International Journal of Mathematical Archive-7(12), 2016, 53-58
Available online through www.ijma.info ISSN 2229 – 5046

SOME THEOREMS IN ANTI MULTI FUZZY NORMAL SUBHEMIRINGS OF A HEMIRING

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(Received On: 28-09-16; Revised & Accepted On: 22-12-16)

ABSTRACT

In this paper, we made an attempt to study the algebraic nature of an anti multi fuzzy subhemiring of a hemiring.

Key Words: *Fuzzy subset, multi fuzzy subset, multi fuzzy subhemiring, anti multi fuzzy subhemiring, multi fuzzy normal subhemiring, anti multi fuzzy normal subhemiring.*

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (*R;* +; .). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a= b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1.a = a = a.1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh [14], several researchers explored on the generalization of the concept of fuzzy sets. The notion of anti fuzzy left h-ideals in hemiring was introduced by Akram.M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan[9]. In this paper, we introduce some theorems in anti multi fuzzy normal subhemiring of a hemiring.

1. PRELIMINARIES

1.1 Definition: Let X be a non-empty set. A **fuzzy subset** A of X is a function A: X $\to$ [0, 1].

1.2 Definition: A **multi fuzzy subset** A of a set X is defined as an object of the form A = $\{\langle x, \mu_{A1}(x), \mu_{A2}(x), \mu_{A3}(x), \ldots, \mu_{An}(x) \rangle / x\in X\}$, where $\mu_{Ai} : X\to[0, 1]$ for all i. It is denoted as A = $\langle \mu_{A1}, \mu_{A2}, \mu_{A3}, \ldots, \mu_{An} \rangle$.

1.3 Definition: Let A and B be any two multi fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \subseteq B$ if and only if $\mu_{Ai}(x) \leq \mu_{Bi}(x)$ for all i and for all x in X.
(ii) A = B if and only if $\mu_{Ai}(x) = \mu_{Bi}(x)$ for all i and for all x in X.
(iii) $A^c = 1-A = \langle 1-\mu_{A1}, 1-\mu_{A2}, 1-\mu_{A3}, \ldots, 1-\mu_{An} \rangle$.
(iv) $A\cap B = \{\langle x, \min\{ \mu_{A1}(x), \mu_{B1}(x)\}, \min\{ \mu_{A2}(x), \mu_{B2}(x)\}, \ldots, \min\{\mu_{An}(x), \mu_{Bn}(x)\}\rangle / x\in X\}$.
(v) $A\cup B = \{\langle x, \max \{\mu_{A1}(x), \mu_{B1}(x)\}, \max\{\mu_{A2}(x), \mu_{B2}(x)\}, \ldots, \max\{\mu_{An}(x), \mu_{Bn}(x)\}\rangle / x\in X\}$.

1.4 Definition: Let (R, +, .) be a hemiring. A multi fuzzy subset A of R is said to be a multi fuzzy subhemiring of R if it satisfies the following conditions:

(i) $\mu_{Ai}(x+y) \geq \min (\mu_{Ai}(x), \mu_{Ai}(y))$ for all i,
(ii) $\mu_{Ai}(xy) \geq \min (\mu_{Ai}(x), \mu_{Ai}(y))$, for all i and for all x, y in R.

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1.5 Definition: Let (R, + , .) be a hemiring. A multi fuzzy subhemiring A of R is said to be a multi fuzzy normal subhemiring (MFNSHR) of R if $\mu_{Ai}(xy) = \mu_{Ai}(yx)$, for all i and for all x, y in R.

1.6 Definition: Let (R, +, .) be a hemiring. A multi fuzzy subset A of R is said to be an anti multi fuzzy subhemiring of R if it satisfies the following conditions:

(i) $\mu_{Ai}(x+y) \leq \max(\mu_{Ai}(x), \mu_{Ai}(y))$ for all i,

(ii) $\mu_{Ai}(xy) \leq \max(\mu_{Ai}(x), \mu_{Ai}(y))$, for all i and for all x, y in R.

