

## ANTI MULTI FUZZY SUBHEMIRINGS OF A HEMIRING

<sup>1</sup>Dr B. ANANDH\*, <sup>2</sup>S. VASANTHA KUMAR

<sup>1</sup>Department of Mathematics,  
H. H. The Rajah's College, Pudukkottai, Tamilnadu, India.

<sup>2</sup>Research Scholar, Department of Mathematics,  
Sudharsan College of arts and science, Perumanadu, Pudukkottai, Tamilnadu, India,

(Received On: 24-08-16; Revised & Accepted On: 22-12-16)

### ABSTRACT

*In this paper, we made an attempt to study the algebraic nature of an anti multi fuzzy subhemiring of a hemiring.*

**Key Words:** Fuzzy subset, multi fuzzy subset, multi fuzzy subhemiring, anti multi fuzzy subhemiring, pseudo anti multi fuzzy coset.

### INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring  $(R; +; \cdot)$ . Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra  $(R; +, \cdot)$  is said to be a semiring if  $(R; +)$  and  $(R; \cdot)$  are semigroups satisfying  $a \cdot (b+c) = a \cdot b + a \cdot c$  and  $(b+c) \cdot a = b \cdot a + c \cdot a$  for all  $a, b$  and  $c$  in  $R$ . A semiring  $R$  is said to be additively commutative if  $a+b = b+a$  for all  $a, b$  and  $c$  in  $R$ . A semiring  $R$  may have an identity 1, defined by  $1 \cdot a = a = a \cdot 1$  and a zero 0, defined by  $0+a = a = a+0$  and  $a \cdot 0 = 0 = 0 \cdot a$  for all  $a$  in  $R$ . A semiring  $R$  is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh[13], several researchers explored on the generalization of the concept of fuzzy sets. The notion of anti fuzzy left h-ideals in hemiring was introduced by Akram.M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan[8]. In this paper, we introduce some Theorems in anti multi fuzzy subhemiring of a hemiring.

### 1.PRELIMINARIES

**1.1 Definition:** Let  $X$  be a non-empty set. A **fuzzy subset**  $A$  of  $X$  is a function  $A : X \rightarrow [0, 1]$ .

**1.2 Definition:** A **multi fuzzy subset**  $A$  of a set  $X$  is defined as an object of the form  $A = \{ \langle x, \mu_{A1}(x), \mu_{A2}(x), \mu_{A3}(x), \dots, \mu_{An}(x) \rangle / x \in X \}$ , where  $\mu_{Ai} : X \rightarrow [0, 1]$  for all  $i$ . It is denoted as  $A = \langle \mu_{A1}, \mu_{A2}, \mu_{A3}, \dots, \mu_{An} \rangle$ .

**1.3 Definition:** Let  $A$  and  $B$  be any two multi fuzzy subsets of a set  $X$ . We define the following relations and operations:

- (i)  $A \subseteq B$  if and only if  $\mu_{Ai}(x) \leq \mu_{Bi}(x)$  for all  $i$  and for all  $x$  in  $X$ .
- (ii)  $A = B$  if and only if  $\mu_{Ai}(x) = \mu_{Bi}(x)$  for all  $i$  and for all  $x$  in  $X$ .
- (iii)  $A^c = 1-A = \langle 1-\mu_{A1}, 1-\mu_{A2}, 1-\mu_{A3}, \dots, 1-\mu_{An} \rangle$ .
- (iv)  $A \cap B = \{ \langle x, \min\{\mu_{A1}(x), \mu_{B1}(x)\}, \min\{\mu_{A2}(x), \mu_{B2}(x)\}, \dots, \min\{\mu_{An}(x), \mu_{Bn}(x)\} \rangle / x \in X \}$ .
- (v)  $A \cup B = \{ \langle x, \max\{\mu_{A1}(x), \mu_{B1}(x)\}, \max\{\mu_{A2}(x), \mu_{B2}(x)\}, \dots, \max\{\mu_{An}(x), \mu_{Bn}(x)\} \rangle / x \in X \}$ .

**1.4 Definition:** Let  $(R, +, \cdot)$  be a hemiring. A multi fuzzy subset  $A$  of  $R$  is said to be a multi fuzzy subhemiring of  $R$  if it satisfies the following conditions:

- (i)  $\mu_{Ai}(x+y) \geq \min(\mu_{Ai}(x), \mu_{Ai}(y))$  for all  $i$ ,
- (ii)  $\mu_{Ai}(xy) \geq \min(\mu_{Ai}(x), \mu_{Ai}(y))$ , for all  $i$  and for all  $x, y$  in  $R$ .

