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TRANSIENT AND NUMERICAL SOLUTION OF A BALKING QUEUEING MODEL WITH FEEDBACK AND HETEROGENEOUS SERVERS

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ABSTRACT

In this paper, the transient-state and numerical solution of an M/M/2 queue is found where the customers are arriving in Poisson fashion and are subject to balking. Servers are having different service rates with service times exponentially distributed. The generating function of the time dependent queue length probabilities and the numerical solution for the system are derived and probabilities are plotted graphically. Some important special cases are also derived.

Keywords: *Feedback, balking, heterogeneous servers, Laplace transformation, generating function.*

INTRODUCTION

Queueing Theory was developed to provide models for predicting behavior of systems which are providing service to randomly arising demands. The research on Queueing Theory has been extensively developed due to its significance in decision making problems. Queueing models are usually limited to steady state analysis. This limits the quality and applicability of queueing models. As in real situations traffic intensity is constantly varying, so the determination of transient solution is very much essential in analyzing the behavior of the system. There are methods that have been developed for obtaining transient solution of the queues some of these are generating function method by Bailey (1954), combinatorial method by Champernowne (1956) and difference equation method by Conolly (1958).

Heterogeneity in service is a common feature of many real multi-server queueing situations. This allows customer to receive different quality of service. As a result, queues with heterogeneous servers received considerable attention in the literature. Balking and feedback are considered to study broader perspective of customer behavior. In the case of balking, immediately on arrival a customer may decide not to join the queue perhaps because of long queue length or because of any other information on the length of the service. The notion of customer’s impatience appears in the queueing theory in the works of Haight (1957). A markovian queueing system with balking and two heterogeneous servers has been discussed by Singh (1970). In case of feedback, a dissatisfied customer retries for service. The dissatisfaction of the customer may be due to poor quality of service or due to incomplete service. In that case customer retries for service. The customer rejoins the tail of original queue for repeating the service for second time. Thangraj and Vanitha (2009) obtained transient solution of M/M/1 feedback queue with catastrophes.

In the present paper, a markovian queueing system with balking and two heterogeneous servers has been discussed, where customers arrive according to Poisson process and served one by one on FIFO basis. All incoming customers are given first essential service whereas some of them may demand service for the second time. The transient state and numerical solution of the problem is obtained.

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The queueing system studied in this paper is governed by the following assumptions

1. Arrivals are Poisson with parameter λ and the service times are exponentially distributed with parameter μ_1 and μ_2 for the first and second channel respectively.
2. When both the servers are empty, an arriving customer joins first channel with probability a_1 and second channel with probability a_2 so that $a_1 + a_2 = 1$.
3. The probability of joining the customer to the system is p and that of leaving the system is q for the customer getting service for the first time, so that $p + q = 1$. However the customer will have to leave the system after getting second service.
4. The probability that the customer departs the service channel for the first time is assumed to be c_1 and that for the second time is c_2, so that $c_1 + c_2 = 1$.
5. The arriving customer joins the queue definitely, if the number of customer in the system is less than 'k'. It may join with probability β or balks the queue with probability (1-β) if the number of customer in the system is ≥ k.
6. The waiting space is infinite.
7. The stochastic processes involved, viz.
   a) arrivals of customer.
   b) departure of customer.are statistically independent.

Definitions

$P_1^{(K)}(1,0.t)$ = Probability that the customer is in the first channel at time t and the customer is to depart for the first time or second time according K= 0 or 1.

$P_1^{(K)}(0,1,t)$ = Probability that the customer is in the second channel at time t and the customer is to depart for the first time or second time according K= 0 or 1.

$P_n^{(K)}(t)$ = Probability that there are ncustomers in the system at time t and the next customer is to depart for the first time or second time according K= 0 or 1.

$P_n(t)$ = Probability that there are ncustomers in the system at any time t.

$$P_n(t) = P_n^{(0)}(t) + P_n^{(1)}(t), \qquad n \geq 0 \tag{1}$$
$$P_1(1,0,t) = P_1^{(0)}(1,0,t) + P_1^{(1)}(1,0,t)$$
$$P_1(0,1,t) = P_1^{(0)}(0,1,t) + P_1^{(1)}(0,1,t)$$
$$P_1^{(K)}(t) = P_1^{(K)}(1,0,t) + P_1^{(K)}(0,1,t), \quad K = 0,1$$
$$P_1(t) = P_1^{(0)}(t) + P_1^{(1)}(t)$$
$$P_1(t) = P_1(1,0,t) + P_1(0,1,t)$$

Initially,

$$P_0^{(0)}(0) = 1 \quad \text{and} \quad P_0^{(1)}(t) = 0, \qquad t \geq 0$$

The difference differential equations describing the system are

$$\frac{d}{dt}P_0^{(0)}(t) = -\lambda P_0^{(0)}(t) + \mu_1\left\{qP_1^{(0)}(1,0,t) + P_1^{(1)}(1,0,t)\right\} + \mu_2\left\{qP_1^{(0)}(0,1,t) + P_1^{(1)}(0,1,t)\right\} \tag{2}$$

$$\frac{d}{dt}P_1^{(0)}(1,0,t) = -(\lambda + \mu_1)P_1^{(0)}(1,0,t) + a_1\lambda P_0^{(0)}(t) + \mu_2 c_1\left\{qP_2^{(0)}(t) + P_2^{(1)}(t)\right\} \tag{3}$$

