L}$ -fuzzy set $\mathcal{A}$ on U is defined as a mapping:U $\rightarrow$ L. For each u in U, $\mathcal{A}(\mathrm{u})$ represents the degree (in L) to which u satisfies $\mathcal{A}$.

Lemma 1.2: Consider the set L^* and the operation $\leq_{\mathrm{L}^*}$ defined by:

$$\mathrm{L}^* = \{(\mathrm{x}_1, \mathrm{x}_2) : (\mathrm{x}_1, \mathrm{x}_2) \in [0,1]^2 \text{ and } \mathrm{x}_1 + \mathrm{x}_2 \leq 1\},$$
$$(\mathrm{x}_1, \mathrm{x}_2) \leq_{\mathrm{L}^*} (\mathrm{y}_1, \mathrm{y}_2) \Leftrightarrow \mathrm{x}_1 \leq \mathrm{y}_1 \text{ and } \mathrm{x}_2 \geq \mathrm{y}_2,$$

for every $(\mathrm{x}_1, \mathrm{x}_2), (\mathrm{y}_1, \mathrm{y}_2) \in \mathrm{L}^*$. Then $(\mathrm{L}^*, \leq_{\mathrm{L}^*})$ is a complete lattice.

Classically, a triangular norm T on $([0,1], \leq)$ is defined as an increasing, commutative, associative mapping $\mathrm{T} : [0,1]^2 \rightarrow [0,1]$ satisfying $\mathrm{T}(1, \mathrm{x}) = \mathrm{x}$, for all $\mathrm{x} \in [0,1]$.

These definitions can be straightforwardly extended to any lattice $\mathcal{L} = (\mathrm{L}, \leq_{\mathrm{L}})$. Define first

$$0_{\mathcal{L}} = \inf \mathrm{L} \text{ and } 1_{\mathcal{L}} = \sup \mathrm{L}.$$

Definition 1.3: A triangular norm (t-norm) on $\mathcal{L}$ is a mapping $\mathrm{T}: \mathrm{L}^2 \to \mathrm{L}$ satisfying the following conditions:
1.3 (i) $(\forall x \in \mathrm{L})(\mathrm{T}(x, 1_{\mathcal{L}}) = x)$
1.3 (ii) $(\forall (x,y) \in \mathrm{L}^2)(\mathrm{T}(x,y) = \mathrm{T}(y,x))$
1.3 (iii) $(\forall (x,y,z) \in \mathrm{L}^3)(\mathrm{T}(x, \mathrm{T}(y,z)) = \mathrm{T}(\mathrm{T}(x,y),z))$
1.3 (iv) $(\forall (x,x',y,y') \in \mathrm{L}^4)(x \leq_{\mathrm{L}} x' \text{ and } y \leq_{\mathrm{L}} y' \Rightarrow \mathrm{T}(x,y) \leq_{\mathrm{L}} \mathrm{T}(x',y'))$.

Definition 1.4: A t-norm T on $\mathcal{L}$ is said to be continuous if for any $x, y \in \mathcal{L}$ and any sequence $\{x_n\}$ and $\{y_n\}$ which converge to x and y we have

$$\lim_{n\to\infty} \mathrm{T}(x_n, y_n) = \mathrm{T}(x,y).$$

Example 1.5: $\mathrm{T}(x,y) = \min(x,y)$ and $\mathrm{T}(x,y) = xy$ are two continuous t- norms on $[0,1]$. A t-norm can also be defined recursively as an $(n+1)$ -ary operation $(n \in \mathrm{N})$ by $\mathrm{T}^1 = \mathrm{T}$ and

$$\mathrm{T}^n(x_1, x_2, \ldots\ldots, x_{n+1}) = \mathrm{T}(\mathrm{T}^{n-1}(x_1, x_2, \ldots\ldots, x_n), x_{n+1}) \text{ for } n \geq 2 \text{ and } x_i \in \mathrm{L},\ 1 \leq i \leq n+1.$$

Definition 1.6: A negation on $\mathcal{L}$ is any decreasing mapping $\mathrm{N}: \mathcal{L} \to \mathcal{L}$ satisfying

$$\mathrm{N}(0_{\mathcal{L}}) = 1_{\mathcal{L}} \text{ and } \mathrm{N}(1_{\mathcal{L}}) = 0_{\mathcal{L}}.$$

If $\mathrm{N}(\mathrm{N}(x)) = x$, for all $x \in \mathcal{L}$, then N is called an involutive negation is fixed.
If, for all $\in [0,1]$, $\mathrm{N}_s(x) = 1 - x$, we say that N_s is the standard negation on $([0,1], \leq)$.