**Corresponding Author:** <sup>1</sup>Dr B. Anandh\*, <sup>1</sup>Department of Mathematics,  
H. H. The Rajah's College, Pudukkottai, Tamilnadu, India.

**1.5 Definition:** Let  $(R, +, \cdot)$  be a hemiring. A multi fuzzy subset  $A$  of  $R$  is said to be an anti multi fuzzy subhemiring of  $R$  if it satisfies the following conditions:

- (i)  $\mu_{Ai}(x+y) \leq \max(\mu_{Ai}(x), \mu_{Ai}(y))$  for all  $i$ ,
- (ii)  $\mu_{Ai}(xy) \leq \max(\mu_{Ai}(x), \mu_{Ai}(y))$ , for all  $i$  and for all  $x, y$  in  $R$ .

**1.6 Definition:** Let  $A$  and  $B$  be multi fuzzy subsets of sets  $G$  and  $H$ , respectively. The anti-product of  $A$  and  $B$ , denoted by  $A \times B$ , is defined as  $A \times B = \{ \langle (x, y), \mu_{A1 \times B1}(x, y), \mu_{A2 \times B2}(x, y), \dots, \mu_{An \times Bn}(x, y) \rangle / \text{for all } x \text{ in } G \text{ and } y \text{ in } H \}$ , where  $\mu_{A1 \times B1}(x, y) = \max\{\mu_{A1}(x), \mu_{B1}(y)\}$  for all  $i$ .

**1.7 Definition:** Let  $A$  be a multi fuzzy subset in a set  $S$ , the anti-strongest multi fuzzy relation on  $S$ , that is a multi fuzzy relation on  $A$  is  $V$  given by  $\mu_{Vi}(x, y) = \max\{\mu_{Ai}(x), \mu_{Ai}(y)\}$ , for all  $i$  and for all  $x, y$  in  $S$ .

**1.8 Definition:** Let  $(R, +, \cdot)$  and  $(R^1, +, \cdot)$  be any two hemirings. Let  $f: R \rightarrow R^1$  be any function and  $A$  be an anti multi fuzzy subhemiring in  $R$ ,  $V$  be an anti multi fuzzy subhemiring in  $f(R) = R^1$ , defined by  $\mu_{Vi}(y) = \inf_{x \in f^{-1}(y)} \mu_{Ai}(x)$ , for all  $i$  and for all  $x$  in  $R$  and  $y$  in  $R^1$ . Then  $A$  is called a preimage of  $V$  under  $f$  and is denoted by  $f^{-1}(V)$ .

**1.9 Definition:** Let  $A$  be an anti multi fuzzy subhemiring of a hemiring  $(R, +, \cdot)$  and  $a$  in  $R$ . Then the pseudo anti multi fuzzy coset  $(aA)^p$  is defined by  $((a\mu_{Ai})^p)(x) = p(a)\mu_{Ai}(x)$ , for all  $i$  and for every  $x$  in  $R$  and for some  $p$  in  $P$ .

## 2. PROPERTIES OF ANTI MULTI FUZZY SUBHEMIRING OF A HEMIRING

**2.1 Theorem:** Union of any two anti multi fuzzy subhemiring of a hemiring  $R$  is an anti multi fuzzy subhemiring of  $R$ .

**Proof:** Let  $A$  and  $B$  be any two anti multi fuzzy subhemirings of a hemiring  $R$  and  $x$  and  $y$  in  $R$ . Let  $A = \{(x, \mu_{A1}(x), \mu_{A2}(x), \dots, \mu_{An}(x)) / x \in R\}$  and  $B = \{(x, \mu_{B1}(x), \mu_{B2}(x), \dots, \mu_{Bn}(x)) / x \in R\}$  and also let  $C = A \cup B = \{(x, \mu_{C1}(x), \mu_{C2}(x), \dots, \mu_{Cn}(x)) / x \in R\}$ , where  $\max\{\mu_{Ai}(x), \mu_{Bi}(x)\} = \mu_{Ci}(x)$  for all  $i$ . Now,  $\mu_{Ci}(x+y) \leq \max\{\max(\mu_{Ai}(x), \mu_{Ai}(y)), \max(\mu_{Bi}(x), \mu_{Bi}(y))\} \leq \max(\mu_{Ci}(x), \mu_{Ci}(y))$ . Therefore,  $\mu_{Ci}(x+y) \leq \max(\mu_{Ci}(x), \mu_{Ci}(y))$ , for all  $i$  and for all  $x, y$  in  $R$ . And,  $\mu_{Ci}(xy) \leq \max\{\max(\mu_{Ai}(x), \mu_{Ai}(y)), \max(\mu_{Bi}(x), \mu_{Bi}(y))\} \leq \max(\mu_{Ci}(x), \mu_{Ci}(y))$ . Therefore  $\mu_{Ci}(xy) \leq \max(\mu_{Ci}(x), \mu_{Ci}(y))$ , for all  $i$  and for all  $x, y$  in  $R$ . Therefore  $C$  is an anti multi fuzzy subhemiring of a hemiring  $R$ .

