International Journal of Mathematical Archive-7(12), 2016, 39-43
Available online through www.ijma.info ISSN 2229 – 5046

TOTAL EDGE IRREGULARITY STRENGTH OF SUBDIVIDED STAR GRAPH, TRIANGULAR SNAKE AND LADDER

INDRA RAJASINGH[1], S. TERESA AROCKIAMARY*[2]

[1]Professor, School of Advanced Sciences, VIT University, Chennai 600 127, India.

[2]Department of Mathematics, Stella Maris College, Chennai 600 086, India.

(Received On: 20-11-16; Revised & Accepted On: 19-12-16)

ABSTRACT

Given a graph G (V, E), a labeling ∂: $V \cup E \rightarrow \{1, 2\ldots k\}$ is called an edge irregular total k-labeling if for every pair of distinct edges uv and xy, $\partial(u)+\partial(uv)+\partial(v) \neq \partial(x)+\partial(y)+\partial(xy)$. The minimum k for which G has an edge irregular total k-labeling is called the total edge irregularity strength of G. In this paper we examine the total edge irregularity strength of Subdivided Star Graph, Triangular snake and Ladder.

Key Words: *Irregular total labeling, Labeling, Star graph, Ladder, Triangular snake, Edge irregularity strength, Subdivided star graph.*

AMS Subject Classification 2010 MSC: *05C78.*

1. INTRODUCTION

For a graph G (V, E), Baca et al. [1] define a labelling ∂: $V \cup E \rightarrow \{1, 2\ldots k\}$ to be an *edge irregular k-labeling* of the graph G if $\partial(u) + \partial(uv) + \partial(v) \neq \partial(x) + \partial(y) + \partial(xy)$ for every pair of distinct edges uv and xy. The minimum k for which the graph G has an edge irregular total k-labeling is called the *total edge irregularity strength* of the graph G, and is denoted by $tes(G)$. For a graph $G(V, E)$, with E not empty, it has been proved that $\left\lceil \frac{|E|+2}{3} \right\rceil \leq tes(G) \leq |E|$; $tes(G) \geq \left\lceil \frac{\Delta(G)+1}{2} \right\rceil$ and $tes(G) \leq |E| - \Delta(G)$ [1]. Brandt *et al.* [2] conjecture that for any graph G other than K_5, $s(G) = \max\left\{\left\lceil \frac{\Delta(G)+1}{2} \right\rceil, \left\lceil \frac{|E|+2}{3} \right\rceil\right\}$. The conjecture has been proved to be true for all trees [3] and for large graphs whose maximum degree is not too large relative to its order and size [2]. Jendrol', Miskul, and Sotak proved that $tes(K_5) = 5$; for $n \geq 6$, $(K_n) = \left\lceil \frac{n^2-n+4}{6} \right\rceil$; and that $tes(K_{m,n}) = \left\lceil \frac{mn+2}{3} \right\rceil$. In this paper we prove that $tes(G) = \left\lceil \frac{|E|+2}{3} \right\rceil$ for subdivided star graph, triangular snake and ladder graph proving Brandt's conjecture.

2. TRIANGULAR SNAKE

Definition: A triangular snake is a connected graph in which all blocks are triangles and the block-cut-point graph is a path. It is also obtained from a path $u_1, u_2,\ldots,u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $i = 1, 2,\ldots, n-1$.

An r-dimensional triangular snake is a triangular snake consisting of r blocks of triangles. It is denoted by $TS(r)$. Let $TS(1)$ be denoted by B_1. $TS(r)$ contains r blocks, each isomorphic to B_1. Let the ith block of $TS(r)$ be denoted by B_i, $1 \leq i \leq r$. The r-dimensional triangular snake has $(2r+1)$ vertices and $3r$ edges.

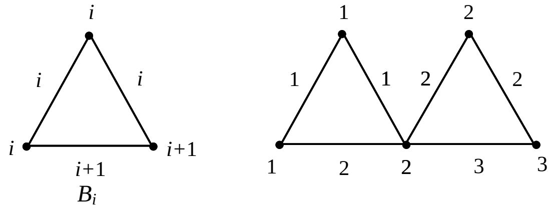
In the sequel, by `edge sum label' of an edge (u, v) in G we mean the sum of the labels of vertices u, v and the edge (u, v).

