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MISTAKES IN H.C.F. AND L.C.M. OF ALGEBRAIC EXPRESSION

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ABSTRACT

In this paper, three problems concerning conceptual and operational mistakes are generally done by the learners in dealing with 'H.C.F. (Highest Common Factor) and L.C.M. (Least Common Multiple) of algebraic expression' and their proofs/solutions & explanation of the proofs/solutions of mistakes have been presented. This study has been done through 'text book scanning', 'class room interaction', 'answer scripts of achievement test' and 'experts' opinions'. This study will help the learners to understand important mathematical concept of 'H.C.F. and L.C.M. of algebraic expression' as well as entire teaching learning process of mathematics.

Key words: *mathematical problem, solution, proof, mistake, mathematical concept, c.f., h.c.f., c.m., l.c.m., algebraic expression.*

MSC AMS Subject Classification 2010: *97A 20, 97C10, 97C70, 97D70, 97H99.*

1. INTRODUCTION

It is known that any misconception is memorized among the learners, then new concept which is based on proper concept of misconception will not be registered in their memories as a result these will not be recalled properly at the time of need for which phobia towards mathematics will be grown that is too much harmful for the learners as well as nation. On the other hand, we know that learners can learn all subjects including mathematics through 'trial and error method'. So, the researcher has also emphasized on how the learners can concretize their mathematics learning through mathematical mistakes since the researcher himself is a mathematics school teacher. In this regard, Linda M. Gojak (2013) said, "Helping students to learn mathematics from their mathematical mistakes can give us insight into their misconceptions and, depending on our instructional reactions, can enable them to develop deeper understanding of the mathematics they are learning. Meaningful assistance encompasses more than fixing a careless error related to poor study habits, such as misreading directions, miscopying a numeral, or forgetting a sign. Although we can easily and explicitly address careless errors in our instruction, it can be more challenging-but potentially much more rewarding-to address errors that fall into the special category of productive mistakes. These are the mistakes that have the potential to promote rich learning [3]."

Some works are seen on this area in a scattered way but not in a systematic way [1, 2, 8, 9, 14]. The author has tried to present the mistakes which done by the learners in school mathematics in a new style so that the learners are entertained as well as linked properly with the mathematical concepts. The author has some contributions on mistakes in mathematics at school level [10, 11, 12, 13].

Now, the author has stressed on mistakes of H.C.F. and L.C.M. of algebraic expression. It is an important concept of algebra which is the step to enter into the several major concepts of algebra. The researcher has already identified and sequenced the elementary concepts of H.C.F. and L.C.M. of algebraic expression. It is seen that a large number of learners do mistake while they solve the problems involving this major concept. He has identified the weakness of the learners where they do mistakes and what types of mistakes they do from his teaching experience in the class room, the answer scripts of achievement test.

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In this paper, considering the above mentioned weakness, two mathematical problems and their solutions' explanation & three mistakes in mathematics related of these problems and their solutions' explanation on 'H.C.F. and L.C.M. of algebraic expression' of school level mathematics have been presented. There is a logical mathematical beauty in this presentation considering the students' need. It is hoped that this study will also help the learners to grow the interest towards mathematics instead of math anxiety and to get more interest among interested students of mathematics.

2. OBJECTIVE OF THE STUDY

To frame the problems involving conceptual mistakes on 'H.C.F. and L.C.M. of algebraic expression' including their solutions/proofs & explain the solutions/proofs of the mistakes.

3. MATERIALS AND METHODS

The following materials and methods have been adopted:

1. Text books of mathematics of different Boards were collected at first. Selection of basic concepts 'H.C.F. and L.C.M. of algebraic expression' and then identification of important elementary concepts of these basic concepts from class room interaction, answer scripts of the achievement test where the learners do mistake, have been done.[4, 5, 6, 7]
2. Two problems of 'H.C.F. and L.C.M. of algebraic expression' have been set and their solutions & proofs and the explanation of the solutions & proofs have been done.
3. Framing the problems of mathematical mistake on 'H.C.F. and L.C.M. of algebraic expression' and their solutions and explanation of the proofs/solutions of the mistakes have been done by the researcher keeping in view the logical order of the subject and the psychological order of learners.
4. Experts' opinions were taken on these. Their opinions were incorporated for the improvement of the study. Then the final forms of mathematical mistake on 'H.C.F. and L.C.M. of algebraic expression' have been done.

4. SALIENT POINTS OF THE STUDY

Adopting the above materials and methods, the two problems and their proper solutions and explanations of the solutions & three mistakes and their solutions /proofs and explanations of the proofs of mistakes on 'H.C.F. and L.C.M. of algebraic expression' have been presented below.

Problem-1: Find the H.C.F. of the following expressions.

$$\mathbf{x^2-2x-15,\ x^2+5x+6}$$

Solution:

1^{st} Expression $= x^2-2x-15$
$= x^2-5x+3x-15$
$= x(x-5)+3(x-5)$
$= (x-5)(x+3)$

2^{nd} Expression $= x^2+5x+6$
$= x^2+3x+2x+6$
$= x(x+3)+2(x+3)$
$= (x+3)(x+2)$

$\therefore$ The required H.C.F is (x+3).

Explanation of the solution of the problem-1:

Explanation: Since, there is a common factor (x+3) in the two expressions: $x^2-2x-15$ and x^2+5x+6.

