

SOME THEOREMS IN ANTI MULTI FUZZY NORMAL SUBHEMIRINGS OF A HEMIRING

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of an anti multi fuzzy subhemiring of a hemiring.

**Key Words:** Fuzzy subset, multi fuzzy subset, multi fuzzy subhemiring, anti multi fuzzy subhemiring, multi fuzzy normal subhemiring, anti multi fuzzy normal subhemiring.

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring  $(R; +; \cdot)$ . Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra  $(R; +, \cdot)$  is said to be a semiring if  $(R; +)$  and  $(R; \cdot)$  are semigroups satisfying  $a \cdot (b+c) = a \cdot b + a \cdot c$  and  $(b+c) \cdot a = b \cdot a + c \cdot a$  for all  $a, b$  and  $c$  in  $R$ . A semiring  $R$  is said to be additively commutative if  $a+b = b+a$  for all  $a, b$  and  $c$  in  $R$ . A semiring  $R$  may have an identity  $1$ , defined by  $1 \cdot a = a = a \cdot 1$  and a zero  $0$ , defined by  $0+a = a = a+0$  and  $a \cdot 0 = 0 = 0 \cdot a$  for all  $a$  in  $R$ . A semiring  $R$  is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh [14], several researchers explored on the generalization of the concept of fuzzy sets. The notion of anti fuzzy left h-ideals in hemiring was introduced by Akram.M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan[9]. In this paper, we introduce some theorems in anti multi fuzzy normal subhemiring of a hemiring.

1. PRELIMINARIES

**1.1 Definition:** Let  $X$  be a non-empty set. A **fuzzy subset**  $A$  of  $X$  is a function  $A: X \rightarrow [0, 1]$ .

**1.2 Definition:** A **multi fuzzy subset**  $A$  of a set  $X$  is defined as an object of the form  $A = \{ \langle x, \mu_{A1}(x), \mu_{A2}(x), \mu_{A3}(x), \dots, \mu_{An}(x) \rangle / x \in X \}$ , where  $\mu_{Ai}: X \rightarrow [0, 1]$  for all  $i$ . It is denoted as  $A = \langle \mu_{A1}, \mu_{A2}, \mu_{A3}, \dots, \mu_{An} \rangle$ .

**1.3 Definition:** Let  $A$  and  $B$  be any two multi fuzzy subsets of a set  $X$ . We define the following relations and operations:

- (i)  $A \subseteq B$  if and only if  $\mu_{Ai}(x) \leq \mu_{Bi}(x)$  for all  $i$  and for all  $x$  in  $X$ .
- (ii)  $A = B$  if and only if  $\mu_{Ai}(x) = \mu_{Bi}(x)$  for all  $i$  and for all  $x$  in  $X$ .
- (iii)  $A^c = 1-A = \langle 1-\mu_{A1}, 1-\mu_{A2}, 1-\mu_{A3}, \dots, 1-\mu_{An} \rangle$ .
- (iv)  $A \cap B = \{ \langle x, \min\{ \mu_{A1}(x), \mu_{B1}(x) \}, \min\{ \mu_{A2}(x), \mu_{B2}(x) \}, \dots, \min\{ \mu_{An}(x), \mu_{Bn}(x) \} \rangle / x \in X \}$ .
- (v)  $A \cup B = \{ \langle x, \max\{ \mu_{A1}(x), \mu_{B1}(x) \}, \max\{ \mu_{A2}(x), \mu_{B2}(x) \}, \dots, \max\{ \mu_{An}(x), \mu_{Bn}(x) \} \rangle / x \in X \}$ .

**1.4 Definition:** Let  $(R, +, \cdot)$  be a hemiring. A multi fuzzy subset  $A$  of  $R$  is said to be a multi fuzzy subhemiring of  $R$  if it satisfies the following conditions:

- (i)  $\mu_{Ai}(x+y) \geq \min(\mu_{Ai}(x), \mu_{Ai}(y))$  for all  $i$ ,
- (ii)  $\mu_{Ai}(xy) \geq \min(\mu_{Ai}(x), \mu_{Ai}(y))$ , for all  $i$  and for all  $x, y$  in  $R$ .