$$\frac{d}{dt}P_1^{(1)}(1,0,t) = -(\lambda + \mu_1)P_1^{(1)}(1,0,t) + a_1\lambda P_0^{(1)}(t) + \mu_2 c_2\left\{qP_2^{(0)}(t) + P_2^{(1)}(t)\right\} + \mu_1 p a_1 P_1^{(0)}(1,0,t) + \mu_2 p a_1 P_1^{(0)}(0,1,t) \tag{4}$$

$$\frac{d}{dt}P_1^{(0)}(0,1,t) = -(\lambda + \mu_2)P_1^{(0)}(0,1,t) + a_2\lambda P_0^{(0)}(t) + \mu_1 c_1\left\{qP_2^{(0)}(t) + P_2^{(1)}(t)\right\} \tag{5}$$

$$\frac{d}{dt}P_1^{(1)}(0,1,t) = -(\lambda + \mu_2)P_1^{(1)}(0,1,t) + a_2\lambda P_0^{(1)}(t) + \mu_1 c_2\left\{qP_2^{(0)}(t) + P_2^{(1)}(t)\right\} + \mu_1 p a_2 P_1^{(0)}(1,0,t) + \mu_2 p a_2 P_1^{(0)}(0,1,t) \tag{6}$$

$$\frac{d}{dt}P_n^{(0)}(t) = -(\lambda + \mu_1 + \mu_2)P_n^{(0)}(t) + \lambda P_{n-1}^{(0)}(t) + (\mu_1 + \mu_2)c_1\left\{qP_{n+1}^{(0)}(t) + P_{n+1}^{(1)}(t)\right\} + (\mu_1 + \mu_2)c_1 p P_n^{(0)}(t), \quad 2 \leq n \leq k-1 \tag{7}$$

$$\frac{d}{dt}P_n^{(1)}(t) = -(\lambda + \mu_1 + \mu_2)P_n^{(1)}(t) + \lambda P_{n-1}^{(1)}(t) + (\mu_1 + \mu_2)c_2\left\{qP_{n+1}^{(0)}(t) + P_{n+1}^{(1)}(t)\right\} + (\mu_1 + \mu_2)c_2 p P_n^{(0)}(t), \quad 2 \leq n \leq k-1 \tag{8}$$

$$\frac{d}{dt}P_n^{(0)}(t) = -(\lambda\beta + \mu_1 + \mu_2)P_n^{(0)}(t) + \left[\lambda\beta + \lambda\delta_{n-k+1,1}(1-\beta)\right]P_{n-1}^{(0)}(t) + (\mu_1 + \mu_2)c_1\left\{qP_{n+1}^{(0)}(t) + P_{n+1}^{(1)}(t)\right\} + (\mu_1 + \mu_2)c_1 p P_n^{(0)}(t), \quad n \geq k \tag{9}$$

$$\frac{d}{dt}P_n^{(1)}(t) = -(\lambda\beta + \mu_1 + \mu_2)P_n^{(1)}(t) + [\lambda\beta + \lambda\delta_{n-k+1,1}(1-\beta)]P_{n-1}^{(1)}(t) + (\mu_1+\mu_2)c_2\left\{qP_{n+1}^{(0)}(t) + P_{n+1}^{(1)}(t)\right\} + (\mu_1+\mu_2)c_2pP_n^{(0)}(t) \qquad n \ge k \tag{10}$$

Where $\delta_{n,1} = \begin{cases} 1, & \text{for } n = 1 \\ 0, & \text{otherwise} \end{cases}$

Taking the Laplace Transformation $\overline{P}_n(s) = \int_0^\infty e^{-st} P_n(t)\,dt$; **Re** $s > 0$ **of (2) - (10) and dividing by** $(\mu_1 + \mu_2)$

$$\left(\rho + \frac{s}{(\mu_1+\mu_2)}\right)\overline{P}_0^{(0)}(s) = \frac{1}{(\mu_1+\mu_2)} + r_1\left\{q\overline{P}_1^{(0)}(1,0,s) + \overline{P}_1^{(1)}(1,0,s)\right\} + r_2\left\{q\overline{P}_1^{(0)}(0,1,s) + \overline{P}_1^{(1)}(0,1,s)\right\} \tag{11}$$

$$\left(\rho + r_1 + \frac{s}{(\mu_1+\mu_2)}\right)\overline{P}_1^{(0)}(1,0,s) = a_1\rho\overline{P}_0^{(0)}(s) + r_2c_1\left\{q\overline{P}_2^{(0)}(s) + \overline{P}_2^{(1)}(s)\right\} \tag{12}$$

$$\left(\rho + r_1 + \frac{s}{(\mu_1+\mu_2)}\right)\overline{P}_1^{(1)}(1,0,s) = r_2c_2\left\{q\overline{P}_2^{(0)}(s) + \overline{P}_2^{(1)}(s)\right\} + r_1pa_1\overline{P}_1^{(0)}(1,0,s) + r_2pa_1\overline{P}_1^{(0)}(0,1,s) \tag{13}$$

$$\left(\rho + r_2 + \frac{s}{(\mu_1+\mu_2)}\right)\overline{P}_1^{(0)}(0,1,s) = a_2\rho\overline{P}_0^{(0)}(s) + r_1c_1\left\{q\overline{P}_2^{(0)}(s) + \overline{P}_2^{(1)}(s)\right\} \tag{14}$$