Therefore, the required H.C.F. is (x+3).

Problem-2: Find the L.C.M. of the following expressions.

$$\mathbf{x^2-2x-15,\ x^2+5x+6}$$

Solution:

1^{st} Expression $= x^2-2x-15$
$= x^2-5x+3x-15$
$= x(x-5)+3(x-5)$
$= (x-5)(x+3)$

2^{nd} Expression = x^2+5x+6
= $x^2+3x+2x+6$
= $x(x+3)+2(x+3)$
= $(x+3)(x+2)$

$\therefore$ The required L.C.M. is $(x+3)(x+2)(x-5)$.

Explanation of the solution of the problem-2:

Explanation: There is a common factor $(x+3)$ in the two expressions: $x^2-2x-15$ and x^2+5x+6 & there is an extra factor $(x-5)$ in the first expression and $(x+2)$ is in the second.

Therefore, the required L.C.M. is $(x+3)(x+2)(x-5)$.

Mistake-1: Show that there is always no difference between C.F. (Common Factor) & H.C.F.

Proof: Let us consider any two algebraic expressions x and x+1.

The factors of x are 1 and x.

The factors of x+1 are 1 and x+1.

Therefore, C.F. of x and x+1 is 1.

And H.C.F of x and x+1 is 1.
$\therefore$ C.F. = H.C.F.

Therefore, there is always no difference between C.F. and H.C.F.

Explanation of the proof of the mistake-1:

Explanation: Factor: A polynomial expression P(x) is said to be a factor of the polynomial expression Q(x) if P(x) divides Q(x) i.e. there is a polynomial expression R(x) such that Q(x)=P(x).R(x)

Equivalent Factor: When the two polynomials P(x) and Q(x) are such that each is a factor of the other, then they are said to be equivalent factor.

For example, $(x-1)$ and $(1-x)$; 5 and -5, etc.

Equivalent factors are treated as same factors.

Therefore, the above statement i.e. the statement of mistake-1 is true when there is only one common factor.

Mistake-2: Show that the H.C.F. of x^2-1, x^3-1 and $1-x^4$ is 1.

Proof: 1^{st} Expression = x^2-1
= $(x)^2-(1)^2$
= $(x+1)(x-1)$

2^{nd} Expression = x^3-1
= $(x)^3-(1)^3$
= $(x-1)(x^2+x+1)$

3^{rd} Expression = $1-x^4$
= $(1)^2-(x^2)^2$
= $(1+x^2)(1-x^2)$
= $(1+x^2)\{(1)^2-(x)^2\}$
= $(1+x^2)(1+x)(1-x)$

$\therefore$ The required H.C.F is 1.

Explanation of the proof of the mistake-2:

Explanation: For factorization (x−1) and (1−x) are equivalent factors and therefore, they are the same factors.

So, the common factor of three algebraic expressions is (x−1) or (1−x).

Therefore, (x−1) {or (1−x)} is a common factor of three algebraic expressions.

Therefore, the required H.C.F. is (x−1) or (1−x).

Mistake-3: Show that the L.C.M. of x^2-1, x^3-1 and $1-x^4$ is $(x^2+x+1)(1+x^2)(x+1)(x-1)(1-x)$.

Proof: 1st Expression = x^2-1
$= (x)^2-(1)^2$
$= (x+1)(x-1)$

2nd Expression = x^3-1
$= (x)^3-(1)^3$
$= (x-1)(x^2+x+1)$

3rd Expression = $1-x^4$
$= (1)^2-(x^2)^2$
$= (1+x^2)(1-x^2)$
$= (1+x^2)\{(1)^2-(x)^2\}$
$= (1+x^2)(1+x)(1-x)$

∴ The required L.C.M is $(x^2+x+1)(1+x^2)(x+1)(x-1)(1-x)$.

Explanation of the proof of the mistake-3:

Explanation: Multiple: If polynomial expression P(x) is a factor of polynomial expression Q(x), then Q(x) is said to be a multiple of P(x).

Equivalent Multiple: If P(x) and Q(x) are equivalent factors, then they are said to be equivalent multiples.

Equivalent multiples are treated as same multiples.

Here, (x−1) and (1−x) are equivalent multiples. Therefore, they are treated as same multiple.

Then, the required L.C.M. is $-(x^2+x+1)(1+x^2)(1+x)(x-1)$. Or $-(x^2+x+1)(1+x^2)(1+x)(1-x)$

5. IMPLEMENTATION

These mistakes may be implemented in the teaching–learning process as well as in the content of H.C.F. and L.C.M. of algebraic expressions of text books of mathematics.

6. CONCLUSION

1. There is a special strategy in this study for learning mathematics which helps the learner to understand important mathematical concept.
2. Three mistakes and their proofs/solutions and the explanation of the proofs/solutions of the mistake on 'H.C.F. and L.C.M. of algebraic expressions' have been presented here.
3. This study will help the learners to understand the mistakes clearly which is generally done by them on 'H.C.F. and L.C.M. of algebraic expressions'.
4. It will also help the learners having different ability levels *i.e.* the above average, average and below average levels for their improvement of mathematics learning.

7. FURTHER RESEARCH

This study may be extended as an application of 'H.C.F. and L.C.M. of algebraic expression' on large number of samples at an appropriate grade level.

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