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$\delta(\delta g)$^-CONTINUOUS FUNCTIONS ON TOPOLOGICAL SPACES AND THEIR CHARACTERISTICS

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ABSTRACT

In this article, a new class of functions namely, $\delta(\delta g)$^-continuous functions on topological spaces is introduced and their relationship with other class of continuous functions are investigated. Further the properties of almost $\delta(\delta g)$^-continuous function and $\delta(\delta g)$^-irresolute function are analysed.

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Key words: *$\delta(\delta g)$^-closed set, $\delta(\delta g)$^-continuous function, $\delta(\delta g)$^-irresolute function, almost $\delta(\delta g)$^continuous function.*

I. INTRODUCTION

The concept of generalised closed sets was introduced and various properties were analysed by Norman Levine [6] in 1970. Velicko [12] introduced δ-open sets in 1968 which are stronger than open sets. Julian Dontech [2] combined the concepts of δ-closedness, g-closedness and defined generalised closed sets called δg-closed sets in 1996 and studied the properties of δg-continuous functions. As an extension of δg-closed sets, Sudha R and Sivakamasundari K introduce a new class of closed sets namely, δg*-closed sets [9] in 2012 and the properties of δg*-continuous functions [10] are analysed in 2013. Combining the concepts of δ-closedness and δg*-closedness, Meena K and Sivakamasundari K introduce a class of closed sets called, δ(δg)*-closed sets [3] and their continuous functions [4] are investigated in 2015. As an extension of the class of g-closed sets, Veerakumar introduced and studied the properties of $\hat{g}$ -closed sets in 2003 [11]. Followed by this Lellis Thivagar [7] defined another class of closed sets called $\delta\hat{g}$ -closed sets and characterised its properties. As an extension, Stella Irene Mary J and Janaranjana Sri S defined a new class of closed sets namely $\delta(\delta g)$^-closed sets and analysed its properties [8] in 2016. A subset, A of (X, τ) is said to be δ(δg)^-closed sets if δcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is $\delta\hat{g}$ -open.

In this paper, we introduce a new class of continuous functions called $\delta(\delta g)$^-continuous functions induced by the class of $\delta(\delta g)$^-closed sets and investigated their properties.

II. PRELIMINARIES

Throughout this paper, (X, τ), (Y, σ) and (Z, η) represent non empty topological spaces on which no separation axioms are mentioned unless otherwise specified. For a subset A of (X, τ), the closure of A and interior of A are denoted by cl(A) and int(A) respectively.

Remark 2.1: The definitions of δ-closed, g-closed, gδ-closed, rg-closed, gpr-closed, δg#-closed, rwg-closed, δgs-closed, πgα-closed, πgs-closed, πgb-closed, πgp-closed and πgsp-closed are mentioned in [7].

Definitions 2.2 [4]: Various classes of continuous functions based on the different classes of closed sets were introduced by many authors. Given below are the definitions of those continuous functions.

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A function f: (X, τ) ⟶ (Y, σ) is called