1.7 Definition: Let A and B be multi fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by A×B, is defined as $A\times B = \{\langle (x, y), \mu_{A1\times B1}(x, y), \mu_{A2\times B2}(x, y), \ldots, \mu_{An\times Bn}(x, y)\rangle$ / for all x in G and y in H}, where $\mu_{Ai\times Bi}(x, y) = \max\{ \mu_{Ai}(x), \mu_{Bi}(y) \}$ for all i.

1.8 Definition: Let A be a multi fuzzy subset in a set S, the anti-strongest multi fuzzy relation on S, that is a multi fuzzy relation on A is V given by $\mu_{Vi}(x, y) = \max\{\mu_{Ai}(x), \mu_{Ai}(y)\}$, for all i and for all x, y in S.

1.9 Definition: Let (R, +, .) and (R^l, +, .) be any two hemirings. Let f: $R \to R^l$ be any function and A be an anti multi fuzzy subhemiring in R, V be an anti multi fuzzy subhemiring in f(R) = R^l, defined by $\mu_{Vi}(y) = \inf_{x\in f^{-1}(y)} \mu_{Ai}(x)$, for all i and for all x in R and y in R^l. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.10 Definition: Let A be an anti multi fuzzy subhemiring of a hemiring (R, +, .) and a in R. Then the pseudo anti multi fuzzy coset $(aA)^p$ is defined by $((a\mu_{Ai})^p)(x) = p(a)\mu_{Ai}(x)$, for all i and for every x in R and for some p in P.

1.11 Definition: Let (R, +, .) be a hemiring. An anti multi fuzzy subhemiring A of R is said to be an anti multi fuzzy normal subhemiring (AMFNSHR) of R if $\mu_{Ai}(xy) = \mu_{Ai}(yx)$, for all i and for all x, y in R.

1.12 Definition: Let A be a multi fuzzy subset of X. For α_i in [0, 1], the lower level subset of A is the set $A_{(\alpha 1, \alpha 2, \ldots, \alpha n)} = \{x\in X : \mu_{Ai}(x) \leq \alpha_i$, for all i}.

2. ANTI MULTI FUZZY SUBHEMIRINGS

2.1 Theorem [2]: Union of any two(a family) of anti multi fuzzy subhemirings of a hemiring R is an anti multi fuzzy subhemiring of R.

2.2 Theorem [2]: If A and B are any two anti multi fuzzy subhemirings of the hemirings R_1 and R_2 respectively, then anti-product A×B is an anti multi fuzzy subhemiring of $R_1\times R_2$.

2.3 Theorem [2]: Let A be a multi fuzzy subset of a hemiring R and V be the anti-strongest multi fuzzy relation of R. Then A is an anti multi fuzzy subhemiring of R if and only if V is an anti multi fuzzy subhemiring of R×R.

2.4 Theorem [2]: Let R and R^l be any two hemirings. The homomorphic image (preimage) of an anti multi fuzzy subhemiring of R is an anti multi fuzzy subhemiring of R^l.

2.5 Theorem [2]: Let R and R^l be any two hemirings. The anti-homomorphic image (preimage) of an anti multi fuzzy subhemiring of R is an anti multi fuzzy subhemiring of R^l.

2.6 Theorem [2]: Let A be an anti multi fuzzy subhemiring of a hemiring H and f is an isomorphism(anti isomorphism) from a hemiring R onto H. Then A∘f is an anti multi fuzzy subhemiring of R.

3. ANTI MULTI FUZZY NORMAL SUBHEMIRINGS

3.1 Theorem: Let (R, +,.) be a hemiring. If any two anti multi fuzzy normal subhemirings of R, then their union is an anti multi fuzzy normal subhemiring of R.

Proof: Let x and y be in R. Let $A = \{(x, \mu_{A1}(x), \mu_{A2}(x), \ldots, \mu_{An}(x)) / x\in R\}$ and $B = \{(x, \mu_{B1}(x), \mu_{B2}(x), \ldots, \mu_{Bn}(x)) / x\in R\}$ and also let $C = A\cup B = \{(x, \mu_{C1}(x), \mu_{C2}(x), \ldots, \mu_{Cn}(x)) / x\in R\}$, where $\max\{ \mu_{Ai}(x), \mu_{Bi}(x) \} = \mu_{Ci}(x)$ for all i. By Theorem 2.1, clearly C is an anti multi fuzzy subhemiring of a hemiring R, since A and B are two anti multi fuzzy subhemirings of the hemiring R. Then, $\mu_{Ci}(xy) = \max\{\mu_{Ai}(xy), \mu_{Bi}(xy)\} = \max\{\mu_{Ai}(yx), \mu_{Bi}(yx)\} = \mu_{Ci}(yx)$, for all i and for all x and y in R. Hence $A \cup B$ is an anti multi fuzzy normal subhemiring of the hemiring R.