**2.2 Theorem:** The union of a family of anti multi fuzzy subhemirings of hemiring  $R$  is an anti multi fuzzy subhemiring of  $R$ .

**Proof:** It is trivial.

**2.3 Theorem:** If  $A$  and  $B$  are any two anti multi fuzzy subhemirings of the hemirings  $R_1$  and  $R_2$  respectively, then anti-product  $A \times B$  is an anti multi fuzzy subhemiring of  $R_1 \times R_2$ .

**Proof:** Let  $A$  and  $B$  be two anti multi fuzzy subhemirings of the hemirings  $R_1$  and  $R_2$  respectively. Let  $x_1$  and  $x_2$  be in  $R_1$ ,  $y_1$  and  $y_2$  be in  $R_2$ . Then  $(x_1, y_1)$  and  $(x_2, y_2)$  are in  $R_1 \times R_2$ . Now,  $\mu_{A1 \times B1}[(x_1, y_1) + (x_2, y_2)] \leq \max\{\max(\mu_{A1}(x_1), \mu_{A1}(x_2)), \max(\mu_{B1}(y_1), \mu_{B1}(y_2))\} \leq \max(\mu_{A1 \times B1}(x_1, y_1), \mu_{A1 \times B1}(x_2, y_2))$ . Therefore  $\mu_{A1 \times B1}[(x_1, y_1) + (x_2, y_2)] \leq \max(\mu_{A1 \times B1}(x_1, y_1), \mu_{A1 \times B1}(x_2, y_2))$  for all  $i$ . Also  $\mu_{A1 \times B1}[(x_1, y_1)(x_2, y_2)] \leq \max\{\max(\mu_{A1}(x_1), \mu_{A1}(x_2)), \max(\mu_{B1}(y_1), \mu_{B1}(y_2))\} \leq \max(\mu_{A1 \times B1}(x_1, y_1), \mu_{A1 \times B1}(x_2, y_2))$ . Therefore  $\mu_{A1 \times B1}[(x_1, y_1)(x_2, y_2)] \leq \max(\mu_{A1 \times B1}(x_1, y_1), \mu_{A1 \times B1}(x_2, y_2))$  for all  $i$ . Hence  $A \times B$  is an anti multi fuzzy subhemiring of hemiring of  $R_1 \times R_2$ .

**2.4 Theorem:** Let  $A$  be a multi fuzzy subset of a hemiring  $R$  and  $V$  be the anti-strongest multi fuzzy relation of  $R$ . Then  $A$  is an anti multi fuzzy subhemiring of  $R$  if and only if  $V$  is an anti multi fuzzy subhemiring of  $R \times R$ .