$$\left(\rho + r_2 + \frac{s}{(\mu_1+\mu_2)}\right)\overline{P}_1^{(1)}(0,1,s) = r_1c_2\left\{q\overline{P}_2^{(0)}(s) + \overline{P}_2^{(1)}(s)\right\} + r_1pa_2\overline{P}_1^{(0)}(1,0,s) + r_2pa_2\overline{P}_1^{(0)}(0,1,s) \tag{15}$$

$$\left(\rho + 1 + \frac{s}{(\mu_1+\mu_2)}\right)\overline{P}_n^{(0)}(s) = \rho\overline{P}_{n-1}^{(0)}(s) + c_1\left\{q\overline{P}_{n+1}^{(0)}(s) + \overline{P}_{n+1}^{(1)}(s)\right\} + c_1p\overline{P}_n^{(0)}(s) \qquad 2 \le n \le k-1 \tag{16}$$

$$\left(\rho + 1 + \frac{s}{(\mu_1+\mu_2)}\right)\overline{P}_n^{(1)}(s) = \rho\overline{P}_{n-1}^{(1)}(s) + c_2\left\{q\overline{P}_{n+1}^{(0)}(s) + \overline{P}_{n+1}^{(1)}(s)\right\} + c_2p\overline{P}_n^{(0)}(s) \qquad 2 \le n \le k-1 \tag{17}$$

$$\left(\rho\beta + 1 + \frac{s}{(\mu_1+\mu_2)}\right)\overline{P}_n^{(0)}(s) = [\rho\beta + \rho\delta_{n-k+1,1}(1-\beta)]\overline{P}_{n-1}^{(0)}(s) + c_1\left\{q\overline{P}_{n+1}^{(0)}(s) + \overline{P}_{n+1}^{(1)}(s)\right\} + c_1p\overline{P}_n^{(0)}(s), n \ge k \tag{18}$$

$$\left(\rho\beta + 1 + \frac{s}{(\mu_1+\mu_2)}\right)\overline{P}_n^{(1)}(s) = [\rho\beta + \rho\delta_{n-k+1,1}(1-\beta)]\overline{P}_{n-1}^{(1)}(s) + c_2\left\{q\overline{P}_{n+1}^{(0)}(s) + \overline{P}_{n+1}^{(1)}(s)\right\} + c_2p\overline{P}_n^{(0)}(s), n \ge k \tag{19}$$

Where $\rho = {}^{\lambda}/_{(\mu_1+\mu_2)}$, $r_1 = {}^{\mu_1}/_{(\mu_1+\mu_2)}$, $r_2 = {}^{\mu_2}/_{(\mu_1+\mu_2)}$

LAPLACE TRANSFORMATION OF PROBABILITY GENERATING FUNCTION OF TRANSIENT-STATE QUEUE LENGTH PROBABILITIES

Definitions

$$P^{(0)}(z,t) = \sum_{n=0}^\infty P_n^{(0)}(t)z^n \qquad \overline{P}^{(0)}(z,s) = \int_0^\infty e^{-st} P^{(0)}(z,t)\,dt$$

$$P^{(0)}(z,t) = \sum_{n=0}^\infty P_n^{(0)}(t)z^n \qquad P^{(0)}(z,s) = \int_0^\infty e^{-st} P^{(0)}(z,t)\,dt$$

$$P^{(1)}(z,t) = \sum_{n=0}^\infty P_n^{(1)}(t)\,z^n \qquad \overline{P}^{(1)}(z,s) = \int_0^\infty e^{-st} P^{(1)}(z,t)\,dt$$

$$P(z,t) = P^{(0)}(z,t) + P^{(1)}(z,t) \qquad \overline{G}(z,s) = \int_0^\infty e^{-st} P(z,t)\,dt \quad \text{with } |z| \le 1$$

$$\overline{P}(z,s) = \frac{\begin{aligned}&\left(\frac{z}{\mu_1+\mu_2}\right)B(z) - (1-z)(qF(z) + c_2pz)\overline{P}_0^{(0)}(s) - zF(z)\left\{(q+pz)\overline{P}_1^{(0)}(s) + \overline{P}_1^{(1)}(s)\right\} \\ &+ z\Big[\{(q+pz)F(z) + p(1-z)(c_1pz - c_2q)\}\left\{r_1\overline{P}_1^{(0)}(1,0,s) + r_2\overline{P}_1^{(0)}(0,1,s)\right\} \\ &+ B(z)\left\{r_1\overline{P}_1^{(1)}(1,0,s) + r_2\overline{P}_1^{(1)}(0,1,s)\right\}\Big] \\ &+ z^2\Big[B(z)\left\{r_2\overline{P}_1^{(0)}(1,0,s) + r_1\overline{P}_1^{(0)}(0,1,s)\right\} + E(z)\left\{r_2\overline{P}_1^{(1)}(1,0,s) + r_1\overline{P}_1^{(1)}(0,1,s)\right\}\Big] \\ &- \big(1-z)(1-\beta)\big)\rho z\left\{B(z)\sum_{n=0}^{k-1}\overline{P}_n^{(0)}(s)z^n + E(z)\sum_{n=0}^{k-1}\overline{P}_n^{(1)}(s)z^n\right\}\end{aligned}}{\{(F(z) - c_1(q+pz))(F(z) - c_2)\} - (q+pz)c_1c_2}$$