Definition 1.7: The 3-tuple $(\mathrm{X}, \mathrm{M}, \mathrm{T})$ is said to be an $\mathcal{L}$ -fuzzy metric space if X is an arbitrary (non-empty) set, T is a continuous t-norm on $\mathcal{L}$ and M is an $\mathcal{L}$ -fuzzy set on $\mathrm{X}^2 \times (0,\infty)$ satisfying the following conditions for all x, y, z $\in$ X and $s, t > 0$.
1.7 (i) $\mathrm{M}(x,y,t) >_{\mathrm{L}} 0_{\mathcal{L}}$
1.7 (ii) $\mathrm{M}(x,y,t) = 1_{\mathcal{L}}$ for all $t > 0$ if and only if $x = y$
1.7 (iii) $\mathrm{M}(x,y,t) = \mathrm{M}(y,x,t)$
1.7 (iv) $\mathrm{T}(\mathrm{M}(x,y,t), \mathrm{M}(y,z,s)) \leq_{\mathrm{L}} \mathrm{M}(x,z,t+s)$ for all $x,y,z \in \mathrm{X}$ and $s,t > 0$
1.7 (v) $\mathrm{M}(x,y,.): (0,\infty) \to \mathrm{L}$ is continuous and $\lim_{t\to\infty} \mathrm{M}(x,y,t) = 1_{\mathcal{L}}$

In this case M is called an $\mathcal{L}$ -fuzzy metric.

Definition 1.8: Let $(\mathrm{X}, \mathrm{M}, \mathrm{T})$ be an $\mathcal{L}$ -fuzzy metric space, For $t \in (0,\infty)$ we define the open ball $\mathrm{B}(x,r,t)$ with center $x \in \mathrm{X}$ and radius $r \in \mathrm{L}\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ is defined by

$$\mathrm{B}(x,r,t) = \{y \in \mathrm{X} : \mathrm{M}(x,y,t) >_{\mathrm{L}} \mathrm{N}(r)\}.$$

A subset $\mathrm{A} \subseteq \mathrm{X}$ is called open if for each $x \in \mathrm{A}$, there exists $t > 0$ and $r \in \mathrm{L}\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$,such that $\mathrm{B}(x,r,t) \subseteq \mathrm{A}$.

Let τ_{M} denote the family of all open subsets of X. Then τ_{M} is called the topology induced by the $\mathcal{L}$ -fuzzy metric M.

Lemma 1.9: Let $(\mathrm{X}, \mathrm{M}, \mathrm{T})$ be an $\mathcal{L}$ -fuzzy metric space. Then, $\mathrm{M}(x,y,t)$ is non-decreasing with respect to t, for all $x, y \in \mathrm{X}$.

Definition 1.10: Let $(\mathrm{X}, \mathrm{M}, \mathrm{T})$ be an $\mathcal{L}$ -fuzzy metric space and $\{x_n\}$ be a sequence in X.
(1) $\{x_n\}$ is said to be convergent to a point $x \in \mathrm{X}$ (denoted by $\lim_{n\to\infty} x_n = x$) if $\lim_{n\to\infty} \mathrm{M}(x, x_n, t) = 1_{\mathcal{L}}$ for all $t > 0$.
(2) $\{x_n\}$ is called a Cauchy sequence if for each $\varepsilon \in \mathrm{L}\backslash\{0_{\mathcal{L}}\}$ and $t > 0$, there exists $n_0 \in \mathbf{N}$, such that $\mathrm{M}(x_n, x_m, t) >_{\mathrm{L}} \mathrm{N}(\varepsilon)$ for all $m \geq n \geq n_0$, $(n \geq m \geq n_0)$.
(3) A $\mathcal{L}$ -fuzzy metric in which every Cauchy sequence is convergent is said to be complete. Hence forth, we assume that T is a continuous t-norm on the lattice L, such that for every $\mu \in \mathrm{L}\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, there is a $\lambda \in \mathrm{L}\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, such that

$$\mathrm{T}^{n-1}(\mathrm{N}(\lambda), \ldots, \mathrm{N}(\lambda)) \geq_{\mathrm{L}} \mathrm{N}(\mu).$$