1. a continuous function if f^{-1}(V) is closed in (X, τ), for every closed set V of (Y, σ).
2. a δ-continuous function if f^{-1}(V) is δ-closed in (X, τ), for every closed set V of (Y, σ).
3. a Δ*-continuous function if f^{-1}(V) is Δ*-closed in (X, τ), for every closed set V of (Y, σ).
4. a δg*-continuous function if f^{-1}(V) is δg*-closed in (X, τ), for every closed set V of (Y, σ).
5. a gδ-continuous function if f^{-1}(V) is gδ-closed in (X, τ) for every closed set V of (Y, σ).
6. a gpr-continuous function if f^{-1}(V) is gpr-closed in (X, τ), for every closed set V of (Y, σ).
7. a gspr-continuous function if f^{-1}(V) is gspr-closed in (X, τ), for every closed set V of (Y, σ).
8. a rg-continuous function if f^{-1}(V) is rg-closed in (X, τ), for every closed set V of (Y, σ).
9. a rwg-continuous function if f^{-1}(V) is rwg-closed in (X, τ), for every closed set V of (Y, σ).
10. a δgs-continuous function if f^{-1}(V) is δgs-closed in (X, τ), for every closed set V of (Y, σ).
11. a πg-continuous function if f^{-1}(V) is πg-closed in (X, τ), for every closed set V of (Y, σ).
12. a πgp-continuous function if f^{-1}(V) is πgp-closed in (X, τ), for every closed set V of (Y, σ).
13. a πgα-continuous function if f^{-1}(V) is πgα-closed in (X, τ), for every closed set V of (Y, σ).
14. a πgs-continuous function if f^{-1}(V) is πgs-closed in (X, τ), for every closed set V of (Y, σ).
15. a πgsp-continuous function if f^{-1}(V) is πgsp-closed in (X, τ), for every closed set V of (Y, σ).
16. a πgb-continuous function if f^{-1}(V) is πgb-closed in (X, τ), for every closed set V of (Y, σ).
17. a sg-continuous function if f^{-1}(V) is sg-closed in (X, τ), for every closed set V of (Y, σ).
18. a gs-continuous function if f^{-1}(V) is gs-closed in (X, τ), for every closed set V of (Y, σ).
19. a *g-continuous function if f^{-1}(V) is *g-closed in (X, τ), for every closed set V of (Y, σ).
20. a αg-continuous function if f^{-1}(V) is αg-closed in (X, τ), for every closed set V of (Y, σ).
21. a δg-continuous function if f^{-1}(V) is δg-closed in (X, τ), for every closed set V of (Y, σ).
22. a g*s-continuous function if f^{-1}(V) is g*s-closed in (X, τ), for every closed set V of (Y, σ).

3. CHARACTERISATION OF $\delta(\delta g)$^-CLOSED SETS

Theorem 3.1: Let A be a $\delta(\delta g)$^-closed set of (X, τ). Then δcl(A)\A does not contain a non-empty $\hat{\delta g}$ -closed set.

Proof: Let A be a δ(δg)^-closed set and suppose F is $\hat{\delta g}$ -closed set contained in δcl(A)\A. It is enough to show that F is an empty set. Since F is $\hat{\delta g}$ -closed, F^C is $\hat{\delta g}$ -open set of (X,τ) and A $\subseteq F^C$. Since A is δ(δg)^-closed set of (X, τ), δcl(A) $\subseteq F^C$. Thus F $\subseteq (\delta cl(A))^C$. Also, F $\subseteq$ δcl(A)-A. Therefore F $\subseteq (\delta cl(A))^C \cap$ (δcl(A)) = φ. Hence F = φ.

Remark 3.2: The following example shows that the converse of the above theorem need not be true.

Example: Let X= {a, b, c} with τ = {X, ϕ, {a}}. Let A = {b, c}. Then δcl(A)\A = ϕ which does not contain a non empty $\hat{\delta g}$ -closed set but A is not a δ(δg)^-closed in (X, τ).

Theorem 3.3: If A is $\delta(\delta g)$^-closed set in a space (X, τ) and A $\subseteq$ B $\subseteq$ δcl(A), then B is also a $\delta(\delta g)$^-closed set.

Proof: Let U be a $\hat{\delta g}$ -open set of (X, τ) such that B $\subseteq$ U. Then A $\subseteq$ U. Since A is $\delta(\delta g)$^-closed set, δcl(A) $\subseteq$ U and by hypothesis, B $\subseteq$ δcl(A). Consequently δcl(B) $\subseteq$ δcl(δcl(A)) = δcl(A). Hence δcl(B) $\subseteq$ δcl(A) $\subseteq$ U and B is $\delta(\delta g)$^-closed set.

Remark 3.4: By Theorem 3.3 in [8], $\delta(\delta g)$^-closed set is not a δ-closed set. The following theorem proves that under some condition, every $\delta(\delta g)$^-closed set is δ-closed.

Theorem 3.5: If A is $\delta\hat{g}$ -open and $\delta(\delta g)$^-closed subset of a topological space (X, τ) then A is δ-closed subset of (X, τ).