$\rho = \lambda/(\mu_1+\mu_2)$; $|z| \le 1$ (20)

Where $F(z) = \left\{-\rho\beta z^2 + \left(\rho\beta + 1 + \frac{s}{(\mu_1+\mu_2)}\right)z\right\}$

$B(z) = \{F(z) - c_2p(1-z)\}$

$E(z) = \{F(z) + c_1p(1-z)\}$

$D = K_1(z) * K_2(z) - c_1c_2(q+pz)$

Where $K_1(z) = \left(-\rho\beta z^2 + \left(\rho\beta + 1 + \frac{s}{(\mu_1+\mu_2)}\right)z - c_1(q+pz)\right)$, $K_2(z) = \left(-\rho\beta z^2 + \left(\rho\beta + 1 + \frac{s}{(\mu_1+\mu_2)}\right)z - c_2\right)$

Obviously $K_1(z)$ and $K_2(z)$ have two zeroes inside the unit circle.

Let $f(z) = K_1(z) * K_2(z)$ and $g(z) = (q+pz)c_1c_2$

$$|f(z)| = |K_1(z)| * |K_2(z)|$$
$$= \left|\left(-\rho\beta z^2 + \left(\rho\beta + 1 + \frac{s}{(\mu_1+\mu_2)}\right)z - c_1(q+pz)\right)\right|\left|-\rho\beta z^2 + \left(\rho\beta + 1 + \frac{s}{(\mu_1+\mu_2)}\right)z - c_2\right|$$
$$\geq (\zeta + c_2)(\zeta + c_1) \text{ for } \frac{s}{(\mu_1+\mu_2)} = \zeta + i\eta,\ |z| = 1$$
$$> c_1c_2 \geq |g(z)|$$

Hence $|f(z)| > |g(z)|$ on $|z| = 1$

Since all the conditions of Rouche's Theorem are satisfied, so D has two zeroes inside the unit circle. Let these zeroes be $z_m(m = 0, 1)$. Numerator must vanish for these two zeroes since $\overline{P}(z,s)$ is an analytical function of z. These two equations along with equation (11) and equations (12), (13), (14), (15) will determine the seven unknowns $\overline{P}_0^{(0)}(s), \overline{P}_1^{(0)}(1,0,s), \overline{P}_1^{(0)}(0,1,s), \overline{P}_1^{(1)}(1,0,s), \overline{P}_1^{(1)}(0,1,s), \overline{P}_2^{(0)}(s), \overline{P}_2^{(1)}(s)$ (in case k=3). Along with equations (11) ,(12), (13), (14), (15) and {equations (16) & (17) for n=2} will determine nine unknowns $\overline{P}_0^{(0)}(s)$, $\overline{P}_1^{(0)}(1,0,s), \overline{P}_1^{(0)}(0,1,s)$, $\overline{P}_1^{(1)}(1,0,s), \overline{P}_1^{(1)}(0,1,s)$, $\overline{P}_2^{(0)}(s)$, $\overline{P}_2^{(1)}(s), \overline{P}_3^{(0)}(s)$ and $\overline{P}_3^{(1)}(s)$ (in case k=4) and along with equations (11) ,(12), (13), (14), (15) and {equations (16) & (17) for n=2,3,4 … k-2} will in general determine (2k+1) unknowns $\overline{P}_0^{(0)}(s), \overline{P}_1^{(0)}(1,0,s), \overline{P}_1^{(0)}(0,1,s)$, $\overline{P}_1^{(1)}(1,0,s), \overline{P}_1^{(1)}(0,1,s), \overline{P}_2^{(0)}(s)$, $\overline{P}_2^{(1)}(s), \ldots$, $\overline{P}_{k-1}^{(0)}(s)$, $\overline{P}_{k-1}^{(1)}(s)$ (when number of customers = k). Hence the generating function $\overline{P}(z,s)$ is completely known. $\overline{P}_n(s)$ can be obtained by using the following formula

$$\overline{P}_n(s) = \frac{1}{n!}\frac{d^{(n)}\overline{P}(z,s)}{dz^{(n)}} \text{ at } z = 0$$

In either case $P_n(t)$ can be found by inverting the Laplace transform $\overline{P}_n(s)$.

Further $\overline{P}(1,s) = \frac{1}{S}$, and $\overline{P}(0,s) = \lim_{z\to 0}\overline{P}(z,s) = \frac{\text{zero}}{\text{zero}}$

