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THE SIMPLE SOLUTION OF THE BIGGEST MATH PROBLEM

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ABSTRACT

In this short note we present a simple proof of the Riemann hypothesis (RH) by using Taylor quadratic approximation.

Key words: *Riemann hypothesis, Dirichlet eta function, Quadratic approximation, Nontrivial roots.*

1. INTRODUCTION

Riemann hypothesis is a conjecture that the Riemann zeta function has its zeros only at the negative even integers and the complex numbers (nontrivial roots) with real part 1/2. It was proposed by Bernhard Riemann (1859), after whom it is named.

The Riemann hypothesis implies results about the distribution of prime numbers. Along with suitable generalizations, some mathematicians consider it the most important unresolved problem in mathematics. The Riemann hypothesis is part of Hilbert's eighth problem in David Hilbert`s list of 23 unsolved problems, it is also one of the Clay Mathematics Institute`s Millennium Prize Problems, through this note we produced the proof by giving an approximate form for the modulus in the Dirichlet eta function to reach the symmetry form around $Re(s) = 0.5$ in the critical strip $Re(s) \in (0,1)$, the property which specialize the nontrivial roots .

2. MATERIAL AND METHODS

It is known that [1]:

$$\zeta(s) = (1-2^{1-s})^{-1}\eta(s),\ \eta(s) = \sum_{n=1}^{\infty}(-1)^{n-1}n^{-s},\ \ Re(s) > 0. \tag{1}$$

Let $s = \sigma + i\gamma$, s is a nontrivial root for Riemann zeta function in the critical strip, $\sigma \in (0,0.5)$, $\gamma \in (-\infty,\infty)$.

$$\therefore \eta(s) = \sum_{n=1}^{\infty}(-1)^{n-1}n^{-s} = \sum_{n=1}^{\infty}(-1)^{n-1}n^{-\sigma}n^{-i\gamma} = 0 \tag{2}$$

The quadratic approximation with fixed point $a = 0.5$:

$$\therefore \sum_{n=1}^{\infty}(-1)^{n-1}\left(n^{-0.5} - n^{-0.5}(\ln n)(\sigma-0.5) + n^{-0.5}\frac{\left((\ln n)(\sigma-0.5)\right)^2}{2!}\right)n^{-i\gamma} \cong 0 \tag{3}$$

In the equation (3) , the error increases as n approaches infinity but this is ineffective on the accuracy of the equation because of Dirichlet eta function convergence[2] , to get the better accuracy :

The same method to define $(n^{\sigma-1} + n^{-\sigma})$ with the fixed point $a = 0.5$:

$$\begin{aligned}\therefore n^{\sigma-1} + n^{-\sigma} \cong\ & (n^{-0.5} + n^{-0.5}) + \left(n^{-0.5}(\ln n)(\sigma-0.5) - n^{-0.5}(\ln n)(\sigma-0.5)\right) \\ & + \left(n^{-0.5}\frac{\left((ln\, n)(\sigma-0.5)\right)^2}{2!} + n^{-0.5}\frac{\left((ln\, n)(\sigma-0.5)\right)^2}{2!}\right)\end{aligned} \tag{4}$$

$$\therefore n^{\sigma-1} + n^{-\sigma} \cong 2n^{-0.5} + n^{-0.5}\left((ln\, n)(\sigma-0.5)\right)^2 \tag{5}$$

$$\therefore \left((ln\, n)(\sigma-0.5)\right)^2 \cong \frac{n^{\sigma-1}+n^{-\sigma}}{n^{-0.5}} - 2 = n^{\sigma-0.5} + n^{0.5-\sigma} - 2 \tag{6}$$

$$\therefore (\ln n)(\sigma-0.5) \cong \sqrt{n^{\sigma-0.5} + n^{0.5-\sigma} - 2} = \sqrt{\left(n^{\frac{\sigma-0.5}{2}} - n^{\frac{0.5-\sigma}{2}}\right)^2} = n^{\frac{\sigma-0.5}{2}} - n^{\frac{0.5-\sigma}{2}} \tag{7}$$

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From equations (6), (7) substitute in equation (3):

$$\therefore \sum_{n=1}^{\infty}(-1)^{n-1}\left(n^{-0.5} - n^{-0.5}\left(n^{\frac{\sigma-0.5}{2}} - n^{\frac{0.5-\sigma}{2}}\right) + n^{-0.5}\left(\frac{\left(n^{\sigma-0.5}+n^{0.5-\sigma}-2\right)}{2}\right)\right)n^{-i\gamma} \cong 0 \qquad (8)$$

Now equation (8) is accurate more than equation (3)

Simplify in the equation (8) leads to:

$$\sum_{n=1}^{\infty}(-1)^{n-1}\left(n^{\frac{0.5-\sigma}{2}-0.5} - n^{-\left(\frac{0.5-\sigma}{2}\right)-0.5} + \frac{n^{-\sigma}+n^{\sigma-1}}{2}\right)n^{-i\gamma} \cong 0 \qquad (9)$$

$$\therefore \sum_{n=1}^{\infty}(-1)^{n-1}n^{\frac{0.5-\sigma}{2}-0.5-i\gamma} - \sum_{n=1}^{\infty}(-1)^{n-1}n^{-\left(\frac{0.5-\sigma}{2}\right)-0.5-i\gamma}$$
$$+0.5\left(\sum_{n=1}^{\infty}(-1)^{n-1}n^{-\sigma-i\gamma} + \sum_{n=1}^{\infty}(-1)^{n-1}n^{\sigma-1-i\gamma}\right) \cong 0 \qquad (10)$$

$$\therefore \eta\left(0.5 - \left(\frac{0.5-\sigma}{2}\right) + i\gamma\right) - \eta\left(0.5 + \left(\frac{0.5-\sigma}{2}\right) + i\gamma\right) + 0.5\left(\eta(\sigma+i\gamma) + \eta(1-\sigma+i\gamma)\right) \cong 0 \qquad (11)$$

From symmetry of nontrivial roots around $\sigma = 0.5$ in the critical strip [3], then both $\sigma + i\gamma$, $1-\sigma+i\gamma$ are roots for Dirichlet eta function.

$$\therefore \eta\left(0.5 - \left(\frac{0.5-\sigma}{2}\right) + i\gamma\right) \cong \eta\left(0.5 + \left(\frac{0.5-\sigma}{2}\right) + i\gamma\right) \qquad (12)$$

Here we can find the symmetry in the Dirichlet eta function around $\sigma = 0.5$, that means equation (12) includes other forms of the roots $\sigma + i\gamma$, $1-\sigma+i\gamma$.

$\therefore 0.5 - \left(\frac{0.5-\sigma}{2}\right) + i\gamma = \sigma + i\gamma$, $0.5 + \left(\frac{0.5-\sigma}{2}\right) + i\gamma = 1-\sigma+i\gamma$, then $\sigma = 0.5$.

Or equation (12) includes one point with two different forms, then $0.5 - \left(\frac{0.5-\sigma}{2}\right) + i\gamma = 0.5 + \left(\frac{0.5-\sigma}{2}\right) + i\gamma$ then $\sigma = 0.5$.

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