**Proof:** Suppose that  $A$  is an anti multi fuzzy subhemiring of a hemiring  $R$ . Then for any  $x = (x_1, x_2)$  and  $y = (y_1, y_2)$  are in  $R \times R$ . We have,  $\mu_{Vi}(x+y) = \max\{\mu_{Ai}(x_1+y_1), \mu_{Ai}(x_2+y_2)\} \leq \max\{\max(\mu_{Ai}(x_1), \mu_{Ai}(y_1)), \max(\mu_{Ai}(x_2), \mu_{Ai}(y_2))\} \leq \max(\mu_{Vi}(x_1, x_2), \mu_{Vi}(y_1, y_2)) = \max(\mu_{Vi}(x), \mu_{Vi}(y))$ . Therefore  $\mu_{Vi}(x+y) \leq \max(\mu_{Vi}(x), \mu_{Vi}(y))$ , for all  $i$  and for all  $x, y$  in  $R \times R$ . And  $\mu_{Vi}(xy) = \max\{\mu_{Ai}(x_1y_1), \mu_{Ai}(x_2y_2)\} \leq \max\{\max(\mu_{Ai}(x_1), \mu_{Ai}(y_1)), \max(\mu_{Ai}(x_2), \mu_{Ai}(y_2))\} \leq \max(\mu_{Vi}(x_1, x_2), \mu_{Vi}(y_1, y_2)) = \max(\mu_{Vi}(x), \mu_{Vi}(y))$ . Therefore  $\mu_{Vi}(xy) \leq \max(\mu_{Vi}(x), \mu_{Vi}(y))$ , for all  $i$  and for all  $x, y$  in  $R \times R$ . This proves that  $V$  is an anti multi fuzzy subhemiring of  $R \times R$ . Conversely assume that  $V$  is an anti multi fuzzy subhemiring of  $R \times R$ , then for any  $x = (x_1, x_2)$  and  $y = (y_1, y_2)$  are in  $R \times R$ , we have  $\max\{\mu_{Ai}(x_1+y_1), \mu_{Ai}(x_2+y_2)\} = \mu_{Vi}(x+y) \leq \max(\mu_{Vi}(x), \mu_{Vi}(y)) = \max(\mu_{Vi}(x_1, x_2), \mu_{Vi}(y_1, y_2)) = \max(\max\{\mu_{Ai}(x_1), \mu_{Ai}(x_2)\}, \max\{\mu_{Ai}(y_1), \mu_{Ai}(y_2)\})$ . If  $x_2 = 0, y_2 = 0$ , we get,  $\mu_{Ai}(x_1+y_1) \leq \max(\mu_{Ai}(x_1), \mu_{Ai}(y_1))$ , for all  $i$  and for all  $x_1, y_1$  in  $R$ . And  $\max\{\mu_{Ai}(x_1y_1), \mu_{Ai}(x_2y_2)\} = \mu_{Vi}(xy) \leq \max(\mu_{Vi}(x), \mu_{Vi}(y)) = \max(\mu_{Vi}(x_1, x_2), \mu_{Vi}(y_1, y_2)) = \max(\max\{\mu_{Ai}(x_1), \mu_{Ai}(x_2)\}, \max\{\mu_{Ai}(y_1), \mu_{Ai}(y_2)\})$ . If  $x_2 = 0, y_2 = 0$ , we get  $\mu_{Ai}(x_1y_1) \leq \max(\mu_{Ai}(x_1), \mu_{Ai}(y_1))$ , for all  $i$  and for all  $x_1, y_1$  in  $R$ . Therefore  $A$  is an anti multi fuzzy subhemiring of  $R$ .

**2.5 Theorem:** A is an anti multi fuzzy subhemiring of a hemiring  $(R, +, \cdot)$  if and only if  $\mu_{A_i}(x+y) \leq \max(\mu_{A_i}(x), \mu_{A_i}(y))$ ,  $\mu_{A_i}(xy) \leq \max(\mu_{A_i}(x), \mu_{A_i}(y))$ , for all  $i$  and for all  $x, y$  in  $R$ .

**Proof:** It is trivial.

**2.6 Theorem:** If A is an anti multi fuzzy subhemiring of a hemiring  $(R, +, \cdot)$ , then  $H = \{x / x \in R: \mu_{A_i}(x) = 0, \text{ for all } i\}$  is either empty or is a subhemiring of  $R$ .

**Proof:** It is trivial.

**2.7 Theorem:** Let A be an anti multi fuzzy subhemiring of a hemiring  $(R, +, \cdot)$ . If  $\mu_{A_i}(x+y) = 1$ , then either  $\mu_{A_i}(x) = 1$  or  $\mu_{A_i}(y) = 1$ , for all  $i$  and for all  $x$  and  $y$  in  $R$ .

**Proof:** It is trivial.

**2.8 Theorem:** Let A be an anti multi fuzzy subhemiring of a hemiring  $(R, +, \cdot)$ , then the pseudo anti multi fuzzy coset  $(aA)^p$  is an anti multi fuzzy subhemiring of a hemiring  $R$ , for every  $a$  in  $R$ .

