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THE ONSET OF CONVECTION IN A ROTATING POROUS LAYER USING A THERMAL NON-EQUILLIBRIUM MODEL WITH A CATTANEO EFFECT IN THE SOLID PHASE

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ABSTRACT

This paper aims to investigate the onset of convection in a layer of fluid saturated Brinkman with rotating porous medium taking into account fluid inertia and local thermal non-equillibrium(LTNE) between the solid and fluid phases with Cattaneo effect in the solid. A two field is used for the energy equations each representing the solid and fluid phases separately. The usual Fourier heat-transfer law is retained in the fluid phase while the solid phase is allowed to transfer heat via Cattaneo heat flux theory.

Keywords: *Cattaneo effect, porous medium, local thermal non-equillibrium, linear stability, thermal convection, rotation.*

INTRODUCTION

In modeling a fluid-saturated porous medium all the above investigations on double diffusive convection have assumed a state of local thermal equilibrium (LTE) between the fluid and the solid phase at any point in the medium. This is a common practice for most of the studies where the temperature gradient at any location between the two phases is assumed to be negligible. For many practical applications, involving high-speed flows or large temperature differences between the fluid and solid phases, the assumption of local thermal equilibrium is inadequate and it is important to take account of the thermal non-equilibrium effects. Due to applications of porous media theory in drying, freezing of foods and other mundane materials and applications in everyday technology such as microwave heating, rapid heat transfer from computer chips via use of porous metal foams and their use in heat pipes, it is believed that local thermal non-equilibrium (LTNE) theory will play a major role in future developments.

Recently, attention has been given to the LTNE model in the study of convection heat transfer in porous media. Much of this work has been reviewed in recent books by Ingham and Pop (1998, 2005). Kuznetsov (1996) studied a perturbation solution for a thermal non-equilibrium fluid flow through a three-dimensional sensible storage packed bed. The review of Kuznetsov (1998) gives detailed information about the most but very latest works on thermal non-equilibrium effects on internal forced convection flows. An excellent review of research on local thermal non-equilibrium phenomena in porous medium convection, primarily free and forced convection boundary layers and free convection within cavities, is given by Rees and Pop (2005). Free convection in a square porous cavity using a thermal non-equilibrium model is studied by Baytas and Pop (2002). A review of thermal non- The problem of two-dimensional steady mixed convection in a vertical porous layer using thermal non-equilibrium model is investigated numerically by Saeid (2004). The effect of thermal non-equilibrium on the onset of convection in a porous layer using the Lapwood-Brinkman model and also including anisotropy in permeability and thermal diffusivity in a densely packed porous layer have been investigated by Malashetty et al. (2005). Straughan (2006) has considered a problem of thermal convection in a fluid-saturated porous layer using a global nonlinear stability analysis with a thermal non-equilibrium model. He has established the equivalence of the linear instability and nonlinear stability boundaries for the thermal convection in a rotating porous layer with the Darcy law using the non-equilibrium model. More recently, Malashetty and co-workers (2009) have studied the LTNE problem including rotation, boundary effect and second diffusive component on thermal convection in a porous layer.

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Many applications in engineering disciplines as well as in circumstances linked to modern porous media involve high permeability porous media and in such situations the Darcy equation fails to give satisfactory results. Therefore use of non-Darcy models, which takes care of boundary and/or inertia effects, is of fundamental and practical interest to obtain accurate results for high permeability porous media. While the rotation effect on double diffusive convection in porous media is studied for the case of local thermal equilibrium with non-Darcian models in recent past (2006) the same is not studied for the case of local thermal non-equilibrium model. In this paper, we perform the linear stability analysis of the onset convection in a porous layer saturated with a binary fluid employing Brinkman model subjected to rotation with the assumption that the fluid and solid phases are not in local thermal equilibrium.

The classical energy equation used in the study of convective instability problems in a fluid/porous layer is a parabolic-type partial differential equation which allows an infinite speed for heat transport. The new theories make use of modified versions that involve hyperbolic-type heat transport equation admitting finite speed for heat transport. Thus, heat transport is viewed as a wave phenomenon rather than a diffusion phenomenon and this is referred to as second sound. In particular, the second sound effect appears greater in solids, especially those involved in porous metallic foams. A key way to introduce this effect is to use Cattaneo (1948) law for the heat flux. Based on this approach, studies have been undertaken in the past to investigate thermal convection in a fluid layer (1984) and also in a fluid-saturated Darcy porous medium using a local thermal equilibrium(LTE) model with Cattaneo-Fox and Cattaneo-Christov effects(1984,2010) The details about the developments on this topic are amply documented in the book by straughan (2011). The Cattaneo effect on thermal convection in a fluid-saturated Darcy porous medium using a local thermal non-equilibrium (LTNE) model is investigated for the first time by Straughan (2013). In addition to performing linear instability analysis, a global non-linear stability threshold is determined. The effect of second sound is delineated in a detailed manner.