Proof: Let A be $\delta\hat{g}$ -open and $\delta(\delta g)$^-closed. By definition, δcl(A) $\subseteq$ A. Hence A is δ-closed.

Remark 3.6: The intersection of two $\delta(\delta g)$^-closed sets need not be δ(δg)^-closed. In the next theorem, we show that intersection of two $\delta(\delta g)$^-closed sets implies $\delta(\delta g)$^-closed where one of them is δ-closed.

Theorem 3.7: In a topological space (X, τ), the intersection of a $\delta(\delta g)$^-closed set and δ-closed set is always $\delta(\delta g)$^-closed.

Proof: Let A be $\delta(\delta g)$^-closed and F be δ-closed set in (X,τ). Since a δ-closed set is $\delta(\delta g)$^-closed (Theorem 3.3 in [8]), F is $\delta(\delta g)$^-closed and hence A $\bigcap$ F is the intersection of two $\delta(\delta g)$^-closed sets. Suppose that U is any $\delta\hat{g}$ -open set with A $\bigcap$ F $\subseteq$ U, it follows that A $\subseteq$ U $\bigcup$ F^C and so δcl(A) $\subseteq$ U $\bigcup$ F^C . Then δcl(A $\bigcap$ F) $\subseteq$ δcl(A) $\bigcap$ F $\subseteq$ U. Hence A $\bigcap$ F is $\delta(\delta g)$^-closed.

Theorem 3.8: Let A $\subseteq$ Y $\subseteq$ X and suppose that A is $\delta(\delta g)$^-closed in X, then A is δ(δg)^-closed relative to Y.

Proof: Given, A $\subseteq$ Y $\subseteq$ X and A is $\delta(\delta g)$^-closed in X. Let A $\subseteq$ Y $\bigcap$ U, where U is $\delta\hat{g}$ -open in X. Since A is $\delta(\delta g)$^-closed, A $\subseteq$ U implies, δcl(A) $\subseteq$ U. It follows that Y $\bigcap$ U. Hence A is $\delta(\delta g)$^-closed relative to Y.

4. $\delta(\delta g)$^-CONTINUOUS MAPS AND IRRESOLUTE MAPS

We introduce the new class of continuous function namely, δ(δg)^-continuous function.

Definition 4.1: $\delta(\delta g)$^-continuous function: A function *f:* (X, τ) ⟶ (Y, σ) is called a $\delta(\delta g)$^-continuous if f^{-1}(V) is $\delta(\delta g)$^-closed in (X,τ) for every closed set V of (Y, σ).

Definition 4.2: $\delta(\delta g)$^-irresolute function: A function f: (X, τ) ⟶ (Y, σ) is called a $\delta(\delta g)$^-irresolute if f^{-1}(V) is $\delta(\delta g)$^-closed in (X, τ) for every $\delta(\delta g)$^-closed set V of (Y, σ).

Theorem 4.3: Let f : (X, τ) ⟶ (Y, σ) be a function. Then the following statements are equivalent.

i. f is $\delta(\delta g)$^-continuous.
ii. The inverse image of every open set in (Y, σ) is $\delta(\delta g)$^-open in (X, τ).

Proof:
$(i) \Rightarrow (ii)$: Let f be a $\delta(\delta g)$^-continuous map and U be any open subset of X. Then (Y−U) is closed in Y. Since f is δ(δg)^-continuous, f^{-1}(Y− U) = X− f^{-1}(U) is $\delta(\delta g)$^-closed in X. Hence f^{-1}(U) is $\delta(\delta g)$^-open in X.
$(ii) \Rightarrow (i)$: Let V be a closed subset of Y . Then (Y − V) is open in Y. Consequently, f^{-1}(Y−V) = X− f^{-1}(V) is $\delta(\delta g)$^-open in X. Then f^{-1}(V) is $\delta(\delta g)$^-closed in X. Therefore, f is $\delta(\delta g)$^-continuous.