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**1.5 Definition:** Let  $(R, +, \cdot)$  be a hemiring. A multi fuzzy subhemiring  $A$  of  $R$  is said to be a multi fuzzy normal subhemiring (MFNSHR) of  $R$  if  $\mu_{A_i}(xy) = \mu_{A_i}(yx)$ , for all  $i$  and for all  $x, y$  in  $R$ .

**1.6 Definition:** Let  $(R, +, \cdot)$  be a hemiring. A multi fuzzy subset  $A$  of  $R$  is said to be an anti multi fuzzy subhemiring of  $R$  if it satisfies the following conditions:

- (i)  $\mu_{A_i}(x+y) \leq \max(\mu_{A_i}(x), \mu_{A_i}(y))$  for all  $i$ ,
- (ii)  $\mu_{A_i}(xy) \leq \max(\mu_{A_i}(x), \mu_{A_i}(y))$ , for all  $i$  and for all  $x, y$  in  $R$ .

**1.7 Definition:** Let  $A$  and  $B$  be multi fuzzy subsets of sets  $G$  and  $H$ , respectively. The anti-product of  $A$  and  $B$ , denoted by  $A \times B$ , is defined as  $A \times B = \{ \langle (x, y), \mu_{A_1 \times B_1}(x, y), \mu_{A_2 \times B_2}(x, y), \dots, \mu_{A_n \times B_n}(x, y) \rangle / \text{for all } x \text{ in } G \text{ and } y \text{ in } H \}$ , where  $\mu_{A_i \times B_i}(x, y) = \max \{ \mu_{A_i}(x), \mu_{B_i}(y) \}$  for all  $i$ .

**1.8 Definition:** Let  $A$  be a multi fuzzy subset in a set  $S$ , the anti-strongest multi fuzzy relation on  $S$ , that is a multi fuzzy relation on  $A$  is  $V$  given by  $\mu_{V_i}(x, y) = \max \{ \mu_{A_i}(x), \mu_{A_i}(y) \}$ , for all  $i$  and for all  $x, y$  in  $S$ .

**1.9 Definition:** Let  $(R, +, \cdot)$  and  $(R^1, +, \cdot)$  be any two hemirings. Let  $f: R \rightarrow R^1$  be any function and  $A$  be an anti multi fuzzy subhemiring in  $R$ ,  $V$  be an anti multi fuzzy subhemiring in  $f(R) = R^1$ , defined by  $\mu_{V_i}(y) = \inf_{x \in f^{-1}(y)} \mu_{A_i}(x)$ , for all  $i$  and for all  $x$  in  $R$  and  $y$  in  $R^1$ . Then  $A$  is called a preimage of  $V$  under  $f$  and is denoted by  $f^{-1}(V)$ .

**1.10 Definition:** Let  $A$  be an anti multi fuzzy subhemiring of a hemiring  $(R, +, \cdot)$  and  $a$  in  $R$ . Then the pseudo anti multi fuzzy coset  $(aA)^p$  is defined by  $((a\mu_{A_i})^p)(x) = p(a)\mu_{A_i}(x)$ , for all  $i$  and for every  $x$  in  $R$  and for some  $p$  in  $P$ .

**1.11 Definition:** Let  $(R, +, \cdot)$  be a hemiring. An anti multi fuzzy subhemiring  $A$  of  $R$  is said to be an anti multi fuzzy normal subhemiring (AMFNSHR) of  $R$  if  $\mu_{A_i}(xy) = \mu_{A_i}(yx)$ , for all  $i$  and for all  $x, y$  in  $R$ .

**1.12 Definition:** Let  $A$  be a multi fuzzy subset of  $X$ . For  $\alpha_i$  in  $[0, 1]$ , the lower level subset of  $A$  is the set  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)} = \{x \in X : \mu_{A_i}(x) \leq \alpha_i, \text{ for all } i\}$ .

## 2. ANTI MULTI FUZZY SUBHEMIRINGS

**2.1 Theorem [2]:** Union of any two(a family) of anti multi fuzzy subhemirings of a hemiring  $R$  is an anti multi fuzzy subhemiring of  $R$ .

**2.2 Theorem [2]:** If  $A$  and  $B$  are any two anti multi fuzzy subhemirings of the hemirings  $R_1$  and  $R_2$  respectively, then anti-product  $A \times B$  is an anti multi fuzzy subhemiring of  $R_1 \times R_2$ .