The intent of the present paper is to investigate this problem in a layer of Newtonian fluid-saturated Brinkman porous medium using a LTNE model, which allows the fluid and solid media to be at different temperatures, and including fluid inetia. As considered by Straughan (2013), the usual Fourier heat transfer law is retained in the fluid phase while temperature waves are allowed in the solid phase via a Cattaneo-like heat flux theory as the second sound effect is dominant in the solid skeleton. Although the linear stability analysis is modified, it is still possible to proceed analytically to find the condition for the onset criterion is affected by the combined effect of rotation and local thermal non-equilibrium.

MATHEMATICAL FORMULATION

We consider an infinite horizontal fluid-saturated porous layer confined between the planes z = 0 and z = d, with the vertically downward gravity force $\mathbf{g}$ acting on it. The lower surface is held at constant temperature, T_l, while the upper surface is at $T_u < T_l$. A Cartesian coordinate system (x,y,z) is chosen such that the origin is at the bottom of the porous layer and the z-axis vertically upward in the presence of gravitational field. The solid and fluid phases of the porous medium are assumed to be in local thermal non-equilibrium (LTNE) with two field model for temperatures. The solid temperature equation is modified to allow the heat transfer via Cattaneo heat flux theory, while the usual Forier heat transfer law is ued in the fluid. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. The porous layer is subjected to the rotation with an angular velocity $\mathbf{\Omega} = (0,0,\Omega)$ about the z-axis. The Brinkman model is used as the momentum equation

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \frac{\nu}{K}\mathbf{q}\mathbf{\Omega}\frac{2}{\varepsilon}\mathbf{q} \times = -\frac{1}{\rho_0}\nabla p + \mathbf{g}\frac{\rho_f}{\rho_0} + \mathbf{q}\nabla^2 \quad . \tag{1}$$

where $\mathbf{q}(u, v, w)$ is the velocity vector, p is the pressure, ε and K denote porosity and permeability, respectively, ρ and ν are density and kinematic viscosity, respectively.

In modeling energy equation for a fluid-saturated porous system, two kinds of theoretical approaches have been used. In the first model, the fluid and solid structures are assumed to be in local thermal equilibrium. This assumption is satisfactory for small-pore media such as geothermal reservoirs and fibrous insulations and small temperature differences between the phases. In the second kind of approach, the fluid and solid structures are assumed to be in thermal non-equilibrium. For many applications involving high-speed flows or large temperature difference between the fluid and solid phases, it is important to take account of the thermal non-equilibrium effects. If the temperatures difference between phases is a very important safety parameter (e.g., fixed bed nuclear propulsion systems and nuclear reactor modeling), the thermal non-equilibrium model in the porous media is an indispensable model.

The local thermal non-equilibrium, which account for the transfer of heat between the fluid and solid phases is considered. A two-field model that represents the fluid and solid phase temperature fields separately, is employed for the energy equation

$$\varepsilon\left(\rho_0\, c\right)_f\,\frac{\partial T_f}{\partial t}+\left(\rho_0\, c\right)_f\,(\mathbf{q}\cdot\nabla)T_f=\varepsilon\, k_f\,\nabla^2 T_f+h\left(T_s-T_f\right), \tag{2}$$

$$(1-\varepsilon)\,\left(\rho_0 c\right)_s\,\frac{\partial T_s}{\partial t}=(1-\varepsilon)\,k_s\nabla^2 T_s-h\left(T_s-T_f\right), \tag{3}$$