On using L' Hospital's rule, it can be shown that

$$\overline{P}(0,s) = \overline{P}_0^{(0)}(s)$$

SPECIAL CASES

1. When there is no balking i.e. M/M/2 feedback queueing model with unequal service rates. Put β=1 in equation (36)

$$\overline{P}(z,s) = \frac{\begin{array}{l}\left(\frac{z}{\mu_1+\mu_2}\right)B^*(z) + (1 - z(qF^*(z) + c_2pz)\overline{P}_0^{(0)}(s) - zF^*(z)\left\{(q+pz)\overline{P}_1^{(0)}(s) + \overline{P}_1^{(1)}(s)\right\} \\ \quad + z\left[\{(q+pz)F^*(z) + p(1-z)(c_1pz - c_2q)\}\left\{r_1\overline{P}_1^{(0)}(1,0,s) + r_2\overline{P}_1^{(0)}(0,1,s)\right\}\right. \\ \quad \left. + B^*(z)\left\{r_1\overline{P}_1^{(1)}(1,0,s) + r_2\overline{P}_1^{(1)}(0,1,s)\right\}\right] \\ \quad + z^2\left[B^*(z)\left\{r_2\overline{P}_1^{(0)}(1,0,s) + r_1\overline{P}_1^{(0)}(0,1,s)\right\} + E^*(z)\left\{r_2\overline{P}_1^{(1)}(1,0,s) + r_1\overline{P}_1^{(1)}(0,1,s)\right\}\right]\end{array}}{\{(F^*(z) - c_1(q+pz))(F^*(z) - c_2)\} - (q+pz)c_1c_2}$$

$$\rho = \lambda/(\mu_1+\mu_2);\ |z| \leq 1 \tag{21}$$

Where $F^*(z) = \left\{-\rho z^2 + \left(\rho + 1 + \frac{s}{(\mu_1+\mu_2)}\right)z\right\}$

$B^*(z) = \{F^*(z) - c_2p(1-z)\}$

$E^*(z) = \{F^*(z) + c_1p(1-z)\}$

which coincides with equation (3.31) of Kumari (2011).

2. When there is no feedback and no balking with unequal service rate

Putting $q = 1, p = 0,\ \beta = 1, c_1 = 1,\ c_2 = 0, \overline{P}^{(0)}(z,s) = \overline{P}(z,s),\ \overline{P}^{(1)}(z,s) = 0$ and $\overline{P}_0^{(0)}(s) = \overline{P}_0(s)$ in (20)

$$\overline{P}(z,s) = \frac{\begin{array}{c} z/(\mu_1+\mu_2) - (1-z)\overline{P}_0(s) - z\overline{P}_1(s) + (r_1z + r_2z^2)\overline{P}_1(1,0,s) \\ +(r_2z + r_1z^2)\overline{P}_1(0,1,s)\end{array}}{\{F^{**}(z) - 1)\}} \tag{22}$$

where $F^{**}(z) = \left\{-\rho z^2 + \left(\rho + 1 + \frac{s}{(\mu_1+\mu_2)}\right)z\right\}\rho = \frac{\lambda}{(\mu_1+\mu_2)};\ |z| \leq 1$

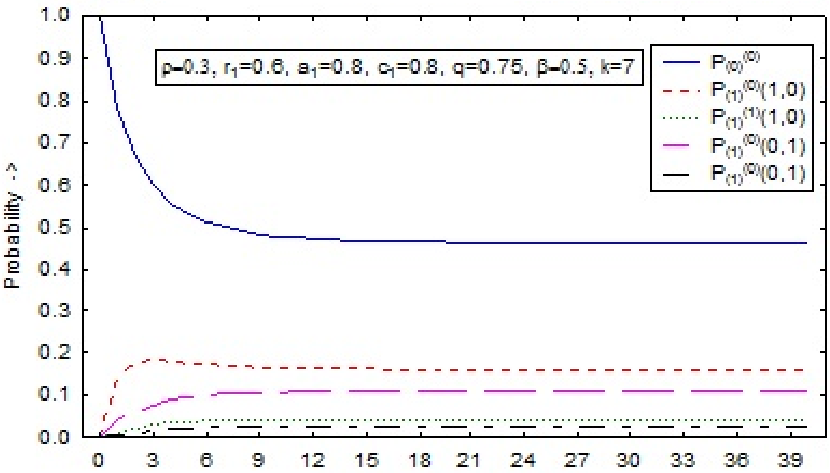
3. When there is no feedback and no balking with equal service rate. i.e. M/M/2 system.

$$\overline{P}(z,s) = \frac{z/(2\mu) - (1-z)\left\{\overline{P}_0(s) + \left(\frac{1}{2}\right) z\overline{P}_1(s)\right\}}{\{F^{\#}(z) - 1\}} \tag{23}$$

where $F^{\#}(z) = \{-\rho z^2 + (\rho + 1 + s/2\mu)z\}$ and $\rho = \frac{\lambda}{2\mu}$; $|z| \le 1$

Numerical Solution: The numerical results are generated using MATLAB programming. Fig.1 shows plot of probabilities $P_0^{(0)}, P_1^{(0)}(1,0)$, $P_1^{(1)}(1,0)$, $P_1^{(0)}(0,1)$, and $P_1^{(1)}(0,1)$with respect to time t (average service times). It is clear from the graph that the probability $P_0^{(0)}$ decreases rapidly in the starting and then becomes almost steady from the initial value one at time t=0. Probability $P_1^{(0)}(1,0)$ increases rapidly in the starting and then decreases slightly before attaining some steady value for higher values of t. Probabilities $P_1^{(1)}(1,0)$, $P_1^{(0)}(0,1)$, and $P_1^{(1)}(0,1)$ increase slowly in the startingand approach some steady state values after some time.