Theorem 4.4: Every $\delta(\delta g)$^-irresolute function is $\delta(\delta g)$^-continuous. The converse need not be true.

Proof: Let f : X ⟶ Y be δ(δg)^-continuous function and V be any closed set in Y. Then V is $\delta(\delta g)$^-closed in Y. Since f is $\delta(\delta g)$^-irresolute function, f^{-1}(V) is $\delta(\delta g)$^-closed in X. Hence f is $\delta(\delta g)$^-continuous.

Example: Let X = Y= {a, b, c} with topologies τ = {X, φ, {a}, {b}, {a, b}} and σ = {Y, φ, {a}}. Consider a map f: (X, τ) ⟶ (Y, σ) defined by f(a) = c, f(b) = b and f(c) = a. Since $f^{-1}(b,c)$ = {a, b} is not a $\delta(\delta g)$^-closed set in (X, τ). Then f is $\delta(\delta g)$^-continuous but not $\delta(\delta g)$^-irresolute.

Theorem 4.5: Every δ-continuous map f : (X, τ) ⟶ (Y, σ) is $\delta(\delta g)$^-continuous. The converse need not be true.

Proof: Let V be a closed set in (Y, σ). Since f is δ-continuous, f^{-1}(V) is δ-closed in (X, τ). By Theorem 3.3 [8], every δ-closed set is $\delta(\delta g)$^-closed set. Consequently f^{-1}(V) is $\delta(\delta g)$^-closed set in (X, τ). Hence f is $\delta(\delta g)$^-continuous.

Example: Let X= Y= {a, b, c} with topologies τ ={X, φ, {a, b},{b, c},{b}} and σ ={Y, φ,{b}}. Let f: (X, τ) ⟶ (Y, σ) be the identity map. Then f is $\delta(\delta g)$^-continuous map but not δ-continuous map, Since the closed set {a, c} is $\delta(\delta g)$^-closed set but not δ-closed set in (X, τ).

Theorem 4.6:

i. Every Δ*-continuous map f: (X, τ) ⟶ (Y, τ) is $\delta(\delta g)$^continuous.
ii. Every δg*-continuous map f: (X, τ) ⟶ (Y, τ) is $\delta(\delta g)$^-continuous.
The converse need not be true.

Proof:

i. Let V be a closed set in (Y, σ). Since f is Δ*-continuous, f^{-1}(V) is Δ*-closed in (X, τ). By Theorem 3.4(ii) [8], every Δ*-closed set is $\delta(\delta g)$^-closed set. Consequently f^{-1}(V) is $\delta(\delta g)$^-closed set in (X, τ). Hence f is $\delta(\delta g)$^-continuous.
ii. Let V be a closed set in (Y, σ). Since f is δg*-continuous, f^{-1}(V) is δg*-closed in (X, τ). By Theorem 3.4(i) [8], every δg*-closed set is δ(δg)^-closed set. Consequently f^{-1}(V) is $\delta(\delta g)$^-closed set in (X, τ). Hence f is δ(δg)^-continuous.

Example:

i. Let X= Y = {a, b, c} with topologies τ ={X, φ, {a, b},{c}} and σ ={Y, φ,{a}}. Let f: (X, τ) ⟶ (Y, τ) be the identity map. Then f is $\delta(\delta g)$^-continuous map but not Δ*-continuous map. Since the closed set {a} is δ(δg)^-closed set but not Δ*-closed set in (X, τ).
ii. Let X =Y = {a, b, c} with topologies τ ={X, φ, {a, b},{b, c},{b}} and σ ={Y, φ,{c}}. Define a map f: (X, τ) ⟶ (Y, τ) by f(a) ={b}, f(b) = {c}, f(c) = {a}. Then f is $\delta(\delta g)$^-continuous map but not δg*-continuous map, Since the closed set {a} is $\delta(\delta g)$^-closed set but not δg*-closed set in (X,τ).