3.2 Theorem: Let (R, +, .) be a hemiring. The union of a family of anti multi fuzzy normal subhemirings of R is an anti multi fuzzy normal subhemiring of R.

Proof: It is trivial.

3.3 Theorem: Let A and B be anti multi fuzzy subhemirings of the hemirings G and H, respectively. If A and B are anti multi fuzzy normal subhemirings, then A×B is an anti multi fuzzy normal subhemiring of G×H.

Proof: Let A and B be anti multi fuzzy normal subhemirings of the hemirings G and H respectively. By Theorem 2.2, clearly A×B is an anti multi fuzzy subhemiring of G×H. Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in G×H. Now, $\mu_{Ai\times Bi}[(x_1, y_1)(x_2, y_2)] = \max\{\mu_{Ai}(x_1x_2), \mu_{Bi}(y_1y_2)\} = \max\{\mu_{Ai}(x_2x_1), \mu_{Bi}(y_2y_1)\} = \mu_{Ai\times Bi}(x_2x_1, y_2y_1) = \mu_{Ai\times Bi}[(x_2, y_2)(x_1, y_1)]$. Hence A×B is an anti multi fuzzy normal subhemiring of G×H.

3.4 Theorem: Let A be a multi fuzzy subset in a hemiring R and V be the anti-strongest multi fuzzy relation on R. Then A is an anti multi fuzzy normal subhemiring of R if and only if V is an anti multi fuzzy normal subhemiring of R×R.

Proof: It is trivial.

3.5 Theorem: Let (R, +, .) and $(R^l, +, .)$ be any two hemirings. The homomorphic image of an anti multi fuzzy normal subhemiring of R is an anti multi fuzzy normal subhemiring of R^l.

Proof: Let $f: R \rightarrow R^l$ be a homomorphism. Then, f(x+y)= f(x)+f(y), f(xy) = f(x)f(y), for all x and y in R. Let V = f(A), where A is an anti multi fuzzy normal subhemiring of a hemiring R. Now, for f(x), f(y) in R^l, By Theorem 2.4, clearly V is an anti multi fuzzy subhemiring of a hemiring R^l, since A is an anti multi fuzzy subhemiring of a hemiring R. Now, $\mu_{vi}(f(x)f(y)) \leq \mu_{Ai}(xy) = \mu_{Ai}(yx) \geq \mu_{vi}(f(yx)) = \mu_{vi}(f(y)f(x))$, which implies that $\mu_{vi}(f(x)f(y)) = \mu_{vi}(f(y) f(x))$, for all i and for all f(x) and f(y) in R^l. Hence V is an anti multi fuzzy normal subhemiring of a hemiring R^l.

3.6 Theorem: Let (R, +, .) and $(R^l, +, .)$ be any two hemirings. The homomorphic preimage of an anti multi fuzzy normal subhemiring of R^l is an anti multi fuzzy normal subhemiring of R.

Proof: Let V = f(A), where V is an anti Multi fuzzy normal subhemiring of a hemiring R^l. Let x and y in R. By Theorem 2.4, clearly A is an anti multi fuzzy subhemiring of a hemiring R, since V is an anti multi fuzzy subhemiring of a hemiring R^l. Now, $\mu_{Ai}(xy) = \mu_{vi}(f(x)f(y)) = \mu_{vi}(f(y)f(x)) = \mu_{vi}(f(yx)) = \mu_{Ai}(yx)$, which implies that $\mu_{Ai}(xy) = \mu_{Ai}(yx)$, for all i and for all x and y in R. Hence A is an anti multi fuzzy normal subhemiring of a hemiring R.