**2.3 Theorem [2]:** Let  $A$  be a multi fuzzy subset of a hemiring  $R$  and  $V$  be the anti-strongest multi fuzzy relation of  $R$ . Then  $A$  is an anti multi fuzzy subhemiring of  $R$  if and only if  $V$  is an anti multi fuzzy subhemiring of  $R \times R$ .

**2.4 Theorem [2]:** Let  $R$  and  $R^1$  be any two hemirings. The homomorphic image (preimage) of an anti multi fuzzy subhemiring of  $R$  is an anti multi fuzzy subhemiring of  $R^1$ .

**2.5 Theorem [2]:** Let  $R$  and  $R^1$  be any two hemirings. The anti-homomorphic image (preimage) of an anti multi fuzzy subhemiring of  $R$  is an anti multi fuzzy subhemiring of  $R^1$ .

**2.6 Theorem [2]:** Let  $A$  be an anti multi fuzzy subhemiring of a hemiring  $H$  and  $f$  is an isomorphism( anti isomorphism) from a hemiring  $R$  onto  $H$ . Then  $A \circ f$  is an anti multi fuzzy subhemiring of  $R$ .

## 3. ANTI MULTI FUZZY NORMAL SUBHEMIRINGS

**3.1 Theorem:** Let  $(R, +, \cdot)$  be a hemiring. If any two anti multi fuzzy normal subhemirings of  $R$ , then their union is an anti multi fuzzy normal subhemiring of  $R$ .

**Proof:** Let  $x$  and  $y$  be in  $R$ . Let  $A = \{(x, \mu_{A_1}(x), \mu_{A_2}(x), \dots, \mu_{A_n}(x)) / x \in R\}$  and  $B = \{(x, \mu_{B_1}(x), \mu_{B_2}(x), \dots, \mu_{B_n}(x)) / x \in R\}$  and also let  $C = A \cup B = \{(x, \mu_{C_1}(x), \mu_{C_2}(x), \dots, \mu_{C_n}(x)) / x \in R\}$ , where  $\max \{ \mu_{A_i}(x), \mu_{B_i}(x) \} = \mu_{C_i}(x)$  for all  $i$ . By Theorem 2.1, clearly  $C$  is an anti multi fuzzy subhemiring of a hemiring  $R$ , since  $A$  and  $B$  are two anti multi fuzzy subhemirings of the hemiring  $R$ . Then,  $\mu_{C_i}(xy) = \max \{ \mu_{A_i}(xy), \mu_{B_i}(xy) \} = \max \{ \mu_{A_i}(yx), \mu_{B_i}(yx) \} = \mu_{C_i}(yx)$ , for all  $i$  and for all  $x$  and  $y$  in  $R$ . Hence  $A \cup B$  is an anti multi fuzzy normal subhemiring of the hemiring  $R$ .

**3.2 Theorem:** Let  $(R, +, \cdot)$  be a hemiring. The union of a family of anti multi fuzzy normal subhemirings of  $R$  is an anti multi fuzzy normal subhemiring of  $R$ .

**Proof:** It is trivial.

**3.3 Theorem:** Let  $A$  and  $B$  be anti multi fuzzy subhemirings of the hemirings  $G$  and  $H$ , respectively. If  $A$  and  $B$  are anti multi fuzzy normal subhemirings, then  $A \times B$  is an anti multi fuzzy normal subhemiring of  $G \times H$ .

**Proof:** Let  $A$  and  $B$  be anti multi fuzzy normal subhemirings of the hemirings  $G$  and  $H$  respectively. By Theorem 2.2, clearly  $A \times B$  is an anti multi fuzzy subhemiring of  $G \times H$ . Let  $x_1$  and  $x_2$  be in  $G$ ,  $y_1$  and  $y_2$  be in  $H$ . Then  $(x_1, y_1)$  and  $(x_2, y_2)$  are in  $G \times H$ . Now,  $\mu_{A_i \times B_i}[(x_1, y_1)(x_2, y_2)] = \max\{\mu_{A_i}(x_1x_2), \mu_{B_i}(y_1y_2)\} = \max\{\mu_{A_i}(x_2x_1), \mu_{B_i}(y_2y_1)\} = \mu_{A_i \times B_i}[(x_2, y_2)(x_1, y_1)]$ . Hence  $A \times B$  is an anti multi fuzzy normal subhemiring of  $G \times H$ .