Theorem 4.7: Let f : (X, τ) ⟶ (Y, τ) be $\delta(\delta g)$^-continuous map. Then f is
i. gδ-continuous ii. gpr-continuous iii. gspr-continuous

Proof:

i. Let V be a closed set in (Y, σ). Since f is $\delta(\delta g)$^-continuous, then f^{-1}(V) is δ(δg)^-closed in (X, τ). By Theorem 3.7(i) [8], every $\delta(\delta g)$^-closed set is gδ-closed set in (X, τ). Consequently f^{-1}(V) is gδ-closed set in (X, τ). Hence f is gδ-continuous.
ii. Let V be a closed set in (Y, σ). Since f is δ(δg)^-continuous, f^{-1}(V) is $\delta(\delta g)$^-closed in (X, τ). By Theorem 3.7(ii) [8], every $\delta(\delta g)$^-closed set is gpr-closed set. It follows that f^{-1}(V) is gpr-closed set in (X, τ) and f is gpr-continuous.
iii. Let V be a closed set in (Y, σ). Since f is δ(δg)^-continuous, f^{-1}(V) is $\delta(\delta g)$^-closed in (X, τ). By Theorem 3.7(iii) [8], every $\delta(\delta g)$^-closed set is gspr-closed set. So, f^{-1}(V) is gspr-closed set in (X, τ). Thus f is gspr-continuous.

Example: Let X =Y = {a, b, c} with topologies τ ={X, φ, {a, c}} and σ ={Y, φ, {c}}. Let f: (X, τ) ⟶ (Y, τ) be the identity map. Then f is gδ-continuous, gpr-continuous and gspr-continuous but not δ(δg)^-continuous map, Since the closed set {c} is gδ-closed, gpr-closed and gspr-closed but not δ(δg)^-closed set in (X,τ).

Theorem 4.8: Let f: (X, τ) ⟶ (Y, τ) be $\delta(\delta g)$^-continuous map. Then f is
i. rg-continuous iii. rwg-continuous ii. δgs-continuous iv. $\delta g^{\#}$-continuous
The converse need not be true.

Proof:

i. Let V be a closed set in (Y, σ). Since f is $\delta(\delta g)$^-continuous, then f^{-1}(V) is $\delta(\delta g)$^-closed in (X, τ). By Theorem 3.9(i) [8], every δ(δg)^-closed set is rg-closed set in (X, τ). Consequently f^{-1}(V) is rg-closed set in (X, τ). Hence f is rg-continuous.
ii. Let V be a closed set in (Y, σ). Since f is $\delta(\delta g)$^-continuous, f^{-1}(V) is δ(δg)^-closed in (X, τ). By Theorem 3.8(ii) [8], every δ(δg)^-closed set is δgs-closed set. So f^{-1}(V) is δgs-closed set in (X, τ) and f is δgs-continuous.

iii. Let V be a closed set in (Y, σ). Since f is $\delta(\delta g)$^-continuous, f^{-1}(V) is δ(δg)^-closed in (X, τ). By Theorem 3.9(ii) [8], every $\delta(\delta g)$^-closed set is rwg-closed set. It follows that f^{-1}(V) is rwg-closed set in (X, τ). Hence f is rwg-continuous.

iv. Let V be a closed set in (Y, σ). Since f is $\delta(\delta g)$^-continuous, f^{-1}(V) is δ(δg)^-closed in (X, τ). By Theorem 3.8(i) [8], every $\delta(\delta g)$^-closed set is $\delta g^{\#}$-closed set. Consequently f^{-1}(V) is $\delta g^{\#}$-closed set in (X, τ). Hence f is gspr-continuous.

Example: Let X = {a, b, c}=Y with topologies τ ={X, φ, {a}} and σ ={Y, φ, {a, b}}. Define a map f: (X, τ) ⟶ (Y, σ) by f(a) ={a}, f(b) ={c}, f(c) ={b}. Then f is rg-continuous, rwg-continuous, δgs-continuous and $\delta g^{\#}$-continuous but not $\delta(\delta g)$^-continuous, Since the closed set {a, c} is rg-closed, rwg-closed, δgs-closed and $\delta g^{\#}$-closed but not $\delta(\delta g)$^-closed in (X, τ).