3.7 Theorem: Let (R, +, .) and $(R^l, +, .)$ be any two hemirings. The anti-homomorphic image of an anti multi fuzzy normal subhemiring of R is an anti multi fuzzy normal subhemiring of R^l.

Proof: Let $f: R \rightarrow R^l$ be an anti-homomorphism. Then, f(x+y) = f(y)+f(x), f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where A is an anti multi fuzzy normal subhemiring of a hemiring R. Now, for f(x) and f(y) in R^l, By Theorem 2.5, clearly V is an anti multi fuzzy subhemiring of a hemiring R^l, since A is an anti multi fuzzy subhemiring of a hemiring R. Now, $\mu_{vi}(f(x)f(y)) \leq \mu_{Ai}(yx) = \mu_{Ai}(xy) \geq \mu_{vi}(f(xy)) = \mu_{vi}(f(y)f(x))$, which implies that $\mu_{vi}(f(x)f(y)) = \mu_{vi}(f(y)f(x))$, for all i and for all f(x) and f(y) in R^l. Hence V is an anti multi fuzzy normal subhemiring of a hemiring R^l.

3.8 Theorem: Let (R, +, .) and $(R^l, +, .)$ be any two hemirings. The anti-homomorphic preimage of an anti multi fuzzy normal subhemiring of R^l is an anti multi fuzzy normal subhemiring of R.

Proof: Let V = f(A), where V is an anti multi fuzzy normal subhemiring of a hemiring R^l. Let x and y in R. By Theorem 2.5, clearly A is an anti multi fuzzy subhemiring of a hemiring R, since V is an anti multi fuzzy subhemiring of a hemiring R^l. Now, $\mu_{Ai}(xy) = \mu_{vi}(f(y)f(x)) = \mu_{vi}(f(x)f(y)) = \mu_{vi}(f(yx)) = \mu_{Ai}(yx)$, which implies that $\mu_{Ai}(xy) = \mu_{Ai}(yx)$, for all i and for all x, y in R. Hence A is an anti multi fuzzy normal subhemiring of a hemiring R.

3.9 Theorem: Let A be an anti multi fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. If A is an anti multi fuzzy normal subhemiring of the hemiring H, then A◦f is an anti multi fuzzy normal subhemiring of the hemiring R.

Proof: Let x and y in R. By Theorem 2.6, clearly A◦f is an anti multi fuzzy subhemiring of a hemiring R. Now, $(\mu_{Ai}\circ f)(xy) = \mu_{Ai}(f(x)f(y)) = \mu_{Ai}(f(y)f(x)) = \mu_{Ai}(f(yx)) = (\mu_{Ai}\circ f)(yx)$, which implies that $(\mu_{Ai}\circ f)(xy) = (\mu_{Ai}\circ f)(yx)$, for all i and for all x, y in R. Hence A◦f is an anti multi fuzzy normal subhemiring of a hemiring R.

3.10 Theorem: Let A be an anti multi fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. If A is an anti multi fuzzy normal subhemiring of the hemiring H, then A∘f is an anti multi fuzzy normal subhemiring of the hemiring R.

Proof: Let x and y in R. By Theorem 2.6, clearly A∘f is an anti multi fuzzy subhemiring of the hemiring R. Now, $(\mu_{Ai}\circ f)(xy) = \mu_{Ai}(f(y)f(x)) = \mu_{Ai}(f(x)f(y)) = \mu_{Ai}(f(yx)) = (\mu_{Ai}\circ f)(yx)$, which implies that $(\mu_{Ai}\circ f)(xy) = (\mu_{Ai}\circ f)(yx)$, for all i and for all x, y in R. Hence A∘f is an anti multi fuzzy normal subhemiring of the hemiring R.

4. LOWER LEVEL SUBHEMIRINGS OF ANTI MULTI FUZZY NORMAL SUBHEMIRINGS

4.1 Theorem: Let A be an anti multi fuzzy subhemiring of a hemiring R. Then for α_i in [0, 1] such that $\mu_{Ai}(0) \le \alpha_i$, for all i, $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$ is a lower level subhemiring of R.