**3.4 Theorem:** Let  $A$  be a multi fuzzy subset in a hemiring  $R$  and  $V$  be the anti-strongest multi fuzzy relation on  $R$ . Then  $A$  is an anti multi fuzzy normal subhemiring of  $R$  if and only if  $V$  is an anti multi fuzzy normal subhemiring of  $R \times R$ .

**Proof:** It is trivial.

**3.5 Theorem:** Let  $(R, +, \cdot)$  and  $(R^1, +, \cdot)$  be any two hemirings. The homomorphic image of an anti multi fuzzy normal subhemiring of  $R$  is an anti multi fuzzy normal subhemiring of  $R^1$ .

**Proof:** Let  $f: R \rightarrow R^1$  be a homomorphism. Then,  $f(x+y) = f(x)+f(y)$ ,  $f(xy) = f(x)f(y)$ , for all  $x$  and  $y$  in  $R$ . Let  $V = f(A)$ , where  $A$  is an anti multi fuzzy normal subhemiring of a hemiring  $R$ . Now, for  $f(x)$ ,  $f(y)$  in  $R^1$ , By Theorem 2.4, clearly  $V$  is an anti multi fuzzy subhemiring of a hemiring  $R^1$ , since  $A$  is an anti multi fuzzy subhemiring of a hemiring  $R$ . Now,  $\mu_{V_i}(f(x)f(y)) \leq \mu_{A_i}(xy) = \mu_{A_i}(yx) \geq \mu_{V_i}(f(y)f(x)) = \mu_{V_i}(f(y)f(x))$ , which implies that  $\mu_{V_i}(f(x)f(y)) = \mu_{V_i}(f(y)f(x))$ , for all  $i$  and for all  $f(x)$  and  $f(y)$  in  $R^1$ . Hence  $V$  is an anti multi fuzzy normal subhemiring of a hemiring  $R^1$ .

**3.6 Theorem:** Let  $(R, +, \cdot)$  and  $(R^1, +, \cdot)$  be any two hemirings. The homomorphic preimage of an anti multi fuzzy normal subhemiring of  $R^1$  is an anti multi fuzzy normal subhemiring of  $R$ .

**Proof:** Let  $V = f(A)$ , where  $V$  is an anti Multi fuzzy normal subhemiring of a hemiring  $R^1$ . Let  $x$  and  $y$  in  $R$ . By Theorem 2.4, clearly  $A$  is an anti multi fuzzy subhemiring of a hemiring  $R$ , since  $V$  is an anti multi fuzzy subhemiring of a hemiring  $R^1$ . Now,  $\mu_{A_i}(xy) = \mu_{V_i}(f(x)f(y)) = \mu_{V_i}(f(y)f(x)) = \mu_{V_i}(f(y)f(x)) = \mu_{A_i}(yx)$ , which implies that  $\mu_{A_i}(xy) = \mu_{A_i}(yx)$ , for all  $i$  and for all  $x$  and  $y$  in  $R$ . Hence  $A$  is an anti multi fuzzy normal subhemiring of a hemiring  $R$ .

**3.7 Theorem:** Let  $(R, +, \cdot)$  and  $(R^1, +, \cdot)$  be any two hemirings. The anti-homomorphic image of an anti multi fuzzy normal subhemiring of  $R$  is an anti multi fuzzy normal subhemiring of  $R^1$ .

**Proof:** Let  $f: R \rightarrow R^1$  be an anti-homomorphism. Then,  $f(x+y) = f(y)+f(x)$ ,  $f(xy) = f(y)f(x)$ , for all  $x$  and  $y$  in  $R$ . Let  $V = f(A)$ , where  $A$  is an anti multi fuzzy normal subhemiring of a hemiring  $R$ . Now, for  $f(x)$  and  $f(y)$  in  $R^1$ , By Theorem 2.5, clearly  $V$  is an anti multi fuzzy subhemiring of a hemiring  $R^1$ , since  $A$  is an anti multi fuzzy subhemiring of a hemiring  $R$ . Now,  $\mu_{V_i}(f(x)f(y)) \leq \mu_{A_i}(yx) = \mu_{A_i}(xy) \geq \mu_{V_i}(f(y)f(x)) = \mu_{V_i}(f(y)f(x))$ , which implies that  $\mu_{V_i}(f(x)f(y)) = \mu_{V_i}(f(y)f(x))$ , for all  $i$  and for all  $f(x)$  and  $f(y)$  in  $R^1$ . Hence  $V$  is an anti multi fuzzy normal subhemiring of a hemiring  $R^1$ .

