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TOTAL EDGE IRREGULARITY STRENGTH OF SUBDIVIDED STAR GRAPH, TRIANGULAR SNAKE AND LADDER

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ABSTRACT

Given a graph G (V, E), a labeling ∂: $V \cup E \rightarrow \{1, 2\ldots k\}$ is called an edge irregular total k-labeling if for every pair of distinct edges uv and xy, $\partial(u)+\partial(uv)+\partial(v) \neq \partial(x) + \partial(y) + \partial(xy)$. The minimum k for which G has an edge irregular total k-labeling is called the total edge irregularity strength of G. In this paper we examine the total edge irregularity strength of Subdivided Star Graph, Triangular snake and Ladder.

Key Words: *Irregular total labeling, Labeling, Star graph, Ladder, Triangular snake, Edge irregularity strength, Subdivided star graph.*

AMS Subject Classification 2010 MSC: *05C78.*

1. INTRODUCTION

For a graph *G* (*V*, *E*), Baca et al. [1] define a labelling ∂: $V \cup E \rightarrow \{1, 2\ldots k\}$ to be an *edge irregular k-labeling* of the graph *G* if $\partial(u) + \partial(uv) + \partial(v) \neq \partial(x) + \partial(y) + \partial(xy)$ for every pair of distinct edges *uv* and *xy*. The minimum *k* for which the graph *G* has an edge irregular total *k*-labeling is called the *total edge irregularity strength* of the graph *G*, and is denoted by *tes*(*G*). For a graph *G*(*V*, *E*), with *E* not empty, it has been proved that $\left\lceil\frac{|E|+2}{3}\right\rceil \leq tes(G) \leq |E|$; $tes(G) \geq \left\lceil\frac{\Delta(G)+1}{2}\right\rceil$ and $tes(G) \leq |E| - \Delta(G)$ [1]. Brandt *et al.* [2] conjecture that for any graph *G* other than K_5, $s(G) = \max\left\lceil\frac{\Delta(G)+1}{2}\right\rceil, \left\lceil\frac{|E|+2}{3}\right\rceil$. The conjecture has been proved to be true for all trees [3] and for large graphs whose maximum degree is not too large relative to its order and size [2]. Jendrol', Miskul, and Sotak proved that $tes(K_5) = 5$; for $n \geq 6$, $(K_n) = \left\lceil\frac{n^2-n+4}{6}\right\rceil$; and that $tes(K_{m,n}) = \left\lceil\frac{mn+2}{3}\right\rceil$. In this paper we prove that $tes(G) = \left\lceil\frac{|E|+2}{3}\right\rceil$ for subdivided star graph, triangular snake and ladder graph proving Brandt's conjecture.

2. TRIANGULAR SNAKE

Definition: A triangular snake is a connected graph in which all blocks are triangles and the block-cut-point graph is a path. It is also obtained from a path $u_1, u_2,\ldots,u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $i = 1, 2,\ldots, n-1$.

An *r*-dimensional triangular snake is a triangular snake consisting of *r* blocks of triangles. It is denoted by *TS*(*r*). Let *TS*(1) be denoted by B_1. *TS*(*r*) contains *r* blocks, each isomorphic to B_1. Let the *i*th block of *TS*(*r*) be denoted by B_i, $1 \leq i \leq r$. The *r*-dimensional triangular snake has (2*r*+1) vertices and 3*r* edges.

In the sequel, by `edge sum label' of an edge (*u*, *v*) in *G* we mean the sum of the labels of vertices *u*, *v* and the edge (*u*, *v*).

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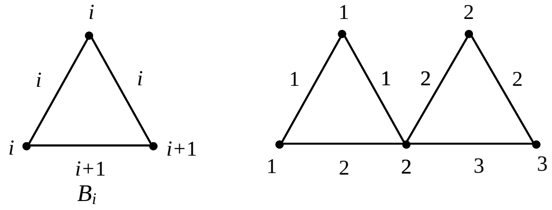
2.1 Lemma *tes*(*TS*(2)) = 3.