Definition 1.11: Let $(\mathrm{X}, \mathrm{M}, \mathrm{T})$ be an $\mathcal{L}$ -fuzzy metric space. M is said to be continuous functions on $\mathrm{X}^2 \times (0,\infty)$ if

$$\lim_{n\to\infty} \mathrm{M}(x_n, y_n, t_n) = \mathrm{M}(x,y,t),$$

whenever a sequence $\{(x_n, y_n, t_n)\}$ in $\mathrm{X}^2 \times (0,\infty)$ converges to a point $(x, y, t \in \mathrm{X}^2 \times (0,\infty)$

i.e. $\lim_{n\to\infty} \mathrm{M}(x_n, x, t) = \lim_{n\to\infty} \mathrm{M}(y_n, y, t) = 1_{\mathcal{L}}$ and $\lim_{n\to\infty} \mathrm{M}(x, y, t_n) = \mathrm{M}(x,y,t)$.

Lemma 1.12: Let (X, M, T) be an $\mathcal{L}$ -fuzzy metric space. Then M is continuous functions on $X^2 \times (0, \infty)$.

Lemma 1.13: Let (X, M, T) be an $\mathcal{L}$ -fuzzy metric space. If we define $E_{\lambda,M}: X^2 \to \mathbf{R}^+ \cup \{0\}$ by

$$E_{\lambda,M}(x, y) = \inf\{t > 0 : M(x, y, t) >_L L\, N(\lambda)\} \text{ for all } \lambda \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}, \text{ and } x, y \in X, \text{ then}$$

(1) For all $\mu \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ there exists $\lambda \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$,, such that

$$E_{\mu,M}(x_1, x_n) \leq E_{\lambda,M}(x_1, x_2) + E_{\lambda,M}(x_2, x_3) + \cdots + E_{\lambda,M}(x_{n-1}, x_n)$$

for all $x_1, x_2 \cdot \cdot \cdot, x_n \in X$.

(2) The sequence$\{x_n\}_{n\in N}$ is convergent to x w.r.t. $\mathcal{L}$ -fuzzy metric M if and only if $E_{\lambda,M}(x_n, x) \to 0$.

Also the sequence $\{x_n\}_{n\in N}$ is Cauchy w.r.t. $\mathcal{L}$ -fuzzy metric M if and only if it is Cauchy with $E_{\lambda,M}$.

We shall need the following lemma for proof of our main theorem:

Lemma 1.14: Let (X, M, T) be a $\mathcal{L}$ -fuzzy metric space. If

$$M(x_n, x_{n+1}, t) \geq_L M(x_0, x_1, k^n t)$$

for some $k > 1$ and for every $n \in \mathbf{N}$. Then sequence $\{x_n\}$ is a Cauchy sequence.

Definition 1.15: We say that the $\mathcal{L}$ -fuzzy metric space (X, M, T) has property(C), if it satisfies the following condition:

$$M(x, y, t) = C,$$

for all $t > 0 \Longrightarrow C = 1_{\mathcal{L}}$.

Definition 1.16: Let S and T be two mappings from an $\mathcal{L}$ -fuzzy metric space (X, M, T) into itself and $\{x_n\}$ be a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$$

for some $z \in X$. Then the mapping S and T are said to be compatible if

$$\lim_{n\to\infty} M(STx_n, TSx_n, t) = 1_{\mathcal{L}} \text{ for all } t > 0$$

Definition 1.17: Let S and T be mappings from an $\mathcal{L}$ -fuzzy metric space (X, M, T) into itself. The maps S and T are said to be weakly compatible if they commute at their coincidence points, i.e. if $Sp = Tp$ for some $p \in X$, then

$$STp = TSp$$

Proposition 1.18: Self mappings S and T of an $\mathcal{L}$ -fuzzy metric space (X, M, T) are compatible then they are weakly compatible.