Corresponding Author: S. Teresa Arockiamary[2]*
[2]Department of Mathematics, Stella Maris College, Chennai 600 086, India.

2.1 Lemma *tes*(*TS*(2)) = 3.

Proof: Let *TS*(2) be labeled as in Figure 1(*b*). It is easy to check that *tes*(*TS*(2)) = 3.

The following algorithm yields the total edge irregularity strength of *TS*(*r*), $r \geq 3$.



(*a*) (*b*)

Figure-1

Procedure *tes*(*TS*(*r*))
Input:
r-dimensional triangular snake,*TS*(*r*).

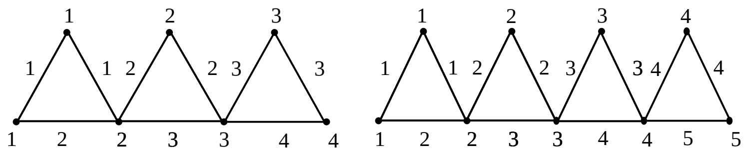
Algorithm:
Label the *i*th block B_i of *TS*(*r*) as shown in Figure 1(*a*), $1 \leq i \leq r$.

End Procedure *tes*(*TS*(*r*)).

Output: *tes*(*TS*(*r*)) = $\lceil(3r+2)/3\rceil$.

Proof of Correctness: The labeling is well defined since the label of the right end vertex of the base B_i is equal to the label of the left end vertex of the base of $B_{i+1}= i+1$, $\forall\ i = 1,..., r-1$. We prove the result by induction on *l*. By lemma 2.1, *tes*(*TS*(2)) = 3. This proves the result when $l = 2$. Assume the result to be true for *TS*(*l*). Consider *TS*(*l*+1). Edge irregular total labeling of *TS*(*l*) are 3,4 5,...,3*l*+2. The three edges of B_{l+1} have edge sum labels $l+1+l+1+l+1$, $l+1+l+1+l+2$, $l+1+l+2+l+2$ which are nothing but $3l+3$, $3l+4$, $3l+5$.

Labeling of *TS*(3) and *TS*(4) are shown in Figure 2(*a*) and 2(*b*). Thus we have the following theorem.



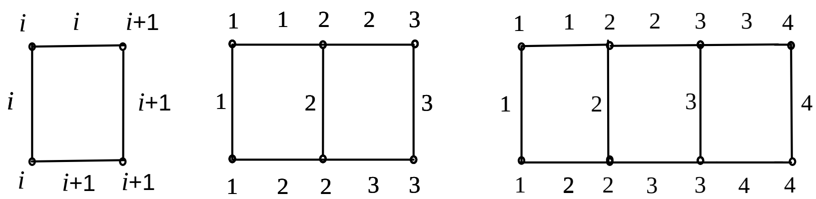
(*a*) (*b*)

Figure-2

2.1 Theorem: Let *TS*(*r*) be an *r*-dimensional triangular snake. Then *tes*(*TS*(*r*)) = $\lceil(3r+2)/3\rceil$.

3. LADDER

Definition: The ladder graph L_n is a planar undirected graph with $2n$ vertices and $n + 2(n - 1)$ edges. It is denoted by $L_{n,1}= P_1 \times P_n$. An *r*-dimensional ladder is a ladder consisting of *r* regions bounded by 4-cycles. It is denoted by *LA*(*r*). By a `block' B_i we mean a region bounded by a 4-cycle. The *r*-dimensional ladder has $(2r + 2)$ vertices and $(3r + 1)$ edges.



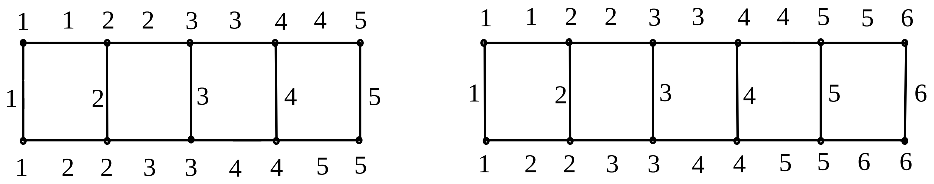
(*a*) (*b*) (*c*)

Figure-3

3.1 Lemma: $tes(LA(2)) = 3$.

Proof: Let $LA(2)$ be labeled as in Figure 3(*b*). It is easy to check that $tes(LA(2)) = 3$.