Proof: Let $TS(2)$ be labeled as in Figure 1(b). It is easy to check that $tes(TS(2)) = 3$.

The following algorithm yields the total edge irregularity strength of $TS(r)$, $r \geq 3$.



(a) (b)

Figure-1

Procedure *tes*(*TS*(*r*))
Input:
r-dimensional triangular snake, $TS(r)$.

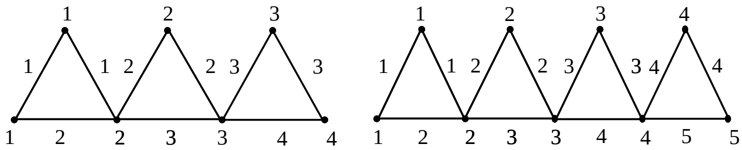
Algorithm:
Label the ith block B_i of $TS(r)$ as shown in Figure 1(a), $1 \leq i \leq r$.

End Procedure *tes*(*TS*(*r*)).

Output: $tes(TS(r)) = \lceil(3r+2)/3\rceil$.

Proof of Correctness: The labeling is well defined since the label of the right end vertex of the base B_i is equal to the label of the left end vertex of the base of $B_{i+1} = i+1$, $\forall\, i = 1,\ldots, r-1$. We prove the result by induction on l. By lemma 2.1, $tes(TS(2)) = 3$. This proves the result when $l = 2$. Assume the result to be true for $TS(l)$. Consider $TS(l+1)$. Edge irregular total labeling of $TS(l)$ are 3,4 5,...,3l+2. The three edges of B_{l+1} have edge sum labels $l+1+l+1+l+1$, $l+1+l+1+l+2$, $l+1+l+2+l+2$ which are nothing but $3l+3$, $3l+4$, $3l+5$.

Labeling of $TS(3)$ and $TS(4)$ are shown in Figure 2(a) and 2(b). Thus we have the following theorem.



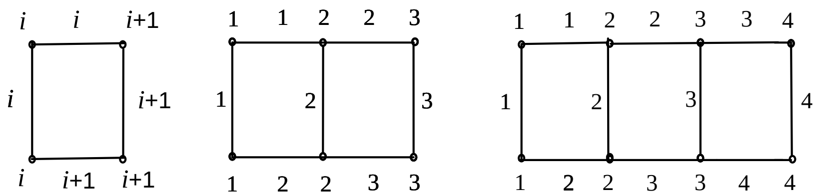
(a) (b)

Figure-2

2.1 Theorem: Let $TS(r)$ be an r-dimensional triangular snake. Then $tes(TS(r)) = \lceil(3r+2)/3\rceil$.

3. LADDER

Definition: The ladder graph L_n is a planar undirected graph with $2n$ vertices and $n + 2(n-1)$ edges. It is denoted by $L_{n,1} = P_1 \times P_n$. An r-dimensional ladder is a ladder consisting of r regions bounded by 4-cycles. It is denoted by $LA(r)$. By a `block' B_i we mean a region bounded by a 4-cycle. The r-dimensional ladder has $(2r + 2)$ vertices and $(3r + 1)$ edges.



(a) (b) (c)

Figure-3

3.1 Lemma: $tes(LA(2)) = 3$.

Proof: Let $LA(2)$ be labeled as in Figure 3(b). It is easy to check that $tes(LA(2)) = 3$.

The following algorithm yields the total edge irregularity strength of $LA(r)$, $r \geq 3$.