Probabilities $P_0^{(0)}, P_1^{(0)}(1,0)$, $P_1^{(1)}(1,0)$, $P_1^{(0)}(0,1)$ and $P_1^{(1)}(0,1)$ Vs time t

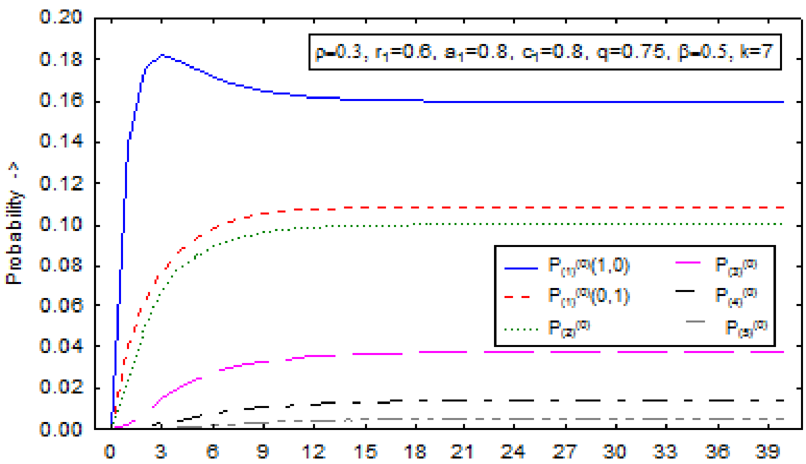


$t(\boldsymbol{average\ service\ time}) \rightarrow$

Figure-1

Comparison among probabilities when the customer at the head of the queue is to join the server for the first time i.e. among $P_0^{(0)}, P_1^{(0)}(1,0)$, $P_1^{(0)}(0,1), P_2^{(0)}, P_3^{(0)}, P_4^{(0)}$ and $P_5^{(0)}$ is done through Fig. 2. It is interpreted that probability decreases as n (number of customers in the system) increases for the case under study. Also $P_1^{(0)}(1,0)$ take higher values than the corresponding $P_1^{(0)}(0,1)$(beacuase of high probability of joining first server when system is empty). It is also seen that all the probabilities finally approach steady values. Fig.3 shows that the probabilities $P_1^{(1)}(1,0)$, $P_1^{(1)}(0,1), P_2^{(1)}, P_3^{(1)}, P_4^{(1)}$ and $P_5^{(1)}$ also have same relationship among themselves as probabilities $P_0^{(0)}$, $P_1^{(0)}(1,0)$, $P_1^{(0)}(0,1)$, $P_2^{(0)}$, $P_3^{(0)}$, $P_4^{(0)}$ and $P_5^{(0)}$ have in Fig.2.

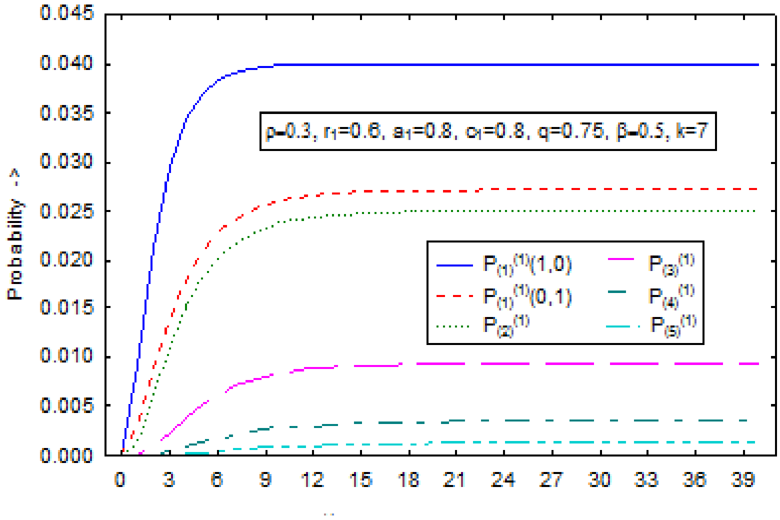
Probabilities $P_0^{(0)}, P_1^{(0)}(1,0)$, $P_1^{(0)}(0,1), P_2^{(0)}, P_3^{(0)}, P_4^{(0)}$ and $P_5^{(0)}$ Vs time t



$t(\boldsymbol{average\ service\ time}) \rightarrow$

Figure-2

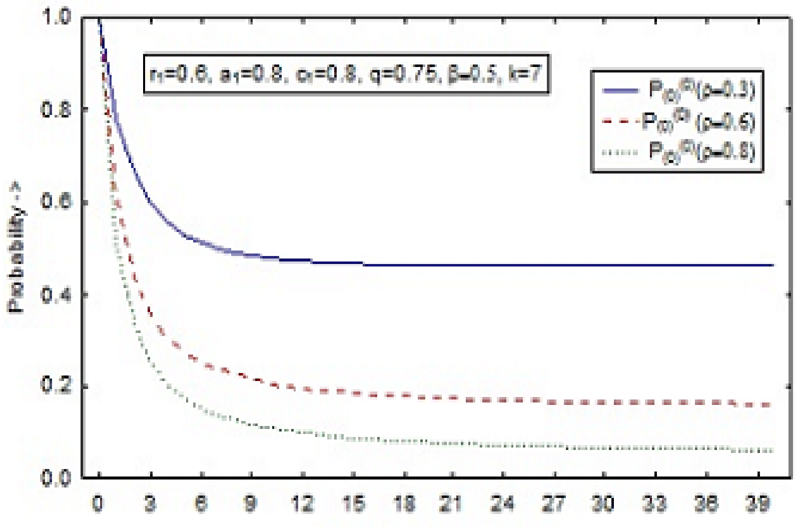
Probability $P_1^{(1)}(1,0)$, $P_1^{(1)}(0,1), P_2^{(1)}, P_3^{(1)}, P_4^{(1)}$ and $P_5^{(1)}$