**3.8 Theorem:** Let  $(R, +, \cdot)$  and  $(R^1, +, \cdot)$  be any two hemirings. The anti-homomorphic preimage of an anti multi fuzzy normal subhemiring of  $R^1$  is an anti multi fuzzy normal subhemiring of  $R$ .

**Proof:** Let  $V = f(A)$ , where  $V$  is an anti multi fuzzy normal subhemiring of a hemiring  $R^1$ . Let  $x$  and  $y$  in  $R$ . By Theorem 2.5, clearly  $A$  is an anti multi fuzzy subhemiring of a hemiring  $R$ , since  $V$  is an anti multi fuzzy subhemiring of a hemiring  $R^1$ . Now,  $\mu_{A_i}(xy) = \mu_{V_i}(f(y)f(x)) = \mu_{V_i}(f(x)f(y)) = \mu_{V_i}(f(x)f(y)) = \mu_{A_i}(yx)$ , which implies that  $\mu_{A_i}(xy) = \mu_{A_i}(yx)$ , for all  $i$  and for all  $x$ ,  $y$  in  $R$ . Hence  $A$  is an anti multi fuzzy normal subhemiring of a hemiring  $R$ .

**3.9 Theorem:** Let  $A$  be an anti multi fuzzy subhemiring of a hemiring  $H$  and  $f$  is an isomorphism from a hemiring  $R$  onto  $H$ . If  $A$  is an anti multi fuzzy normal subhemiring of the hemiring  $H$ , then  $A \circ f$  is an anti multi fuzzy normal subhemiring of the hemiring  $R$ .

**Proof:** Let  $x$  and  $y$  in  $R$ . By Theorem 2.6, clearly  $A \circ f$  is an anti multi fuzzy subhemiring of a hemiring  $R$ . Now,  $(\mu_{A_i \circ f})(xy) = \mu_{A_i}(f(x)f(y)) = \mu_{A_i}(f(y)f(x)) = \mu_{A_i}(f(yx)) = (\mu_{A_i \circ f})(yx)$ , which implies that  $(\mu_{A_i \circ f})(xy) = (\mu_{A_i \circ f})(yx)$ , for all  $i$  and for all  $x$ ,  $y$  in  $R$ . Hence  $A \circ f$  is an anti multi fuzzy normal subhemiring of a hemiring  $R$ .

**3.10 Theorem:** Let A be an anti multi fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. If A is an anti multi fuzzy normal subhemiring of the hemiring H, then  $A \circ f$  is an anti multi fuzzy normal subhemiring of the hemiring R.

**Proof:** Let x and y in R. By Theorem 2.6, clearly  $A \circ f$  is an anti multi fuzzy subhemiring of the hemiring R. Now,  $(\mu_{A_i \circ f})(xy) = \mu_{A_i}(f(y)f(x)) = \mu_{A_i}(f(x)f(y)) = \mu_{A_i}(f(yx)) = (\mu_{A_i \circ f})(yx)$ , which implies that  $(\mu_{A_i \circ f})(xy) = (\mu_{A_i \circ f})(yx)$ , for all i and for all x, y in R. Hence  $A \circ f$  is an anti multi fuzzy normal subhemiring of the hemiring R.

#### 4. LOWER LEVEL SUBHEMIRINGS OF ANTI MULTI FUZZY NORMAL SUBHEMIRINGS

**4.1 Theorem:** Let A be an anti multi fuzzy subhemiring of a hemiring R. Then for  $\alpha_i$  in  $[0, 1]$  such that  $\mu_{A_i}(0) \leq \alpha_i$ , for all i,  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$  is a lower level subhemiring of R.

