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INTEGER PROGRAMMING PROBLEM

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ABSTRACT

This paper contain the solution of integer programming problem with the help of Transportation Problem use the basic initial methods to compute the result. The most important and successful application in the optimization refer to transportation problem that is the special case of the integer programming problem

Keywords: *Transportation Problem, Integer Programming Problem, Methods for finding initial basic solution.*

I. INTRODUCTION

As the name implied INTEGER PROGRAMMING PROBLEM are the special case of linear programming problem, where all or some of the variable in the optimal solution are restricted to non-negative integer values. The linear programming problem that have been discussed thus for all have been continuous, in the sense that the decision variable are allowed to be fractional often this is a realistic assumption. For example we might easily produce $104\frac{3}{4}$ gallons of a divisible good such as wine.

Transportation problem firstly in second world war by Triwantowich and presented by F.L.Hitchcock [(2)] in his paper. The history of Transportation problem is discussed in "Optimum Utilization of Transportation system" by T.C.Koopmans [(1)] Transportation problem is one of the fundamental problem network flow problem which is usually use to minimize the transportation cost for industries with number of destination and number of sources which satisfying their limit of demand and supply.

In this paper linear programming problem convert into the integer programming problem by the Tora software and also convert into the transportation problem, find the minimum cost of integer programming problem.

II. FORMULATION OF TRANSPORTATION PROBLEM

Formulate the world problem into the transportation problem.
S_i= amount to be shipped from shipping origin. ($S_i \geq 0$)
D_j= amount to be received at the destination. ($D_j \geq 0$)
C_{ij}= cost of per unit object from origin i to destination j. ($C_{ij} \geq 0$)
X_{ij}= amount to be shipped from origin i to destination j with minimize cost. ($X_{ij} \geq 0$)

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Plants (origin)	Warehouse (Destinations)				Supply (availability)
	W1	W2	wj	wn	
P1	X11 C11	X12 C12	X1j C1j	X1n C1n	S1
P2	X21 C21	X22 C22	X2j C2j	X2 n C2n	S2
Pi	Xi1 Ci 1	Xi2 Ci2	Xij Cij	Xin Cin	Si
Pm	Xm1 Cm 1	Xm2 Cm2	Xmj Cmj	xmn Cmn	Sm
Demand (requirement)	D1	D2	Dj	Dn	$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$

Mathematically, transportation problem can be expressed as linear programing problem.

Minimize total cost $Z= x_{11}c_{11} + x_{12}c_{12} +\ldots+ x_{21}c_{21}+\ldots+ x_{mn}\, c_{mn}$

Subject to constraints

$$x_{i1}+x_{i2}+\ldots+x_{im} = s_i \tag{1}$$

$$x_{1j}+x_{2j}+\ldots+x_{mj} = d_j \tag{2}$$

$$X_{ij} \geq 0 \text{ for all i and j} \tag{3}$$

Or it can be written as:

Minimize $z = \sum_{i,j=1}^{m,n} c_{ij} x_{ij}$

Subject to constraints $\sum_{j=1}^{n} x_{ij} = s_i \quad (i = 1, 2, 3, \ldots m)$

$\sum_{i=1}^{m} x_{ij} = d_i \quad (i = 1, 2, 3, \ldots n)$ and $X_{ij} \geq 0$

1. If $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_i$, the given transportation problem is balanced. In this total supply is equal to total demand.

2. If $\sum_{i=1}^{m} s_i \neq \sum_{j=1}^{n} d_i$, the given is unbalanced.

III. NUMERICAL EXAMPLE

Find the optimal cost for the transportation problem.

ORIGIN/DESTNATION	A	B	C	D	CAPACITY
E	1	2	1	4	30
F	3	3	2	1	50
G	4	2	5	9	20
DEMAND	20	40	30	10	

Solution: Given transportation problem is balanced, then IBFS by the LEAST COST ENTRY method.

ORIGIN/DESTNATION	A	B	C	D	CAPACITY
E	1	2	1[30]	4	30
F	3[20]	3[20]	2	1[10]	50
G	4	2[20]	5	9	20
DEMAND	20	40	30	10	

Number of allocation $\neq$ m+n-1

(m = no. of rows, n= no. of column)

Here is degeneracy to remove the degeneracy we put Δ in minimum cost cell. Take value of Δ = 0.