Theorem 4.9: Let f: (X, τ) ⟶ (Y, σ) be $\delta(\delta g)$^-continuous map. Then f is
i. πg-continuous iv. πgp-continuous ii. πgα-continuous iii. πgs-continuous
vi. πgsp-continuous v. πgb -continuous

Proof:

i. Let V be a closed set in (Y, σ). Since f is δ(δg)^-continuous, then f^{-1}(V) is $\delta(\delta g)$^-closed in (X, τ). By Theorem 3.10(i) [8], every $\delta(\delta g)$^-closed set is πg-closed set in (X, τ). Consequently f^{-1}(V) is πg-closed set in (X, τ). Hence f is πg-continuous.

ii. Let V be a closed set in (Y, σ). Since f is δ(δg)^-continuous, f^{-1}(V) is $\delta(\delta g)$^-closed in (X, τ). By Theorem 3.11(i) [8], every $\delta(\delta g)$^-closed set is πgα-closed set. Consequently f^{-1}(V) is πgα-closed set in (X, τ). Hence f is πgα-continuous.

iii. Let V be a closed set in (Y, σ). Since f is δ(δg)^-continuous, f^{-1}(V) is $\delta(\delta g)$^-closed in (X, τ). By Theorem 3.11(ii) [8], every $\delta(\delta g)$^-closed set is πgs-closed set. Consequently f^{-1}(V) is πgs-closed set in (X, τ). Hence f is πgs-continuous.

iv. Let V be a closed set in (Y, σ). Since f is $\delta(\delta g)$^-continuous, f^{-1}(V) is $\delta(\delta g)$^-closed in (X, τ). By Theorem 3.10(ii) [8], every $\delta(\delta g)$^-closed set is πgp-closed set. Consequently f^{-1}(V) is πgp-closed set in (X, τ). Hence f is πgp-continuous.

v. Let V be a closed set in (Y, σ). Since f is $\delta(\delta g)$^-continuous, f^{-1}(V) is $\delta(\delta g)$^-closed in (X, τ). By Theorem 3.10(iii) [8], every $\delta(\delta g)$^-closed set is πgb-closed set. Consequently f^{-1}(V) is πgb-closed set in (X, τ). Hence f is πgb-continuous.

vi. Let V be a closed set in (Y, σ). Since f is $\delta(\delta g)$^-continuous, f^{-1}(V) is δ(δg)^-closed in (X, τ). By Theorem 3.11(iii) [8], every $\delta(\delta g)$^-closed set is πgsp-closed set. Consequently f^{-1}(V) is πgsp-closed set in (X, τ). Hence f is πgsp-continuous.

Remark 4.10: The converse of the above Theorem need not hold which shown in the following example.

Example: Let X = {a, b, c} =Y with topologies τ = {X, φ, {a, c}} and σ ={Y, φ, {b}}. Define a map f: (X, τ) ⟶ (Y, τ) by f(a) ={c}, f(b) = {a}, f(c) = {b}. Then f is πg-continuous, πgp-continuous, πgα-continuous, πgb –continuous, πgs-continuous, πgsp-continuous but not δ(δg)^-continuous map, Since the closed set {c} is πg-closed, πgp-closed, πgα-closed, πgb-closed, πgs-closed and πgsp-closed but not $\delta(\delta g)$^-closed set in (X, τ).

Definition 4.11: We introduce the following definition.

A function f: (X, τ) ⟶ (Y, τ) is called **almost $\boldsymbol{\delta(\delta g)}$^-continuous,** if f^{-1}(V) is $\delta(\delta g)$^-closed in X for every regular closed set V of Y.

Theorem 4.12: Let f: X ⟶ Y be a function. The following statements are equivalent.
i. *f* is almost $\delta(\delta g)$^-continuous.
ii. $f^{-1}(V)$ is $\delta(\delta g)$^-open in X for every regular open set V of Y.

