International Journal of Mathematical Archive-7(12), 2016, 30-32
Available online through www.ijma.info ISSN 2229 – 5046

CONTRIBUTION OF SOME INTEGRAL TRANSFORMS IN SOLVING ORDINARY DIFFERENTIAL EQUATIONS

H. K. UNDEGAONKAR*
Assistant Professor, Bahirji Smarak Mahavidyalaya, Basmathnagar, Dist; Hingoli, India.

R. N. INGLE
Associate Professor, Bahirji Smarak Mahavidyalaya, Basmathnagar, Dist. Hingoli, India.

(Received On: 14-11-16; Revised & Accepted On: 16-12-16)

ABSTRACT

In this paper both transforms namely Laplace and Sumudu were applied to solve second order differential equations and tried to show contribution of Laplace transform and Sumudu transform to solve ordinary linear differential equations of second order with constant coefficients.

Keywords: *Laplace transform, ordinary differential equations, Sumudu transform.*

1. INTRODUCTION

There are some mathematical tools like Fourier transform, Laplace transform, Sumudu transform, etc which are convenient to solve ordinary and partial differential equations. Laplace transform is the integral transform which is applicable in solving ordinary and partial differential equations [1]. This transform is also applicable in engineering sciences [2]. There is one more integral transform namely Sumudu transform proposed by Gamage K.Watugala to solve differential equations in 1993 [3]

Major derivations in Mathematical sciences are in the form of ordinary or partial differential equations which we can solve by applying these integral transforms. Sumudu transforms are applicable in solve partial differential equations [7]. In this paper we have applied both transforms Laplace and Sumudu for obtaining solution of ordinary linear differential equations of second order with constant coefficients to demonstrate contribution of integral transforms in differential equations.

2(A) SOME DEFINITIONS AND THEOREMS OF LAPLACE TRANSFORM DESCRIBED IN [2]

Definition 2.1: Laplace transform: The Laplace transform of f (t) is defined by L [f (t)] = F (p) =$\int_0^\infty f(t)e^{-pt}$ dt

Theorem 2.1: Let F(t) and its derivatives F'(t), F"t),…….,F^{n-1}(t) be continuous functions for t≥0 and be of exponential orders as t→ ∞ and F^n(t) is of class A then the Laplace transform of F^n(t) exists when p>α and is given by

$$L\{F^n(t)\} = P^n L\{F(t)\} - p^{n-1}F(0) - pn\text{-}2\text{-}F'(0)\ldots\ \ldots\ \ldots F^{n-1}(0)$$

Some inverse Laplace transforms of standard functions which are discussed in [2]

If L {f (t)} = F(s) then

(a) $L^{-1}\left[\frac{dF(s)}{ds}\right] = -t\,f(t)$
(b) $L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$
(c) $L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{\sin at}{a}$

*Corresponding Author: H. K. Undegaonkar**
Assistant Professor, Bahirji Smarak Mahavidyalaya, Basmathnagar, Dist. Hingoli, India.

2(B). SOME DEFINITIONS AND THEOREMS OF SUMUDU TRANSFORM WHICH ARE DESCRIBED IN [3], [4]

Definition 2.2: Sumudu transform: Consider the set B = {F (g) such that ∃N, t1, t2 >0,$|f(g)|$ < N$e^{|g|/tj}$, if j∈ $(-1)^j$ X [0, ∞)} then over this set the Sumudu transform is defined by S[f(t)] $=\frac{1}{u}\int_0^\infty e^{\frac{-t}{u}}$f(t)dt

Theorem 2.2: Let f (t) be in B and let $G_n(u)$ denote the sumudu transform of the nth derivative $f^n(t)$ of f(t) then for n≥1

$$G_n(u) = \frac{G(u)}{u^n} = \sum_{k=0}^{n-1}\frac{f^k(0)}{u^{n-k}}$$

Some inverse Sumudu transforms of standard functions described in [5]

(d) $S^{-1}\left[\frac{u^2}{(1+a^2u^2)^2}\right] = \frac{t\sin at}{2a}$

(e) $S^{-1}\left[\frac{1}{1+a^2u^2}\right] = \cos at$

3. MAIN PROBLEM THE OF PAPER

Consider the O.D.E Y''(t) + n^2y(t) = 2ncos(nt) with initial conditions y(0) = 2, y'(0) = 0 (3.1)

First we will solve the above differential equation by Laplace transform method.