The following algorithm yields the total edge irregularity strength of $LA(r)$, $r \geq 3$.



(*a*) (*b*)

Figure-4

Procedure *tes*(*LA*(*r*))
Input:
r-dimensional ladder, $LA(r)$.
Algorithm:

Label the *i*th block B_i of $LA(r)$ as shown in Figure 3(*a*), $1 \leq i \leq r$.

End Procedure *tes*(*LA*(*r*)).

Output: $tes(LA(r)) = \lceil (3r+3)/3 \rceil$.

Proof of Correctness: It is easy to check the result for $LA(2)$. Assume the result to be true for $LA(r)$. Consider $LA(r+1)$. Edge irregular total labeling of $LA(r)$ are *3, 4 5,..., 3r+3*. The three edges of B_{r+1} have edge sum labels $r+1+r+1+r+2$, $r+2+r+2+r+1$, $r+2+r+2+r+2$ which are nothing but $3r+4$, $3r+5$, $3r+6$.

Labeling of $LA(3)$ is shown in Figure 3(*c*). Thus we have the following theorem.

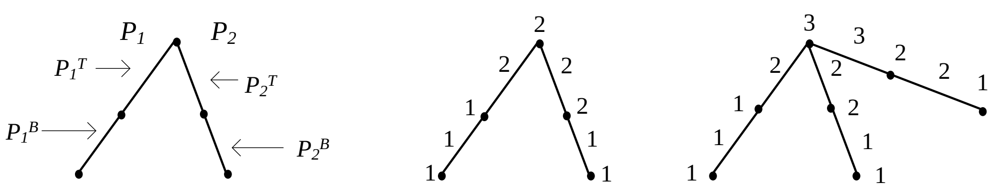
3.1 Theorem: Let $LA(r)$ be an r-dimensional ladder. Then $tes(LA(r)) = \lceil (3r+3)/3 \rceil$.

Labeling of $LA(4)$ and $LA(5)$ are shown in Figure 4.

4. SUBDIVIDED STAR GRAPH

In graph theory, a star S_k is the complete bipartite graph $K_{1,k}$ which is nothing but a tree with one internal node and k leaves. Alternatively, S_k is defined to be the tree of order $k+1$ with maximum diameter 2; in which case a star of $k > 2$ has k leaves. A star with 3 edges is called a claw.

The star graph S_k is an edge-transitive matchstick graph, and has diameter 2 (when $k > 1$), girth ∞ (it has no cycles), chromatic index k, and chromatic number 2 (when $k > 0$). Stars may also be described as the only connected graphs in which at most one vertex has degree greater than one.



(*a*) (*b*) (*c*)

Figure-5

Definition: Subdivide $K_{1,n}$ by introducing a new vertex on each edge of $K_{1,n}$. The graph so obtained is denoted by $K^*_{1,n}$. $K^*_{1,n}$ has $(2n+1)$ vertices and $2n$ edges.

Notation: Denote the i paths of $K^*_{1,i}$ as $P_1, P_2, ..., P_i$ and the edges of $K^*_{1,i}$ as P_i^T and P_i^B. See Figure 5(a).

4.1 Lemma: $tes(K^*_{1,2}) = 2$.

Proof: Let $K^*_{1,2}$ be labeled as in Figure 5(*b*). It is easy to check that $tes(K^*_{1,2}) = 2$.

We now consider $K^*_{1,n}$, $n \geq 3$.

Procedure $tes(K^*_{1,n})$

Input:
Subdivided star graph, $K^*_{1,n}$.

Algorithm:
(1) Label the vertices and edges of $K^*_{1,2}$ as in Lemma 4.1.
(2) Having labeled $K^*_{1,2}$, label $K^*_{1,i}$, $i \geq 3$ as follows:

Denote the vertex which is incident to all *i* paths of $K^*_{1,i}$ as the root vertex *u*. To label the *i* paths P_i we proceed as follows. First we label $P_i^B(K^*_{1,i})$, then label $P_i^T(K^*_{1,i})$ from left to right.