where c is the specific heat, k, the thermal conductivity and h being the inter-phase heat transfer coefficient and subscripts f and s denote fluid and solid phase, respectively. In two-field model the energy equations are coupled by means of the terms, which accounts for the heat lost to or gained from the other phase. The inter-phase heat transfer coefficient h depends on the nature of the porous matrix and the saturating fluid and the value of this coefficient has been the subject of intense experimental interest. Large values of h correspond to a rapid transfer of heat between the phases (LTE) and small values of h gives rise to relatively strong thermal non-equilibrium effects. In Eqs. (2)-(3) T_f and T_s are intrinsic average of the temperature fields and this allows one to set $T_f=T_s=T_w$ whenever the boundary of the porous medium is maintained at the temperature T_w. The equation of continuity, solute concentration and state are

$$\nabla\cdot\mathbf{q}=0\,, \tag{4}$$

$$\tau_s\,\frac{\partial\mathbf{Q}}{\partial t}=-\mathbf{Q}-\kappa_s\nabla T_s\,, \tag{5}$$

$$\rho_f=\rho_0\left[1-\alpha_t\left(T_f-T_l\right)\right].,\tag{6}$$

Where $\mathbf{Q},\tau_s,\alpha_t,$ and κ_s are the heat flux in the solid, the solid thermal relaxation time, thermal and solute expansion coefficients and solute diffusivity respectively.

The basic state is assumed to be quiescent and is given by

$$\mathbf{q}_b=(0,0,0)\;\;T_f=T_{fb}(z),\;\;T_s=T_{sb}(z),\;\;S=S_b\left(z\right). \tag{7}$$

The basic state temperatures and concentration satisfy the equations

$$\frac{d^2T_{fb}}{dz^2}=0\,,\;\frac{d^2T_{sb}}{dz^2}=0. \tag{8}$$

with boundary conditions

$$T_{fb}=T_{sb}=T_l\quad at\quad z=0\,, \tag{9}$$

$$T_{fb}=T_{sb}=T_u\quad at\quad z=d\,, \tag{10}$$

so that the conduction state solutions are given by

$$T_{fb}=T_{sb}=-\frac{\Delta T}{d}\,z+T_l\,, \tag{11}$$

Let us non dimensionalize using the following transformations,

$$\left(x,\;y,\;z\right)=\left(x^*,\;y^*,\;z^*\right)d,\;\;t=\frac{\left(\rho_0 c\right)_f d^2}{k_f}t^*,\;\;\left(u',v',w'\right)=\frac{\varepsilon\, k_f}{\left(\rho_0 c\right)_f d}\left(u^*,v^*,w^*\right),$$

$$T_f=(\Delta T)T_f^*,\;\;T_s=(\Delta T)T_s^*,\;\;\mathbf{Q}=\frac{\kappa_s\Delta T}{d}\mathbf{Q}^*. \tag{12}$$

to obtain non-dimensional equations in the form (on dropping the asterisks for simplicity),

$$\left[\left(\frac{1}{Va}\frac{\partial}{\partial t}-Da\nabla^2+1\right)^2\nabla^2+Ta\frac{\partial^2}{\partial z^2}\right]w=\left(\frac{1}{Va}\frac{\partial}{\partial t}-Da\nabla^2+1\right)\left(Ra_T\nabla_1^2T_f-Ra_S\nabla_1^2S\right), \tag{13}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T_f+(\mathbf{q}\cdot\nabla)T_f=w+H\left(T_s-T_f\right), \tag{14}$$

$$\alpha\frac{\partial \mathrm{T}_s}{\partial t}=-\nabla.\mathbf{Q}-\gamma\, H\left(T_f-T_s\right), \tag{15}$$

$$\left(\tau\frac{\partial}{\partial t}+1\right)\mathbf{Q}=-\nabla T_s, \tag{16}$$

where $Va=\varepsilon\Pr/Da$, the Vadasz number, $Ta=\left(2\Omega K/\varepsilon\nu\right)^2$, the Taylor number, $Ra_T=\beta_T gd(\Delta T)K/\varepsilon\nu\kappa_f$, the thermal Rayleigh number, $Ra_S=\beta_S g\Delta SdK/\varepsilon\nu\kappa_f$, solute Rayleigh number, $H=hd^2/\varepsilon k_f$, the inter-phase heat transfer coefficient, $\alpha=\kappa_f/\kappa_s$, the ratio of diffusivities, $\gamma=\varepsilon k_f/(1-\varepsilon)k_s$, the porosity modified conductivity ratio, $\tau=\dfrac{\tau_s\kappa_f}{d^2}$ the non dimensional solid thermal relaxation time parameter, $Pr=\nu/\kappa_f$, $Da=K/d^2$ and $\kappa_f=\dfrac{k_f}{(\rho_0 c)_f}$ being the effective Prandtl number, Darcy number and thermal diffusivity of the fluid respectively.