Proof:
$(i) \Rightarrow (ii)$: Given, f is almost $\delta(\delta g)$^-continuous. Let V be a regular open subset of Y. Then (Y − V) regular closed. It follows that, $f^{-1}(Y-V)$ = X − $f^{-1}(V)$ is δ(δg)^-closed in X. Hence $f^{-1}(V)$ is $\delta(\delta g)$^-open in X.

$(ii) \Rightarrow (i)$: Let V be a regular closed subset of Y. Then (Y−V) is regular open. By hypothesis, $f^{-1}(Y-V)$ = X − $f^{-1}(V)$ is $\delta(\delta g)$^-open in X. Therefore, $f^{-1}(V)$ is $\delta(\delta g)$^-closed. Hence f is almost $\delta(\delta g)$^-continuous.

Theorem 4.13: Every $\delta(\delta g)$^-continuous function is almost $\delta(\delta g)$^-continuous.

Proof: Let f : X → Y be a $\delta(\delta g)$^-continuous function and V be any regular-closed in Y, then V is closed in Y. Since f is $\delta(\delta g)$^-continuous function, $f^{-1}(V)$ is $\delta(\delta g)$^-closed in X. Hence f is almost $\delta(\delta g)$^-continuous.

Remark 4.14: The composition of two $\delta(\delta g)$^-continuous need not be $\delta(\delta g)$^-continuous.

Example: Let X = {a, b, c} = Y = Z with topologies τ = {X, φ, {a, c}}, σ = {Y, φ, {a}} and η = {Z, φ, {c}}. Let f: (X, τ) ⟶ (Y, σ) defined by f(a) = b, f(b) = c and f(c) = a. Let g: (Y, σ) ⟶ (Z, η) defined by g(a) = c, g(b) = a and g(c) =b. Then both f and g are $\delta(\delta g)$^-continuous. But $(g \circ f)^{-1}$({a, c}) = $f^{-1}(g^{-1}(\{a,c\}))$ = $f^{-1}(\{a,b\}) = \{a,c\}$ which is not $\delta(\delta g)$^-closed in (X, τ).

Theorem 4.15: Let f : (X, τ) ⟶ (Y, σ) be $\delta(\delta g)$^-continuous and g : (Y, σ) ⟶ (Z, η) be δ-continuous function. Then ($g \circ f$) : (X, τ) ⟶ (Z, η) is $\delta(\delta g)$^-continuous function.

Proof: Let V be any closed set in (Z, η). Since g is δ-continuous, it follows that $g^{-1}(V)$ is δ-closed in (Y, σ). Since every δ-closed set is closed, $g^{-1}(V)$ is closed in (Y, σ). Since f is $\delta(\delta g)$^-continuous, which implies that $f^{-1}(g^{-1}(V))$ = $(g \circ f)^{-1}$(V) is $\delta(\delta g)$^-closed in (X, τ). Hence ($g \circ f$) is $\delta(\delta g)$^-continuous.

Theorem 4.16: Let f : (X, τ) ⟶ (Y, σ) and g : (Y, σ) ⟶ (Z, η) be δ-continuous maps, then their composition map ($g \circ f$) : (X, τ) ⟶ (Z, η) is $\delta(\delta g)$^-continuous map.

Proof: Let V be any closed set in (Z, η). Since g is δ-continuous, $g^{-1}(V)$ is δ-closed and it is closed in (Y, σ). By hypothesis, f is δ-continuous. Consequently, $f^{-1}(g^{-1}(V))$ = $(g \circ f)^{-1}$(V) is δ-closed. Since every δ-closed set is $\delta(\delta g)$^-closed (Theorem 3.3 in [7]), $(g \circ f)^{-1}$(V) is $\delta(\delta g)$^-closed in (X, τ). Hence ($g \circ f$) is $\delta(\delta g)$^-continuous.

Remark 4.17: $\delta(\delta g)$^-continuity is independent from sg-continuous, gs-continuous, *g-continuous, αg-continuous and δg-continuous.