ORIGIN/DESTNATION	A	B	C	D	CAPACITY
E	1[Δ=0].	2	1[30]	4	30
F	3[20]	3[20]	2	1[10]	50
G	4	2[20]	5	9	20
DEMAND	20	40	30	10	

Then the cost = 3*20+3*20+2*20+1*30+1*10+Δ*1=200Rs.

IV. BY VOGEL APPROXIMATION METHOD

ORIGIN/DESTNATION	A	B	C	D	CAPACITY
E	1[20]	2[10]	1	4	30
F	3	3[10]	2[30]	1[10]	50
G	4	2[20]	5	9	20
DEMAND	20	40	30	10	

Number of allocation = m+n-1=06 (m=no. of rows, n=no. of column).

So the solution is non-degenerate.

Then IBFS
1*20+2*10+3*10+2*20+2*30+1*10 =180 Rs
.
x_{11}=20, x_{12}=10, x_{22}=10, x_{23}=30, x_{24}=10, x_{32}=20

V. FIND THE OPTIMUM SOLUTION OF GIVEN PROBLEM BY MODI METHOD

Calculate ui, vj by [cij=ui+vj]

ORIGIN/DESTNATION	A	B	C	D	ui
E	1[20]	2[10]	1	4	-1
F	3	3[10]	2[30]	1[10]	0
G	4	2[20]	5	9	-1
Vj	0	3	2	1	

Find Δij for occupied cells: Δij=cij-[ui+vj]

Δ13=+ve, Δ14=+ve, Δ21=+ve, Δ31=+ve, Δ33=+ve, Δ34=+ve

All Δij are positive so the solution is optimum. Thus the minimum cost of the given problem = 180 Rs.

Now convert the transportation problem into the linear programming problem:

Min z = $x_{11}+2x_{12}+x_{13}+4x_{14}+3x_{21}+3x_{22}+2x_{23}+x_{24}+4x_{31}+2x_{32}+5x_{33}+9x_{34}$

Subject to constraints:
$x_{11}+x_{12}+x_{13}+x_{14} \leq 30$
$x_{21}+x_{22}+x_{23}+x_{24} \leq 50$
$x_{31}+x_{32}+x_{33}+x_{34} \leq 20$
$x_{11}+x_{21}+x_{31} \geq 20$
$x_{12}+x_{22}+x_{32} \geq 40$
$x_{13}+x_{23}+x_{33} \geq 30$
$x_{14}+x_{24}+x_{34} \geq 10$
$x_{11},x_{12},x_{13},x_{14},x_{21},x_{22},x_{23}.x_{24},x_{31},x_{32},x_{33},x_{34} \geq 0$

Change the above linear programing problem into the integer {linear} programing problem by using the Tora software of transportation problem. We are getting the optimal solution of integer programming problem minimum cost z=180 Rs., with variable, $x_{13=30}$, $x_{22=20}$,$x_{32=20}$, $x_{24=10}$.

VI. CONCLUSION

Running the above problem, the result of the problem are equal to integer programming problem by using VAM and MODI method of transportation problem faster and easier than the Branch and Bound, Gomory method for solving the I.P.P. this has been brought out through developed transportation problem into the integer programming problem and compare the result. There is scope for further development of these topics.

VII. REFERENCES

1. Hitchcock, F.L. “The distribution of product from several sources to numerous localities” J. Math.Phy.vol20 (1941), pp 224 to 230
2. Koopmans, T.C. “optimum utilization of transportation system”, Proc. Intern. Statics. Conf. Washington D.C (1947)
3. Sharma, Gaurav; Abbas, S.H.; Gupta, Vijay “Optimum Solution of Transportation Problem with the help of phase-II method of Simplex Method”, Indian journal of applied life science, Vol.6, No.1 & 2(2011), pp. 49-54.
4. R. Palaniyappa and V.Vinob /Anew type transportation problem using object oriented model/IJMA-4(11), NOV 2013.
5. Sharma, Gaurav; Abbas, S.H.; Gupta. “Solving Transportation Problem with the help of Integer Programming Problem” IOSR Journal of Engineering June 2012, Vol.2 (6), pp1274-1277.

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