**Proof:** For all x and y in  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$ . Now,  $\mu_{A_i}(x+y) \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\} \leq \alpha_i$ , for all i, which implies that  $\mu_{A_i}(x+y) \leq \alpha_i$ , for all i. And,  $\mu_{A_i}(xy) \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\} \leq \alpha_i$ , for all i, which implies that  $\mu_{A_i}(xy) \leq \alpha_i$ , for all i. Hence  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$  is a lower level subhemiring of a hemiring R.

**4.2 Theorem:** Let A be an anti multi fuzzy subhemiring of a hemiring R. Then two lower level subhemiring  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$ ,  $A_{(\beta_1, \beta_2, \dots, \beta_n)}$  and  $\alpha_i, \beta_i$  in  $[0, 1]$  such that  $\mu_{A_i}(0) \leq \alpha_i, \mu_{A_i}(0) \leq \beta_i$  with  $\alpha_i < \beta_i$  of A are equal if and only if there is no x in R such that  $\beta_i > \mu_{A_i}(x) > \alpha_i$ , for all i.

**Proof:** Assume that  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)} = A_{(\beta_1, \beta_2, \dots, \beta_n)}$ . Suppose there exists x in R such that  $\beta_i > \mu_{A_i}(x) > \alpha_i$ . Then  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)} \subseteq A_{(\beta_1, \beta_2, \dots, \beta_n)}$  implies x belongs to  $A_{(\beta_1, \beta_2, \dots, \beta_n)}$ , but not in  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$ . This is contradiction to  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)} = A_{(\beta_1, \beta_2, \dots, \beta_n)}$ . Therefore there is no x in R such that  $\beta_i > \mu_{A_i}(x) > \alpha_i$ , for all i. Conversely if there is no x in R such that  $\beta_i > \mu_{A_i}(x) > \alpha_i$ , for all i. Then  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)} = A_{(\beta_1, \beta_2, \dots, \beta_n)}$ .

**4.3 Theorem:** Let R be a hemiring and A be a multi fuzzy subset of R such that  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$  be a subhemiring of R. If  $\alpha_i$  in  $[0, 1]$ , then A is an anti multi fuzzy subhemiring of R.

**Proof:** Let x and y in R and  $\mu_{A_i}(x) = \alpha_i$  and  $\mu_{A_i}(y) = \beta_i$ , for all i. If  $\alpha_i < \beta_i$ , then  $x, y \in A_{(\beta_1, \beta_2, \dots, \beta_n)}$ ,  $\mu_{A_i}(x+y) \leq \beta_i = \max\{\mu_{A_i}(x), \mu_{A_i}(y)\} \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ , which implies that  $\mu_{A_i}(x+y) \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ , for all i and for all x, y in R and  $\mu_{A_i}(xy) \leq \beta_i = \max\{\mu_{A_i}(x), \mu_{A_i}(y)\} \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ , which implies that  $\mu_{A_i}(xy) \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ , for all i and for all x and y in R. If  $\alpha_i > \beta_i$ , then x and y in  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$ ,  $\mu_{A_i}(x+y) \leq \alpha_i = \max\{\mu_{A_i}(y), \mu_{A_i}(x)\} \leq \max\{\mu_{A_i}(y), \mu_{A_i}(x)\}$ , which implies that  $\mu_{A_i}(x+y) \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ , for all i and for all x, y in R and  $\mu_{A_i}(xy) \leq \beta_i = \max\{\mu_{A_i}(y), \mu_{A_i}(x)\} \leq \max\{\mu_{A_i}(y), \mu_{A_i}(x)\}$ , which implies that  $\mu_{A_i}(xy) \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ , for all i and for all x, y in R. Hence A is an anti multi fuzzy subhemiring of the hemiring R.

**4.4 Theorem:** Let A be an anti multi fuzzy subhemiring of a hemiring R. If any two lower level subhemirings of A belongs to R, then their intersection is also lower level subhemiring of A in R.