It is worth mentioning that the Rayleigh number Ra_T defined above is based on the properties of the fluid while

$$Ra_{T_{LTE}}=\left(\frac{\gamma}{1+\gamma}\right)Ra_T=\frac{\rho_f g\beta_T(\Delta T)Kd}{\mu\left[\varepsilon k_f+(1-\varepsilon)k_s\right]}, \tag{17}$$

is the Rayleigh number based on the mean properties of the porous medium and it is this latter definition, which is used in the thermal equilibrium model.

Since the fluid and solid phases are not in thermal equilibrium, the use of appropriate thermal boundary conditions may pose a difficulty. However, the assumption that the solid and fluid phases share the same temperatures at that of the boundary temperatures helps in overcoming this difficulty. Accordingly, Eqs. (13)- (16) are solved for stress free isothermal isosolutal boundaries. Hence the boundary conditions for the perturbed variables are given by

$$w=\frac{\partial^2 w}{\partial z^2}=T_f=T_s=0 \text{ at } z=0,1. \tag{18}$$

The stress free boundary conditions are chosen for mathematical simplicity, without qualitatively important physical effect being lost. The use of stress-free boundary conditions is a useful mathematical simplification but is not physically sound. The correct boundary conditions for a viscous binary fluid are to impose rigid-rigid boundary conditions but then the problem is not tractable analytically.

LINEAR STABILITY ANALYSIS

The linear stability of the static solution can be discussed by neglecting all quadratic terms and seeking solutions of the form exp(ωt), where ω is the complex quantity. No instability arise from the decoupled equation for C,E and G while the remaining equations yield an expression for the Darcy- Rayleigh number.

$$Ra_T=\frac{\delta^2}{a^2}\left(\frac{\left[\delta^2+H(1+\gamma)\right]+\omega\left[\alpha\omega+\delta^2(1+\alpha)+H(\alpha+\gamma)(\tau\omega+1)\right]-H^2\gamma(\tau\omega+1)}{(\alpha\omega+\delta^2+\gamma H)(\tau\omega+1)}\right)$$
$$\left[\delta^2\left(\frac{\omega}{Va}+1+Da\,\delta^2\right)+\frac{\pi^2 Ta}{1+Da\,\delta^2+\omega/Va}\right] \tag{19}$$

STATIONARY CONVECTION

The direct bifurcation (steady onset) corresponds to $\omega_i=0$ and the steady convection occurs at

$$Ra_T^{St}=\frac{\left[\left(\pi^2+a^2\right)+H(\gamma+1)\right]\left(\left(\pi^2+a^2\right)^2+Da\left(\pi^2+a^2\right)^3+\dfrac{\left(\pi^2+a^2\right)\pi^2 Ta}{Da\left(\pi^2+a^2\right)+1}\right)}{a^2\left[\left(\pi^2+a^2\right)+\gamma H\right]}. \tag{20}$$

CONCLUSION

The Brinkman model was considered to describe the flow in a porous medium and the problem was analysed for stress free isothermal boundaries. Analytical expressions for the occurrence of steady and oscillatory convection were obtained. The Cattaneo effect is found to alter the nature of convective instability when compared to its absence. It is observed that the presence of Cattaneo effect is to investigate the occurrence of convective instability via oscillatory motions when the solid thermal relaxation time parameter τ exceeds sufficiently high value. The effect of Taylor number, *Ta* on the onset of convection is the minimum of Rayleigh number for both stationary and oscillatory states increases with the Taylor number, indicating that the effect of rotation is to enhance the stability of the system in both stationary and oscillatory modes. The effect of inter-phase heat transfer coefficient *H* is found to delay the onset of stationary convection but it exhibits the opposite trend on the onset of oscillatory convection. Moreover, the threshold value of *H* decreases with increasing τ noticeably and marginally with increasing the Vadasz number Va and Darcy number Da, while it increases with increasing the ratio of conductivities α. The effect of increasing Da is to delay the onset of stationary and oscillatory convection, while increasing τ, *Ta*, Va and decreasing α is to hasten the onset of oscillatory convection. The size of the convection cells is enlarged with increasing τ, *Ta*, Va and Da, but it is narrowed with increasing α.

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