Proof: For all x and y in $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$. Now, $\mu_{Ai}(x+y) \le \max\{\mu_{Ai}(x), \mu_{Ai}(y)\} \le \alpha_i$, for all i, which implies that $\mu_{Ai}(x+y) \le \alpha_i$, for all i. And, $\mu_{Ai}(xy) \le \max\{\mu_{Ai}(x), \mu_{Ai}(y)\} \le \alpha_i$, for all i, which implies that $\mu_{Ai}(xy) \le \alpha_i$, for all i. Hence $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$ is a lower level subhemiring of a hemiring R.

4.2 Theorem: Let A be an anti multi fuzzy subhemiring of a hemiring R. Then two lower level subhemiring $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$, $A_{(\beta1, \beta2, \ldots,\beta n)}$ and α_i, β_i in [0, 1] such that $\mu_{Ai}(0) \le \alpha_i$, $\mu_{Ai}(0) \le \beta_i$ with $\alpha_i < \beta_i$ of A are equal if and only if there is no x in R such that $\beta_i > \mu_{Ai}(x) > \alpha_i$, for all i.

Proof: Assume that $A_{(\alpha1, \alpha2, \ldots,\alpha n)} = A_{(\beta1, \beta2, \ldots,\beta n)}$. Suppose there exists x in R such that $\beta_i > \mu_{Ai}(x) > \alpha_i$. Then $A_{(\alpha1, \alpha2, \ldots,\alpha n)} \subseteq A_{(\beta1, \beta2, \ldots,\beta n)}$ implies x belongs to $A_{(\beta1, \beta2, \ldots,\beta n)}$, but not in $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$. This is contradiction to $A_{(\alpha1, \alpha2, \ldots,\alpha n)} = A_{(\beta1, \beta2, \ldots,\beta n)}$. Therefore there is no $x\in R$ such that $\beta_i > \mu_{Ai}(x) > \alpha_i$, for all i. Conversely if there is no $x\in R$ such that $\beta_i > \mu_{Ai}(x) > \alpha_i$, for all i. Then $A_{(\alpha1, \alpha2, \ldots,\alpha n)} = A_{(\beta1, \beta2, \ldots,\beta n)}$.

4.3 Theorem: Let R be a hemiring and A be a multi fuzzy subset of R such that $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$ be a subhemiring of R. If α_i in [0, 1], then A is an anti multi fuzzy subhemiring of R.

Proof: Let x and y in R and $\mu_{Ai}(x) = \alpha_i$ and $\mu_{Ai}(y) = \beta_i$, for all i. If $\alpha_i < \beta_i$, then x, y $\in A_{(\beta1, \beta2, \ldots,\beta n)}$, $\mu_{Ai}(x+y) \le \beta_i = \max\{\mu_{Ai}(x), \mu_{Ai}(y)\} \le \max\{\mu_{Ai}(x), \mu_{Ai}(y)\}$, which implies that $\mu_{Ai}(x+y) \le \max\{\mu_{Ai}(x), \mu_{Ai}(y)\}$, for all i and for all x, y in R and $\mu_{Ai}(xy) \le \beta_i = \max\{\mu_{Ai}(x), \mu_{Ai}(y)\} \le \max\{\mu_{Ai}(x), \mu_{Ai}(y)\}$, which implies that $\mu_{Ai}(xy) \le \max\{\mu_{Ai}(x), \mu_{Ai}(y))$, for all i and for all x and y in R. If $\alpha_i > \beta_i$, then x and y in $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$, $\mu_{Ai}(x+y) \le \alpha_i = \max\{\mu_{Ai}(y), \mu_{Ai}(x)\} \le \max\{\mu_{Ai}(y), \mu_{Ai}(x)\}$, which implies that $\mu_{Ai}(x+y) \le \max\{\mu_{Ai}(x), \mu_{Ai}(y)\}$, for all i and for all x, y in R and $\mu_{Ai}(xy) \le \beta_i = \max\{\mu_{Ai}(y), \mu_{Ai}(x)\} \le \max\{\mu_{Ai}(y), \mu_{Ai}(x)\}$, which implies that $\mu_{Ai}(xy) \le \max\{\mu_{Ai}(x), \mu_{Ai}(y))$, for all i and for all x, y in R. Hence A is an anti multi fuzzy subhemiring of the hemiring R.