Example:
i. Let X = Y = {a, b, c} with τ = {X, φ, {a}} and σ = {Y, φ, {a, c}}. Let f : (X, τ) ⟶ (Y, σ) be an identity map. Then f is sg-continuous, gs-continuous, *g-continuous, αg-continuous and δg-continuous but not δ(δg)^ -continuous, Since the closed set {b} in (Y, σ), $f^{-1}(b)$ = {b} is not δ(δg)^-closed in (X, τ).
ii. Let X = Y = {a, b, c} with τ = {X, φ, {a, b}, {b, c}, {b}} and σ = {Y, φ, {a, c}}. Let f : (X, τ) ⟶ (Y, σ) be an identity map. Then f is δ(δg)^ -continuous but not sg-continuous, gs-continuous, *g-continuous, αg-continuous and δg-continuous, Since the closed set {a,b} in (Y, σ), $f^{-1}(a,b)$ = {a, b} is not sg-closed, gs-closed, *g-closed, αg-closed and δg-closed.

Remark 4.18: δ(δg)^-continuity is independent from g*s-continuous.

Example:
i. Let X = Y = {a, b, c} with τ = {X, φ, {a}, {a, b}} and σ = {Y, φ, {a}, {a, b}}. Let f: (X, τ) ⟶ (Y, σ) be a function defined by f(a) = c, f(b) = a, f(c) = c. Then f is δ(δg)^-continuous but not g*s-continuous, Since the closed set {c} in (Y, σ), $f^{-1}(c)$ = {a, c} is δ(δg)^-closed but not g*s-closed in (X, τ).

ii. Let X = Y = {a, b, c} with τ = {X, φ, {a}, {a, b}} and σ = {Y, φ, {a}, {a, b}}. Let f: (X, τ) ⟶ (Y, σ) be a function defined by f(a) = a, f(b) = c, f(c) = b. Then f is g*s-continuous but not δ(δg)^-continuous, Since the closed set {c} in (Y, σ), $f^{-1}(c)$ = {b} is g*s-closed but not $\delta(\delta g)$^-closed in (X, τ).

REFERENCES

1. Balachandran K and Sundaram P *On generalised continuous maps in topological spaces*, Mem. Fac. Sci. Kochi. Univ. Ser.A. Math, 12 (1991), 5-13.
2. Dontchev J and Ganster M *On δ-generalised closed sets and* $T_{3/4}$*-spaces*, mem. Fac. Sci. Kochi. Univ. Ser. A, Math., 17 (1996), 15-31.
3. Meena K and Sivakamasundari K, *δ(δg)*-closed sets in topological spaces*, International Journal of Research in Science, Engineering and Technology, 3 (2014), 49-54.
4. Meena K and Sivakamasundari K, *Δ*-Continuous functions in topological spaces*, International conference on recent advances in engineering science and management, 4 (2015), 48-56.
5. Noiri T, *On δ-continuous functions*, J.Koreanmath.soc., 16 (1980), 161-166.
6. Norman Levine, *Generalised closed sets in topology*, Rend.circ.Mat.palermo, 19 (1970), 89-96.
7. Lellis Thivagar M, $\delta\hat{g}$ *-closed sets in Topological spaces*, Gen. Math. Notes, 2 (2010), 17-25.
8. Stella Irene Mary J and Janaranjana Sri S On $\delta(\delta g)$^*-closed sets in Topological spaces*, Int. journal of Mathematical Archive, 7(10) (2016), 103-107.
9. Sudha R and Sivakamasundari K *δg*-closed sets in topological spaces*, Int. journal of Mathematical Archive, 3 (2012), 1222-1230.
10. Sudha R and Sivakamasundari K, *δg*-continuous functions in topological spaces*, Int.journal of Computer applications, 18(74) (2013), 21-24.
11. Veerakumar, $\hat{g}$ *-closed sets in topological spaces*, Bull. Alllah. Math. Soc., 18 (2003), 99-112.
12. Velicko, *H-closed spaces* Amer. Math. Soc. Transl., 78 (1968), 103-118.

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