**Proof:** Let A be an anti multi fuzzy subhemiring of a hemiring  $R$ . For every  $x$  and  $y$  in  $R$ , we have,  $((a\mu_{A_i})^p)(x+y) \leq p(a) \max(\mu_{A_i}(x), \mu_{A_i}(y)) \leq \max(p(a)\mu_{A_i}(x), p(a)\mu_{A_i}(y)) = \max(((a\mu_{A_i})^p)(x), ((a\mu_{A_i})^p)(y))$ . Therefore,  $((a\mu_{A_i})^p)(x+y) \leq \max(((a\mu_{A_i})^p)(x), ((a\mu_{A_i})^p)(y))$  for all  $i$ . Now,  $((a\mu_{A_i})^p)(xy) \leq p(a) \max(\mu_{A_i}(x), \mu_{A_i}(y)) \leq \max(p(a)\mu_{A_i}(x), p(a)\mu_{A_i}(y)) = \max(((a\mu_{A_i})^p)(x), ((a\mu_{A_i})^p)(y))$ . Therefore,  $((a\mu_{A_i})^p)(xy) \leq \max(((a\mu_{A_i})^p)(x), ((a\mu_{A_i})^p)(y))$  for all  $i$ . Hence  $(aA)^p$  is an anti multi fuzzy subhemiring of a hemiring  $R$ .

**2.9 Theorem:** Let  $(R, +, \cdot)$  and  $(R^1, +, \cdot)$  be any two hemirings. The homomorphic image of an anti multi fuzzy subhemiring of  $R$  is an anti multi fuzzy subhemiring of  $R^1$ .

**Proof:** Let  $f: R \rightarrow R^1$  be a homomorphism. Then,  $f(x+y) = f(x) + f(y)$  and  $f(xy) = f(x)f(y)$ , for all  $x$  and  $y$  in  $R$ . Let  $V = f(A)$ , where A is an anti multi fuzzy subhemiring of  $R$ . Now, for  $f(x), f(y)$  in  $R^1$ ,  $\mu_{V_i}(f(x)+f(y)) \leq \mu_{A_i}(x+y) \leq \max(\mu_{A_i}(x), \mu_{A_i}(y))$ , which implies that  $\mu_{V_i}(f(x)+f(y)) \leq \max(\mu_{V_i}(f(x)), \mu_{V_i}(f(y)))$  for all  $i$ . Again,  $\mu_{V_i}(f(x)f(y)) \leq \mu_{A_i}(xy) \leq \max(\mu_{A_i}(x), \mu_{A_i}(y))$ , which implies that  $\mu_{V_i}(f(x)f(y)) \leq \max(\mu_{V_i}(f(x)), \mu_{V_i}(f(y)))$  for all  $i$ . Hence V is an anti multi fuzzy subhemiring of  $R^1$ .

**2.10 Theorem:** Let  $(R, +, \cdot)$  and  $(R^1, +, \cdot)$  be any two hemirings. The homomorphic preimage of an anti multi fuzzy subhemiring of  $R^1$  is an anti multi fuzzy subhemiring of  $R$ .

**Proof:** Let  $V = f(A)$ , where V is an anti multi fuzzy subhemiring of  $R^1$ . Let  $x$  and  $y$  in  $R$ . Then,  $\mu_{A_i}(x+y) = \mu_{V_i}(f(x+y)) \leq \max(\mu_{V_i}(f(x)), \mu_{V_i}(f(y))) = \max(\mu_{A_i}(x), \mu_{A_i}(y))$ , which implies that  $\mu_{A_i}(x+y) \leq \max(\mu_{A_i}(x), \mu_{A_i}(y))$  for all  $i$ . Again,  $\mu_{A_i}(xy) = \mu_{V_i}(f(xy)) \leq \max(\mu_{V_i}(f(x)), \mu_{V_i}(f(y))) = \max(\mu_{A_i}(x), \mu_{A_i}(y))$  which implies that  $\mu_{A_i}(xy) \leq \max(\mu_{A_i}(x), \mu_{A_i}(y))$  for all  $i$ . Hence A is an anti multi fuzzy subhemiring of  $R$ .

**2.11 Theorem:** Let  $(R, +, \cdot)$  and  $(R^1, +, \cdot)$  be any two hemirings. The anti-homomorphic image of an anti multi fuzzy subhemiring of  $R$  is an anti multi fuzzy subhemiring of  $R^1$ .