(i) $l(P_r^B(K^*_{1,i})) = l(P_r^B(K^*_{1,i-1}))$, $1 \leq r \leq i-1$ and

$l(P_i^B(K^*_{1,i})) = l(P_1^T(K^*_{1,i-1}))$.

(ii) $l(u) = tes(K^*_{1,i})$.

(iii) Now label the unlabeled edges as follows:

If $P_i^B(K^*_{1,i}) = (u_i,v_i)$ and $P_r^T(K^*_{1,i}) = (u,w_i)$, $1 \leq r \leq i$, with vertex labels and edge labels $l(u_i)$, $l(v_i)$, $l(u)$, $l(w_i)$ and $l(u_iv_i)$, then

$$l(P_r^T(K^*_{1,i})) = l(u_i) + l(v_i) + l(u_iv_i) + r - (l(u) + l(w_i)).$$

End Procedure $tes(K^*_{1,n})$.

Output: $tes(K^*_{1,n}) = \lceil(2n+2)/3\rceil$.

Proof of Correctness: We prove the result by induction on *i*. When *i* = 2, the result is true by Lemma 4.1. Assume the result for *i*.

Consider $K^*_{1,i-1}$. Since the labeling of $K^*_{1,i}$ is an edge irregular *k*-labeling, it is clear that the labeling of vertices and edges of $K^*_{1,i+1}$ obtained by adding consecutive integers as in step 2 (iii) is also an edge irregular *k*-labeling. We know by actual verification that the edge sum labels obtained in Lemma 4.1 are distinct. Hence the edge sum labels of the edges of $K^*_{1,i+1}$ obtained by adding consecutive integers as in step 2 (iii) is also an edge irregular *k*-labeling.

We know by actual verification that the edge sum labels obtained in Lemma 4.1 are distinct. Hence the edge sum labels of the edges of $K^*_{1,i+1}$ are also distinct.

Labeling of $K^*_{1,3}$ is shown in Figure 5(*c*). Thus we have the following theorem.

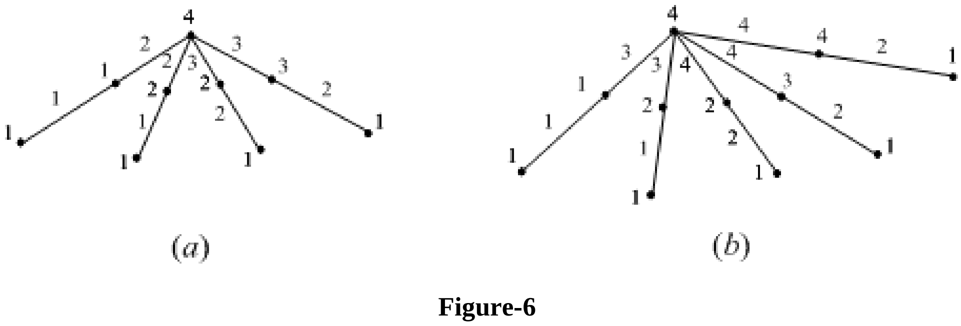


Figure-6

4.1 Theorem: Let $K^*_{1,n}$ be a subdivision of $K_{1,n}$. Then $tes(K^*_{1,n}) = \lceil(2n+2)/3\rceil$.

Labeling of $K^*_{1,4}$ and $K^*_{1,5}$ are shown in Figure 6(*a*) and Figure 6(*b*).

5. CONCLUSION

In this paper, we considered triangular snake, subdivided star graph and ladder graph and proved that they are total edge irregular. Our study is extended to circulant networks.

6. REFERENCES

1. M. Baca, S.Jendrol, M.Miller and J.Ryan, On irregular total labelings, Discrete Math, 307(2007), 1378-1388.
2. S. Brandt, J. Miskuf and D. Rautenbach, On a conjecture about edge irregular total labelings, J.Graph Theory, 57 (2008), 333-343.
3. S. Jendrol, J. Miškuf and R. Soták, Total edge irregularity strength of complete graphs and complete bipartite graphs, Electronic Notes Discrete Mathematics, 28(2007), 281-285.

Source of support: Nil, Conflict of interest: None Declared