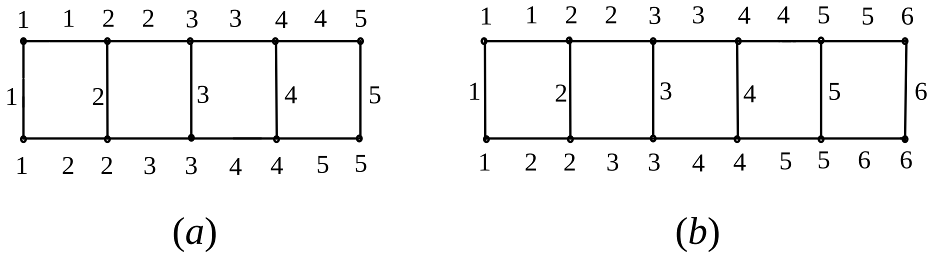


Figure-4

Procedure *tes*(*LA*(*r*))
Input:
r-dimensional ladder, $LA(r)$.
Algorithm:

Label the ith block B_i of $LA(r)$ as shown in Figure 3(a), $1 \leq i \leq r$.

End Procedure *tes*(*LA*(*r*)).

Output: $tes(LA(r)) = \lceil (3r+3)/3 \rceil$.

Proof of Correctness: It is easy to check the result for $LA(2)$. Assume the result to be true for $LA(r)$. Consider $LA(r+1)$. Edge irregular total labeling of $LA(r)$ are *3, 4 5,..., 3r+3*. The three edges of B_{r+1} have edge sum labels $r+1+r+1+r+2$, $r+ 2+r+2+r+1$, $r+2+r+2+r+2$ which are nothing but $3r+4$, $3r+5$, $3r+6$.

Labeling of $LA(3)$ is shown in Figure 3(c). Thus we have the following theorem.

3.1 Theorem: Let $LA(r)$ be an r-dimensional ladder. Then $tes(LA(r)) = \lceil (3r+3)/3 \rceil$.

Labeling of $LA(4)$ and $LA(5)$ are shown in Figure 4.

4. SUBDIVIDED STAR GRAPH

In graph theory, a star S_k is the complete bipartite graph $K_{1,k}$ which is nothing but a tree with one internal node and k leaves. Alternatively, S_k is defined to be the tree of order $k+1$ with maximum diameter 2; in which case a star of $k > 2$ has k leaves. A star with 3 edges is called a claw.

The star graph S_k is an edge-transitive matchstick graph, and has diameter 2 (when $k > 1$), girth ∞ (it has no cycles), chromatic index k, and chromatic number 2 (when $k > 0$). Stars may also be described as the only connected graphs in which at most one vertex has degree greater than one.

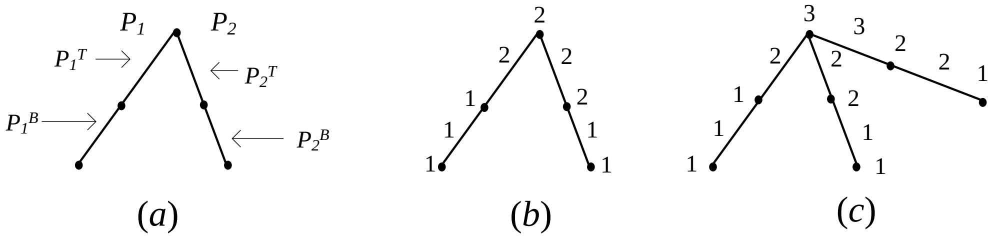


Figure-5

Definition: Subdivide $K_{1,n}$ by introducing a new vertex on each edge of $K_{1,n}$. The graph so obtained is denoted by $K^*_{1,n}$. $K^*_{1,n}$ has $(2n+1)$ vertices and $2n$ edges.

Notation: Denote the i paths of $K^*_{1,i}$ as $P_1, P_2, ..., P_i$ and the edges of $K^*_{1,i}$ as P_i^T and P_i^B. See Figure 5(a).

4.1 Lemma: $tes(K^*_{1,2}) = 2$.

Proof: Let $K^*_{1,2}$ be labeled as in Figure 5(*b*). It is easy to check that $tes(K^*_{1,2}) = 2$.

We now consider $K^*_{1,n}$, $n \geq 3$.

Procedure $tes(K^*_{1,n})$

Input:
Subdivided star graph, $K^*_{1,n}$.