**Proof:** Let  $\alpha_i, \beta_i \in [0, 1]$ . If for all i,  $\alpha_i < \mu_{A_i}(x) < \beta_i$ , then  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)} \subseteq A_{(\beta_1, \beta_2, \dots, \beta_n)}$ . Therefore,  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)} \cap A_{(\beta_1, \beta_2, \dots, \beta_n)} = A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$ , but  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$  is a lower level subhemiring of A. If for all i,  $\alpha_i > \mu_{A_i}(x) > \beta_i$ , then  $A_{(\beta_1, \beta_2, \dots, \beta_n)} \subseteq A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$ . Therefore,  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)} \cap A_{(\beta_1, \beta_2, \dots, \beta_n)} = A_{(\beta_1, \beta_2, \dots, \beta_n)}$ , but  $A_{(\beta_1, \beta_2, \dots, \beta_n)}$  is a lower level subhemiring of A. If  $\alpha_i = \beta_i$ , then  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)} = A_{(\beta_1, \beta_2, \dots, \beta_n)}$ . Hence intersection of any two lower level subhemirings is also a lower level subhemiring of A.

**4.5 Theorem:** Let A be an anti multi fuzzy subhemiring of a hemiring R. If a collection of lower level subhemirings of A, then their intersection is also a lower level subhemiring of A.

**Proof:** It is trivial.

**4.6 Theorem:** Let A be an anti multi fuzzy subhemiring of a hemiring R. If any two lower level subhemirings of A belongs to R, then their union is also a lower level subhemiring of A in R.

**Proof:** It is trivial.

**4.7 Theorem:** Let A be an anti multi fuzzy subhemiring of a hemiring R. If a collection of lower level subhemirings of A, then their union is also a lower level subhemiring of A.

**Proof:** It is trivial.

**4.8 Theorem:** The homomorphic image of a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring R is a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring  $R^1$ .

**Proof:** Let  $f: R \rightarrow R^1$  be a homomorphism. Then  $f(x+y)=f(x)+f(y)$ ,  $f(xy)=f(x)f(y)$ , for all  $x$  and  $y$  in  $R$ . Let  $V = f(A)$ , where  $A$  is an anti multi fuzzy subhemiring of a hemiring  $R$ . By Theorem 2.4, clearly  $V$  is an anti multi fuzzy subhemiring of a hemiring  $R^1$ . Let  $x$  and  $y$  in  $R$ , implies  $f(x)$  and  $f(y)$  in  $R^1$ . Let  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$  is a lower level subhemiring of  $A$ . Now,  $\mu_{V_i}(f(x)) \leq \mu_{A_i}(x) \leq \alpha_i$ , which implies that  $\mu_{V_i}(f(x)) \leq \alpha_i$  and  $\mu_{V_i}(f(y)) \leq \mu_{A_i}(y) \leq \alpha_i$ , which implies that  $\mu_{V_i}(f(y)) \leq \alpha_i$  and  $\mu_{V_i}(f(x)+f(y)) \leq \mu_{A_i}(x+y) \leq \alpha_i$ , which implies that  $\mu_{V_i}(f(x)+f(y)) \leq \alpha_i$ , for all  $i$ . Also,  $\mu_{V_i}(f(x)f(y)) \leq \mu_{A_i}(xy) \leq \alpha_i$ , which implies that  $\mu_{V_i}(f(x)f(y)) \leq \alpha_i$ , for all  $i$ . Hence  $f(A_{(\alpha_1, \alpha_2, \dots, \alpha_n)})$  is a lower level subhemiring of an anti multi fuzzy subhemiring  $V$  of a hemiring  $R^1$ .

**4.9 Theorem:** The homomorphic pre-image of a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring  $R^1$  is a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring  $R$ .

**Proof:** Let  $V = f(A)$ , where  $V$  is an anti multi fuzzy subhemiring of a hemiring  $R^1$ . By Theorem 2.4, clearly  $A$  is an anti multi fuzzy subhemiring of a hemiring  $R$ . Let  $f(x)$  and  $f(y)$  in  $R^1$ , implies  $x$  and  $y$  in  $R$ . Let  $f(A_{(\alpha_1, \alpha_2, \dots, \alpha_n)})$  is a lower level subhemiring of  $V$ . Now,  $\mu_{A_i}(x) = \mu_{V_i}(f(x)) \leq \alpha_i$ , implies that  $\mu_{A_i}(x) \leq \alpha_i$ , for all  $i$ ,  $\mu_{A_i}(y) = \mu_{V_i}(f(y)) \leq \alpha_i$ , implies that  $\mu_{A_i}(y) \leq \alpha_i$  and  $\mu_{A_i}(x+y) = \mu_{V_i}(f(x)+f(y)) \leq \alpha_i$ , which implies that  $\mu_{A_i}(x+y) \leq \alpha_i$ , for all  $i$ . Also,  $\mu_{A_i}(xy) = \mu_{V_i}(f(x)f(y)) \leq \alpha_i$ , which implies that  $\mu_{A_i}(xy) \leq \alpha_i$ , for all  $i$ . Hence,  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$  is a lower level subhemiring of an anti multi fuzzy subhemiring  $A$  of  $R$ .