In fact Branciari give a following Integral contractive type condition

For a given $\varepsilon > 0$, there exists a real number $c \in (0, 1)$ and a locally Lebesgue-integrable function $g: [0, \infty) \to [0, \infty)$ Such that

$$\int_0^{d(fx,fy,t)} g(t)dt \leq_c \int_0^{d(x,y)} g(t)dt$$

and $\int_0^{\varepsilon} g(t)dt > 0$ for all $x, y \in X$ and for each $\varepsilon > 0$.

Also, Branciari-Integral contractive type condition is a generalization of Banach contraction map if $g(t) = 1$ for all $t \geq 0$.

2 MAIN RESULTS

Theorem 2.1: Let A, B, S and T be self mappings of a compete $\mathcal{L}$ fuzzy metric space (X, M, T) which has property (C), satisfying:

2.1 (I) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$ and $T(X)$, $S(X)$ are two closed subsets of X.

2.1 (II)The pairs (A, S) and (B, T) are weak compatible.

2.1 (III) $\int_0^{M(Ax,By,t)} \emptyset(t)dt \geq_L \int_0^{M(Sx,Ty,kt)} \emptyset(t)dt$

for every $x, y \in X$ and some $k > 1$. Where $\varphi: R^+ \to R$ is a Lebesgue-integrable mapping which is summable, nonnegative and such that

$$\int_0^{\varepsilon} \emptyset(t)dt > 0, \varepsilon > 0$$

Then A, B, S and T have a unique common fixed point in X.

Proof: Let $x_0 \in X$ be an arbitrary point in X. By 2.1(I), there is $x_1, x_2 \in X$ such that

$$y_0 = Ax_0 = Tx_1,$$
$$y_1 = Bx_1 = Sx_2$$

Inductively, construct sequences $\{y_n\}$and $\{x_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Tx_{2n+1},$$
$$y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$$

for n= 0, 1, 2…………

Now, we prove that $\{y_n\}$ is a Cauchy sequence.

Let $d_m(t) = M(y_m, y_{m+1}, t)$, $t > 0$. Then, we have

$$\int_0^{d2n(t)} \emptyset(t)dt = \int_0^{M(y_{2n}, y_{2n+1}, t)} \emptyset(t)dt$$
$$= \int_0^{M(Ax_{2n}, Bx_{2n+1}, t)} \emptyset(t)dt$$
$$\geq_L \int_0^{M(Sx_{2n}, Tx_{2n+1}, kt)} \emptyset(t)dt$$
$$= \int_0^{M(y_{2n-1}, y_{2n}, kt)} \emptyset(t)dt$$
$$= \int_0^{d_{2n-1}(kt)} \emptyset(t)dt.$$

Thus

$$d_{2n}(t) \geq_L d_{2n-1}(kt)$$

for every $m = 2n \in \mathbb{N}$ and $\forall t > 0$.

Similarly for an odd integer m = 2n+1, we have $d_{2n+1}(t) \geq_L d_{2n}(kt)$.

Hence for every $n \in \mathbb{N}$, we have

$$d_n(t) \geq_L d_{n-1}(kt).$$

That is,

$$\int_0^{M(y_n, y_{n+1}, t)} \emptyset(t)dt \geq_L \int_0^{M(y_{n-1}, y_n, kt)} \emptyset(t)dt \geq_L$$
$$\geq_L \int_0^{M(y_0, y_1, k^n t)} \emptyset(t)dt..$$

So, by Lemma 1.14, $\{y_n\}$ is Cauchy and the completeness of X implies $\{y_n\}$ converges to y in X. That is

$$\lim_{n\to\infty} y_n = y$$
$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Sx_{2n+2} = y.$$

As $B(X) \subseteq S(X)$, there is $u \in X$ such that $Su = y$.