Applying Laplace transform to equation (3.1) we have

$$L\{y''(t)\} + n^2L\{y(t)\} = 2nL\{cosnt\}$$

By applying theorem (2.1) we have

$$s^2L\{y(t)\} - s\,y(0) - y'(0) + n^2L\{y(t)\} = 2n\left[\frac{s}{s^2+n^2}\right]$$

$$\{s^2+n^2\}L\{y(t)\}\text{ L-2s} = 2n\left[\frac{s}{s^2+n^2}\right]$$

Therefore $L\{y(t)\} = \frac{2ns}{(n^2+s^2)^2} + \frac{2s}{s^2+n^2}$ (3.2)

Operating inverse Laplace transform to equation (3.2) we have

$$y(t) = 2nL^{-1}\left[\frac{S}{(n^2+s^2)^2}\right] + 2L^{-1}\left[\frac{S}{s^2+n^2}\right]$$

By applying formulae (a), (b), (c) we obtain

$$y(t) = \text{t}\sin nt + 2\cos nt \quad (3.3)$$

Now we will try to solve O.D.E (3.1) by applying sumudu transform Operating sumudu transform to equation (3.1) we have

$$S\{y''(t)\} + n^2S\{y(t)\} = 2nS\{cosnt\}$$

By applying theorem (2.2) we obtain

$$\frac{G(u)-y(0)}{u^2} - \frac{y'(0)}{u} + n^2G(u) = \frac{2n}{1+n^2u^2}$$

By applying initial conditions we have

$$\frac{G(u)-2}{u^2} + n^2G(u) = \frac{2n}{1+n^2u^2}$$

Therefore we have

$$G(u)\left[\frac{1+n^2u^2}{u^2}\right] = \frac{2n}{1+n^2u^2} + \frac{2}{u^2}$$

$$G(u) = 2n\frac{u^2}{(1+n^2u^2)^2} + 2\frac{1}{1+n^2u^2} \quad (3.4)$$

Applying inverse sumudu transform to equation (3.4) and applying formulae (d), (e) we obtain

$$y(t) = \text{t}\sin nt + 2\cos nt$$

4. Example: solve y'' (t) +9y (t) = 6 cos $3t$, y (0) = 2.y' (0) = 0 (4.1)

Applying Laplace transform to equation (4.1) we have

$$s^2 L\{y(t)\} - s\, y(0) - y'(0) + 9\, L\{y(t)\} = 6\left[\frac{s}{s^2+3^2}\right]$$

By applying initial conditions we obtain

$$\{s^2 + 9\}L\{y(t)\}\ L\text{-}2s = 2n\left[\frac{s}{s^2+n^2}\right]$$

Therefore $$L\{y(t)\} = \frac{6s}{(3^2+s^2)^2} + \frac{2s}{s^2+3^2} \quad (4.2)$$

Operating inverse Laplace transform to equation (4.2) & applying formulae (a), (b), (c) we get

$$y(t) = t\sin 3t + 2\cos 3t$$

Now applying sumudu transform to equation (4.1) we have

$$S\{y''(t)\} + 9\, S\{y(t)\} = 6S\{cos3t\}$$

By applying theorem (2.2) & applying initial conditions we have

$$\frac{G(u) - 2)}{u^2} + 9G(u) = \frac{6}{1+9u^2}$$

Therefore we have

$$G(u)\left[\frac{1+9u^2}{u^2}\right] = \frac{2}{1+n^2u^2} + \frac{2}{u^2}$$

$$G(u) = \frac{6u^2}{(1+n^2u^2)^2} + \frac{2}{1+n^2u^2} \quad (4.3)$$

Applying inverse sumudu transform to equation (4.3) and applying formulae (d), (e) we obtain

$$y(t) = t\sin 3t + 2\cos 3t$$

5. CONCLUSIONS

In this paper we have applied Laplace transform & Sumudu transform to solve ordinary linear differential equation with constant coefficients successfully After applying this we came to conclusion that these two transforms have a great contribution in solving such differential equations which are useful to determine some derivations in mathematical sciences.

6. ACKNOWLEDGEMENTS

Authors are thankful to the coordinator (Research Centre), Principal of N.E.S. Science College Nanded to provide facilities of library, internet to complete this research work.

REFERENCES

1. Widder D.V. 1946. The Laplace transforms, Princeton University press, USA.
2. Jaegar J. C. 1961, An Introduction to the Laplace transformation with engineering applications, Methuen London.
3. G. K. Watugala, Sumudu transforms: a new integral transform to solve differential equations and control engineering problems, International Journal of Mathematical Education in Science and Technology, 24(1993), vol no 1, 35-43.
4. Belgacem, F.B.M., karaballi, A.A. and Kala L.S. 2007. Analytical Investigations of the Sumudu transform and applications to integral production equations, Math. Probl. Engr: 3:103-118.
5. M.R. Spigel, Theory and problems of Laplace transforms, Schaums outline series, McGraw-Hill, Newyork, 1965.
6. Complex inversion formula for Sumudu transforms, International Journal of Mathematical Education in Science and Technology 29(1998), Vol. no 618-621.
7. S. weerakoon, Application of Sumudu transform to partial differential equations, International Journal of Mathematical Education in Science and Technology 25(1994), Vol. no 2, 277-283.

Source of support: Nil, Conflict of interest: None Declared