4.4 Theorem: Let A be an anti multi fuzzy subhemiring of a hemiring R. If any two lower level subhemirings of A belongs to R, then their intersection is also lower level subhemiring of A in R.

Proof: Let α_i, $\beta_i \in [0, 1]$. If for all i, $\alpha_i < \mu_{Ai}(x) < \beta_i$, then $A_{(\alpha1, \alpha2, \ldots,\alpha n)} \subseteq A_{(\beta1, \beta2, \ldots,\beta n)}$. Therefore, $A_{(\alpha1, \alpha2, \ldots,\alpha n)} \cap A_{(\beta1, \beta2, \ldots,\beta n)} = A_{(\alpha1, \alpha2, \ldots,\alpha n)}$, but $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$ is a lower level subhemiring of A. If for all i, $\alpha_i > \mu_{Ai}(x) > \beta_i$, then $A_{(\beta1, \beta2, \ldots,\beta n)} \subseteq A_{(\alpha1, \alpha2, \ldots,\alpha n)}$. Therefore, $A_{(\alpha1, \alpha2, \ldots,\alpha n)} \cap A_{(\beta1, \beta2, \ldots,\beta n)} = A_{(\beta1, \beta2, \ldots,\beta n)}$, but $A_{(\beta1, \beta2, \ldots,\beta n)}$ is a lower level subhemiring of A. If $\alpha_i = \beta_i$, then $A_{(\alpha1, \alpha2, \ldots,\alpha n)} = A_{(\beta1, \beta2, \ldots,\beta n)}$. Hence intersection of any two lower level subhemirings is also a lower level subhemiring of A.

4.5 Theorem: Let A be an anti multi fuzzy subhemiring of a hemiring R. If a collection of lower level subhemirings of A, then their intersection is also a lower level subhemiring of A.

Proof: It is trivial.

4.6 Theorem: Let A be an anti multi fuzzy subhemiring of a hemiring R. If any two lower level subhemirings of A belongs to R, then their union is also a lower level subhemiring of A in R.

Proof: It is trivial.

4.7 Theorem: Let A be an anti multi fuzzy subhemiring of a hemiring R. If a collection of lower level subhemirings of A, then their union is also a lower level subhemiring of A.

Proof: It is trivial.

4.8 Theorem: The homomorphic image of a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring R is a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring R^{I}.

Proof: Let f: $R \rightarrow R^{I}$ be a homomorphism. Then f(x+y)=f(x)+f(y), f(xy)=f(x)f(y), for all x and y in R. Let V = f(A), where A is an anti multi fuzzy subhemiring of a hemiring R. By Theorem 2.4, clearly V is an anti multi fuzzy subhemiring of a hemiring R^{I}. Let x and y in R, implies f(x) and f(y) in R^{I}. Let $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$ is a lower level subhemiring of A. Now, $\mu_{Vi}(f(x)) \leq \mu_{Ai}(x) \leq \alpha_i$, which implies that $\mu_{Vi}(f(x)) \leq \alpha_i$ and $\mu_{Vi}(f(y)) \leq \mu_{Ai}(y) \leq \alpha_i$, which implies that $\mu_{Vi}(f(y)) \leq \alpha_i$ and $\mu_{Vi}(f(x)+f(y)) \leq \mu_{Ai}(x+y) \leq \alpha_i$, which implies that $\mu_{Vi}(f(x)+f(y)) \leq \alpha_i$, for all i. Also, $\mu_{Vi}(f(x)f(y)) \leq \mu_{Ai}(xy) \leq \alpha_i$, which implies that $\mu_{Vi}(f(x)f(y)) \leq \alpha_i$, for all i. Hence $f(A_{(\alpha1, \alpha2, \ldots,\alpha n)\alpha})$ is a lower level subhemiring of an anti multi fuzzy subhemiring V of a hemiring R^{I}.

4.9 Theorem: The homomorphic pre-image of a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring R^{I} is a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring R.