By (iii), we have

$$\int_0^{M(Au, Bx_{2n+1}, t)} \emptyset(t)dt \geq_L \int_0^{M(Su, Tx_{2n+1}, kt)} \emptyset(t)dt.$$

Since M is continuous, we get(whenever $n \to \infty$ in the above inequality):

$$\int_0^{M(Au, y, t)} \emptyset(t)dt \geq_L \int_0^{M(y, y, kt)} \emptyset(t)dt = 1_{\mathcal{L}} .$$

Thus $M(Au, y, t) = 1_{\mathcal{L}}$,

i.e. $Au = y$.

Therefore, $Au = Su = y$.

Since $A(X) \subseteq T(X)$, there is $v \in X$ such that $Tv = y$.

Thus,

$$\int_0^{M(y, Bv, t)} \emptyset(t)dt = \int_0^{M(Au, Bv, t)} \emptyset(t)dt.$$
$$\geq_L \int_0^{M(Su, Tv, kt)} \emptyset(t)dt$$
$$= 1_{\mathcal{L}}$$

Hence $Tv = Bv = Su = y$.

Since $(A, S)$is weak compatible, we conclude that

$$ASu = SAu,$$

that is

$$Ay = Sy.$$

Also (B, T) is weak compatible then,

$$TBv = BTv$$

that is

$$Ty = By$$

We now prove that

$$Ay = y$$

By 5.2.1(III), we have

$$\int_0^{M(Ay,y,t)} \emptyset(t)dt = \int_0^{M(Ay,Bv,t)} \emptyset(t)dt. \geq_L \int_0^{M(Sy,Tv,kt)} \emptyset(t)dt \geq_L \int_0^{M(Ay,y,k^n t)} \emptyset(t)dt.$$

On the other hand, from Lemma 1.9 we have that

$$\int_0^{M(Ay,y,t)} \emptyset(t)dt \leq_L \int_0^{M(Ay,y,k^n t)} \emptyset(t)dt.$$

Hence,

$$M(Ay, y, t) = C \text{ for all } t > 0.$$

Since (X, M, T) has property(C) it follows that

$$C = 1_{\mathcal{L}},$$

i.e.,

$$Ay = y,$$

Therefore

$$Ay = Sy = y.$$

Similarly we prove that

$$By = y.$$

By 2.1(III), we have

$$\int_0^{M(y,By,t)} \emptyset(t)dt = \int_0^{M(Ay,By,t)} \emptyset(t)dt. \geq_L \int_0^{M(Sy,Ty,kt)} \emptyset(t)dt = \int_0^{M(y,By,kt)} \emptyset(t)dt \geq_L \int_0^{M(y,By,k^n t)} \emptyset(t)dt.$$

On the other hand, from Lemma 1.9 we have that

$$\int_0^{M(y,By,t)} \emptyset(t)dt \leq_L \int_0^{M(y,by,k^n t)} \emptyset(t)dt.$$

Hence, $M(y, By, t) = C \ \forall\, t > 0$

Since (X, M, T) has property (C), it follows that $C = 1_{\mathcal{L}}$,
i.e. $By = y$.

Therefore

$$Ay = By = Sy = Ty = y.$$

i.e., y is a comman fixed point of A, B , S and T.

Uniqueness: Let x be another comman fixed point of A, B , S and T

i.e., $x = Ax = Bx = Sx = Tx$.
Hence

$$\int_0^{M(y,x,t)} \emptyset(t)dt = \int_0^{M(y,Bx,t)} \emptyset(t)dt. \geq_L \int_0^{M(Sy,Tx,kt)} \emptyset(t)dt = \int_0^{M(y,x,kt)} \emptyset(t)dt \geq_L \int_0^{M(y,x,k^n t)} \emptyset(t)dt.$$

On the other hand, from Lemma 1.9 we have that

$$\int_0^{M(y,x,t)} \emptyset(t)dt \leq_L \int_0^{M(y,x,k^n t)} \emptyset(t)dt.$$

Hence, $M(y,x,t) = C \ \ \forall\, t > 0.$

Since (X, M, T) has property (C), it follows that $C = 1_{\mathcal{L}}$,

i.e. $y = x.$

Therefore, y is the unique comman fixed point of self maps A, B, S and T.

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