**4.10 Theorem:** The anti-homomorphic image of a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring  $R$  is a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring  $R^1$ .

**Proof:** Let  $f: R \rightarrow R^1$  be an anti-homomorphism. Then  $f(x+y) = f(y) + f(x)$ ,  $f(xy) = f(y)f(x)$ , for all  $x$  and  $y$  in  $R$ . Let  $V = f(A)$ , where  $A$  is an anti multi fuzzy subhemiring of  $R$ . By Theorem 2.5, clearly  $V$  is an anti multi fuzzy subhemiring of  $R^1$ . Let  $x$  and  $y$  in  $R$ , implies  $f(x)$  and  $f(y)$  in  $R^1$ . Let  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$  is a lower level subhemiring of  $A$ . Now,  $\mu_{V_i}(f(x)) \leq \mu_{A_i}(x) \leq \alpha_i$ , which implies that  $\mu_{V_i}(f(x)) \leq \alpha_i$ ;  $\mu_{V_i}(f(y)) \leq \mu_{A_i}(y) \leq \alpha_i$ , which implies that  $\mu_{V_i}(f(y)) \leq \alpha_i$ , for all  $i$ . Now,  $\mu_{V_i}(f(x)+f(y)) \leq \mu_{A_i}(y+x) \leq \alpha_i$ , which implies that,  $\mu_{V_i}(f(x)+f(y)) \leq \alpha_i$ . Also,  $\mu_{V_i}(f(x)f(y)) \leq \mu_{A_i}(yx) \leq \alpha_i$ , which implies that  $\mu_{V_i}(f(x)f(y)) \leq \alpha_i$ , for all  $i$ . Hence  $f(A_{(\alpha_1, \alpha_2, \dots, \alpha_n)})$  is a lower level subhemiring of an anti multi fuzzy subhemiring  $V$  of  $R^1$ .

**4.11 Theorem:** The anti-homomorphic pre-image of a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring  $R^1$  is a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring  $R$ .

**Proof:** Let  $V = f(A)$ , where  $V$  is an anti multi fuzzy subhemiring of a hemiring  $R^1$ . By Theorem 2.5, clearly  $A$  is an anti multi fuzzy subhemiring of a hemiring  $R$ . Let  $f(x)$  and  $f(y)$  in  $R^1$ , implies  $x$  and  $y$  in  $R$ . Let  $f(A_{(\alpha_1, \alpha_2, \dots, \alpha_n)})$  is a lower level subhemiring of  $V$ . Now,  $\mu_{A_i}(x) = \mu_{V_i}(f(x)) \leq \alpha_i$ , which implies that  $\mu_{A_i}(x) \leq \alpha_i$ ;  $\mu_{A_i}(y) = \mu_{V_i}(f(y)) \leq \alpha_i$ , which implies that  $\mu_{A_i}(y) \leq \alpha_i$ , for all  $i$ . Now,  $\mu_{A_i}(x+y) = \mu_{V_i}(f(y)+f(x)) \leq \alpha_i$ , which implies that  $\mu_{A_i}(x+y) \leq \alpha_i$ , for all  $i$ . Also,  $\mu_{A_i}(xy) = \mu_{V_i}(f(y)f(x)) \leq \alpha_i$ , which implies that  $\mu_{A_i}(xy) \leq \alpha_i$ , for all  $i$ . Hence  $A_{(\alpha_1, \alpha_2, \dots, \alpha_n)}$  is a lower level subhemiring of an anti multi fuzzy subhemiring  $A$  of  $R$ .

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