$t(average\ service\ time) \rightarrow$
Figure-3

To study the effect of parameter ρ on different probabilities of the model, the data of various probabilities is generated for different values of ρ keeping other parameters constant. The set of values that ρ took is {0.3, 0.6, 0.8}. The other parameters were fixed at $\beta = 0.5$, $k = 7$, $c_1 = 0.8$, $q = 0.75$, $r_1 = 0.6$, $a_2 = 0.8$,. In Fig. 4, the probability$P_0^{(0)}$ is plotted against time for different values of ρ. From the figure it is concluded that as ρ increases $P_0^{(0)}$ decreases. So more the traffic intensity i.e. more customers are arriving per unit average service time less is the probability of zero customers in the system.

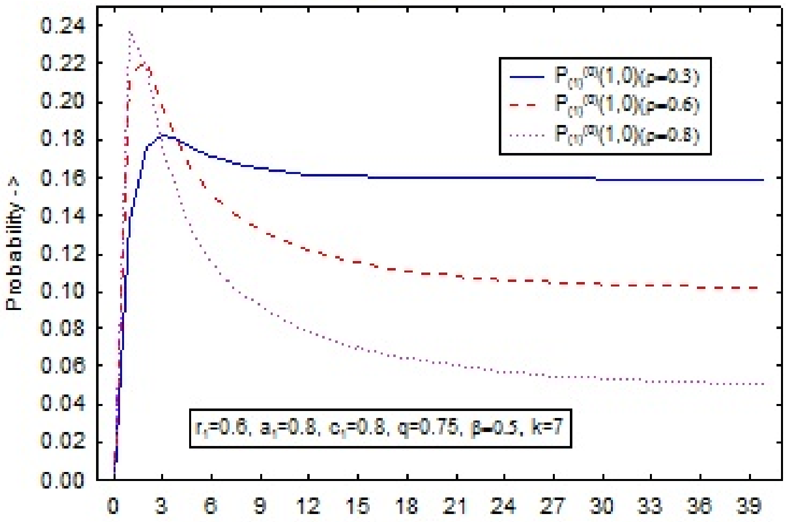
$P_0^{(0)}$ Vs time for different values of traffic intensity $\rho = {}^{\lambda}/_{(\mu_1 + \mu_2)}$



$t(average\ service\ time) \rightarrow$
Figure-4

Behavior of $P_1^{(0)}(1,0)$ and $P_1^{(0)}(0,1)$ with respect to time and changing ρ is apparent from Fig. 5 and Fig.6. Probability $P_1^{(0)}(1,0)$ is higher for $\rho = 0.6$ than for $\rho = 0.3$ for all the values of t. $\rho = 0.8$ takes high value in the starting but its value decreases as t increases and finally attains some steady state value. Probability $P_1^{(0)}(0,1)$ is higher for $\rho = 0.6$ than for $\rho = 0.3$ for all the values of t. $\rho = 0.8$ takes high value in the starting with steep decrease for higher values of t. In all the cases probabilities $P_1^{(0)}(1,0)$ and $P_1^{(0)}(0,1)$ approaches some steady values finally.

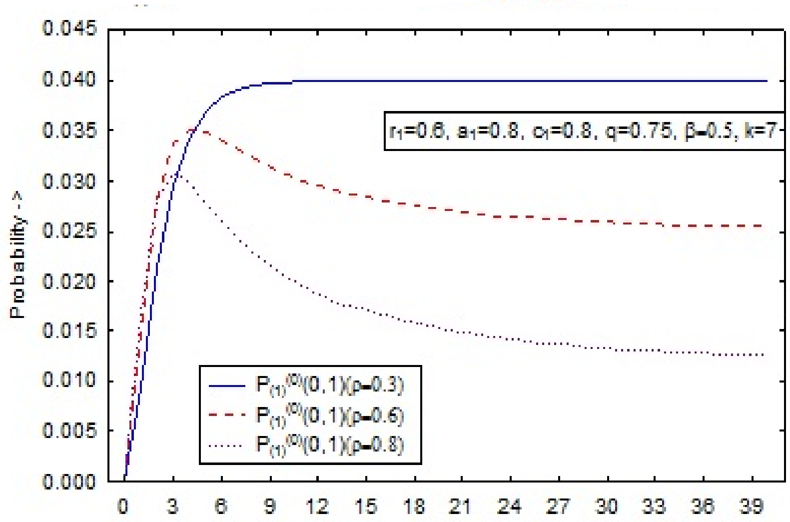
$P_1^{(0)}(1,0)$ Vs time for different values of traffic intensity $\rho = \lambda/(\mu_1 + \mu_2)$



$t(average\ service\ time) \rightarrow$

Figure-5

$P_1^{(0)}(0,1)$ Vs time for different values of traffic intensity $\rho = \lambda/(\mu_1 + \mu_2)$