Proof: Let V = f(A), where V is an anti multi fuzzy subhemiring of a hemiring R^{I}. By Theorem 2.4, clearly A is an anti multi fuzzy subhemiring of a hemiring R. Let f(x) and f(y) in R^{I}, implies x and y in R. Let $f(A_{(\alpha1, \alpha2, \ldots,\alpha n)})$ is a lower level subhemiring of V. Now, $\mu_{Ai}(x) = \mu_{Vi}(f(x)) \leq \alpha_i$, implies that $\mu_{Ai}(x) \leq \alpha_i$, for all i, $\mu_{Ai}(y) = \mu_{Vi}(f(y)) \leq \alpha_i$, implies that $\mu_{Ai}(y) \leq \alpha_i$ and $\mu_{Ai}(x+y) = \mu_{Vi}(f(x)+f(y)) \leq \alpha_i$, which implies that $\mu_{Ai}(x+y) \leq \alpha_i$, for all i. Also, $\mu_{Ai}(xy) = \mu_{Vi}(f(x)f(y)) \leq \alpha_i$, which implies that $\mu_{Ai}(xy) \leq \alpha_i$, for all i. Hence, $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$ is a lower level subhemiring of an anti multi fuzzy subhemiring A of R.

4.10 Theorem: The anti-homomorphic image of a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring R is a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring R^{I}.

Proof: Let f: $R \rightarrow R^{I}$ be an anti-homomorphism. Then f(x+y) = f(y) + f(x), f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where A is an anti multi fuzzy subhemiring of R. By Theorem 2.5, clearly V is an anti multi fuzzy subhemiring of R^{I}. Let x and y in R, implies f(x) and f(y) in R^{I}. Let $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$ is a lower level subhemiring of A. Now, $\mu_{Vi}(f(x)) \leq \mu_{Ai}(x) \leq \alpha_i$, which implies that $\mu_{Vi}(f(x)) \leq \alpha_i$; $\mu_{Vi}(f(y)) \leq \mu_{Ai}(y) \leq \alpha_i$, which implies that $\mu_{Vi}(f(y)) \leq \alpha_i$, for all i. Now, $\mu_{Vi}(f(x)+f(y)) \leq \mu_{Ai}(y+x) \leq \alpha_i$, which implies that, $\mu_{Vi}(f(x)+f(y)) \leq \alpha_i$. Also, $\mu_{Vi}(f(x)f(y)) \leq \mu_{Ai}(yx) \leq \alpha_i$, which implies that $\mu_{Vi}(f(x)f(y)) \leq \alpha_i$, for all i. Hence $f(A_{(\alpha1, \alpha2, \ldots,\alpha n)})$ is a lower level subhemiring of an anti multi fuzzy subhemiring V of R^{I}.

4.11 Theorem: The anti-homomorphic pre-image of a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring R^{I} is a lower level subhemiring of an anti multi fuzzy subhemiring of a hemiring R.

Proof: Let V = f(A), where V is an anti multi fuzzy subhemiring of a hemiring R^{I}. By Theorem 2.5, clearly A is an anti multi fuzzy subhemiring of a hemiring R. Let f(x) and f(y) in R^{I}, implies x and y in R. Let $f(A_{(\alpha1, \alpha2, \ldots,\alpha n)})$ is a lower level subhemiring of V. Now, $\mu_{Ai}(x) = \mu_{Vi}(f(x)) \leq \alpha_i$, which implies that $\mu_{Ai}(x) \leq \alpha_i$; $\mu_{Ai}(y) = \mu_{Vi}(f(y)) \leq \alpha_i$, which implies that $\mu_{Ai}(y) \leq \alpha_i$, for all i. Now, $\mu_{Ai}(x+y) = \mu_{Vi}(f(y)+f(x)) \leq \alpha_i$, which implies that $\mu_{Ai}(x+y) \leq \alpha_i$, for all i. Also, $\mu_{Ai}(xy) = \mu_{Vi}(f(y)f(x)) \leq \alpha_i$, which implies that $\mu_{Ai}(xy) \leq \alpha_i$, for all i. Hence $A_{(\alpha1, \alpha2, \ldots,\alpha n)}$ is a lower level subhemiring of an anti multi fuzzy subhemiring A of R.

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Source of support: Nil, Conflict of interest: None Declared.