Algorithm:
(1) Label the vertices and edges of $K^*_{1,2}$ as in Lemma 4.1.
(2) Having labeled $K^*_{1,2}$, label $K^*_{1,i}$, $i \geq 3$ as follows:

Denote the vertex which is incident to all *i* paths of $K^*_{1,i}$ as the root vertex *u*. To label the *i* paths P_i we proceed as follows. First we label $P_i^B(K^*_{1,i})$, then label $P_i^T(K^*_{1,i})$ from left to right.

(i) $l(P_r^B(K^*_{1,i})) = l(P_r^B(K^*_{1,i-1}))$, $1 \leq r \leq i-1$ and

$l(P_i^B(K^*_{1,i})) = l(P_1^T(K^*_{1,i-1}))$.

(ii) $l(u) = tes(K^*_{1,i})$.

(iii) Now label the unlabeled edges as follows:

If $P_i^B(K^*_{1,i}) = (u_i,v_i)$ and $P_r^T(K^*_{1,i}) = (u,w_i)$, $1\leq r \leq i$, with vertex labels and edge labels $l(u_i)$, $l(v_i)$, $l(u)$, $l(w_i)$ and $l(u_iv_i)$, then

$$l(P_r^T(K^*_{1,i})) = l(u_i) + l(v_i) + l(u_iv_i) + r - (l(u) + l(w_i)).$$

End Procedure $tes(K^*_{1,n})$.

Output: $tes(K^*_{1,n}) = \lceil(2n+2)/3\rceil$.

Proof of Correctness: We prove the result by induction on *i*. When *i* = 2, the result is true by Lemma 4.1. Assume the result for *i*.

Consider $K^*_{1,i-1}$. Since the labeling of $K^*_{1,i}$ is an edge irregular *k*-labeling, it is clear that the labeling of vertices and edges of $K^*_{1,i+1}$ obtained by adding consecutive integers as in step 2 (iii) is also an edge irregular *k*-labeling. We know by actual verification that the edge sum labels obtained in Lemma 4.1 are distinct. Hence the edge sum labels of the edges of $K^*_{1,i+1}$ obtained by adding consecutive integers as in step 2 (iii) is also an edge irregular *k*-labeling.

We know by actual verification that the edge sum labels obtained in Lemma 4.1 are distinct. Hence the edge sum labels of the edges of $K^*_{1,i+1}$ are also distinct.

Labeling of $K^*_{1,3}$ is shown in Figure 5(*c*). Thus we have the following theorem.

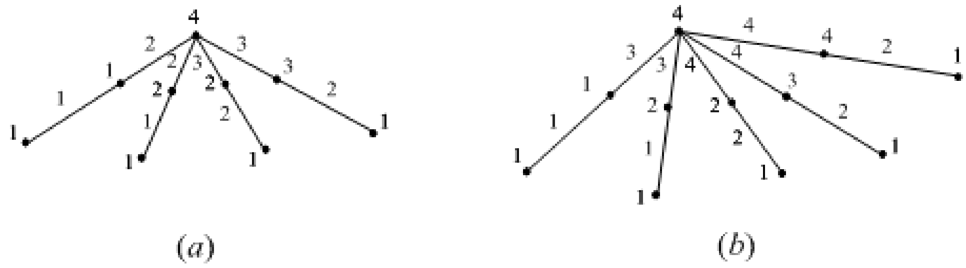


Figure-6

4.1 Theorem: Let $K^*_{1,n}$ be a subdivision of $K_{1,n}$. Then $tes(K^*_{1,n}) = \lceil(2n+2)/3\rceil$.

Labeling of $K^*_{1,4}$ and $K^*_{1,5}$ are shown in Figure 6(*a*) and Figure 6(*b*).

5. CONCLUSION

In this paper, we considered triangular snake, subdivided star graph and ladder graph and proved that they are total edge irregular. Our study is extended to circulant networks.

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