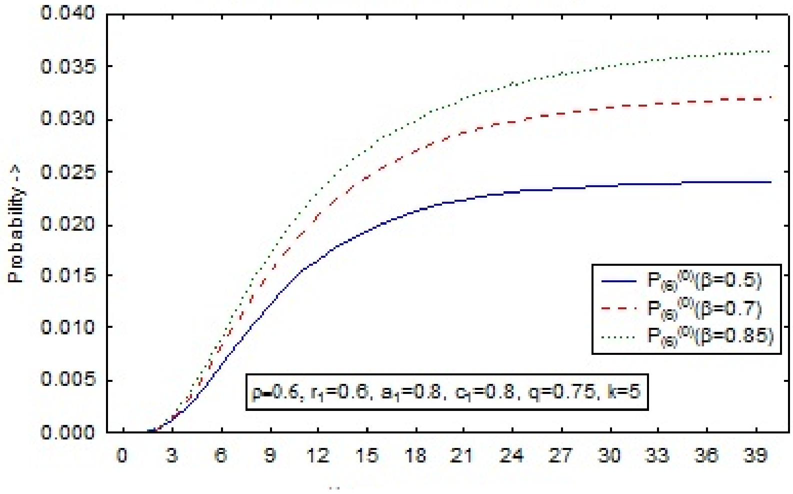


$t(average\ service\ time) \rightarrow$

Figure-6

To study the effect of changing β (probability of joining the customer to system after there are k or more than k customers in the system) is done by generating data of various probabilities for different values of β keeping other parameters constant. The set of values that β took is {0.5, 0.7, 0.85}. The other parameters were fixed at $a_1 = 0.8$, $\rho = 0.6, k = 7,\ r_1 = 0.6,\ c_1 = 0.8, q = 0.75$. In Fig.7 and Fig.8, the probability$P_6^{(0)}$ and $P_6^{(1)}$ is plotted against time for different values of β . From the Fig.7 it is concluded that as probability $P_6^{(0)}$ is higher when $\beta = 0.85$ than when $\beta = 0.7$ for all values of t. For $\beta = 0.5$ the $P_6^{(0)}$ takes high value initially but its value decreases as t increases and but gradually it attains the steady value. Similarly, in Fig.8 probability $P_6^{(1)}$ shows the same behavior as $P_6^{(0)}$. As the value of β increases the value of probability $P_6^{(1)}$ decreases.

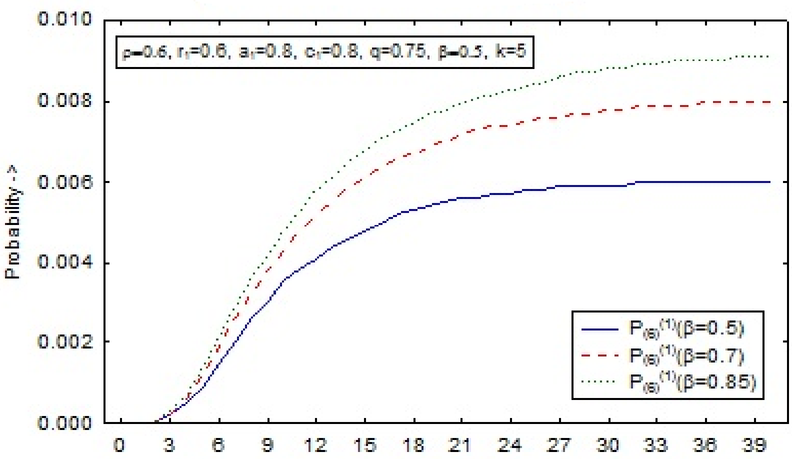
Probability $P_6^{(0)}$ Vs time for different values of β



$t(average\ service\ time) \rightarrow$

Figure-7

$P_6^{(1)}$(t) Vs time for different values of β

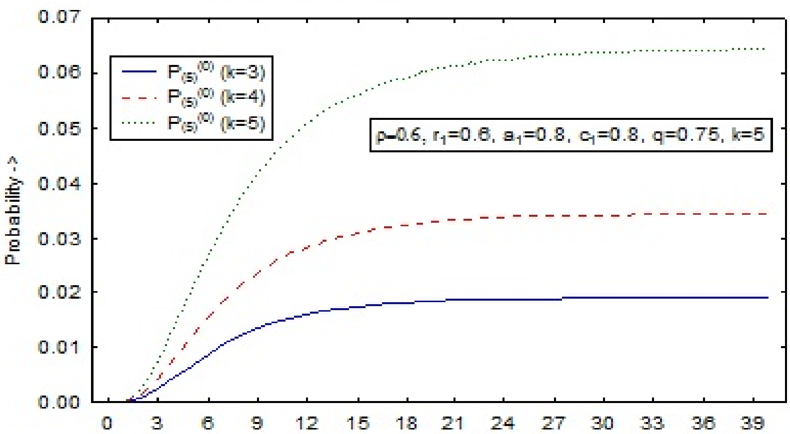


$t(average\ service\ time) \rightarrow$

Figure-8

In Fig.9 by keeping all other parameters constant and varying k i.e. the threshold value at which the customer thinks either to join the queue or to balk. We have plotted the probability $P_5^{(0)}$against different values of time. We can see as the value of ‘k’ increases the probability$P_5^{(0)}$ also increases.

$P_5^{(0)}(t)$ Vs time for different values of k



$t(average\ service\ time) \rightarrow$

Figure-9

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