**Proof:** Let  $f: R \rightarrow R^1$  be an anti-homomorphism. Then,  $f(x+y) = f(y)+f(x)$  and  $f(xy) = f(y)f(x)$ , for all  $x$  and  $y$  in  $R$ . Let  $V = f(A)$ , where A is an anti multi fuzzy subhemiring of  $R$ . Now, for  $f(x), f(y)$  in  $R^1$ ,  $\mu_{V_i}(f(x)+f(y)) \leq \mu_{A_i}(y+x) \leq \max(\mu_{A_i}(y), \mu_{A_i}(x)) = \max(\mu_{A_i}(x), \mu_{A_i}(y))$  which implies that  $\mu_{V_i}(f(x)+f(y)) \leq \max(\mu_{V_i}(f(x)), \mu_{V_i}(f(y)))$  for all  $i$ . Again,  $\mu_{V_i}(f(x)f(y)) \leq \mu_{A_i}(yx) \leq \max(\mu_{A_i}(y), \mu_{A_i}(x)) = \max(\mu_{A_i}(x), \mu_{A_i}(y))$  which implies that  $\mu_{V_i}(f(x)f(y)) \leq \max(\mu_{V_i}(f(x)), \mu_{V_i}(f(y)))$  for all  $i$ . Hence V is an anti multi fuzzy subhemiring of  $R^1$ .

**2.12 Theorem:** Let  $(R, +, \cdot)$  and  $(R^1, +, \cdot)$  be any two hemirings. The anti-homomorphic preimage of an anti multi fuzzy subhemiring of  $R^1$  is an anti multi fuzzy subhemiring of  $R$ .

**Proof:** Let  $V = f(A)$ , where V is an anti multi fuzzy subhemiring of  $R^1$ . Let  $x$  and  $y$  in  $R$ . Then,  $\mu_{A_i}(x+y) = \mu_{V_i}(f(x+y)) \leq \max(\mu_{V_i}(f(y)), \mu_{V_i}(f(x))) = \max(\mu_{A_i}(x), \mu_{A_i}(y))$  which implies that  $\mu_{A_i}(x+y) \leq \max(\mu_{A_i}(x), \mu_{A_i}(y))$  for all  $i$ . Again,  $\mu_{A_i}(xy) = \mu_{V_i}(f(xy)) \leq \max(\mu_{V_i}(f(y)), \mu_{V_i}(f(x))) = \max(\mu_{A_i}(x), \mu_{A_i}(y))$  which implies that  $\mu_{A_i}(xy) \leq \max(\mu_{A_i}(x), \mu_{A_i}(y))$  for all  $i$ . Hence A is an anti multi fuzzy subhemiring of  $R$ .

**2.13 Theorem:** Let A be an anti multi fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then  $A \circ f$  is an anti multi fuzzy subhemiring of  $R$ .

**Proof:** Let  $x$  and  $y$  in  $R$ . Then we have,  $(\mu_{A_i} \circ f)(x+y) = \mu_{A_i}(f(x) + f(y)) \leq \max(\mu_{A_i}(f(x)), \mu_{A_i}(f(y))) \leq \max((\mu_{A_i} \circ f)(x), (\mu_{A_i} \circ f)(y))$  which implies that  $(\mu_{A_i} \circ f)(x+y) \leq \max((\mu_{A_i} \circ f)(x), (\mu_{A_i} \circ f)(y))$  for all  $i$ . And  $(\mu_{A_i} \circ f)(xy) = \mu_{A_i}(f(x)f(y)) \leq \max(\mu_{A_i}(f(x)), \mu_{A_i}(f(y))) \leq \max((\mu_{A_i} \circ f)(x), (\mu_{A_i} \circ f)(y))$  which implies that  $(\mu_{A_i} \circ f)(xy) \leq \max((\mu_{A_i} \circ f)(x), (\mu_{A_i} \circ f)(y))$  for all  $i$ . Therefore  $A \circ f$  is an anti multi fuzzy subhemiring of a hemiring  $R$ .

**2.14 Theorem:** Let  $A$  be an anti multi fuzzy subhemiring of a hemiring  $H$  and  $f$  is an anti-isomorphism from a hemiring  $R$  onto  $H$ . Then  $A \circ f$  is an anti multi fuzzy subhemiring of  $R$ .

**Proof:** Let  $x$  and  $y$  in  $R$ . Then we have,  $(\mu_{A_i} \circ f)(x+y) = \mu_{A_i}(f(y)+f(x)) \leq \max(\mu_{A_i}(f(x)), \mu_{A_i}(f(y))) \leq \max((\mu_{A_i} \circ f)(x), (\mu_{A_i} \circ f)(y))$ , which implies that  $(\mu_{A_i} \circ f)(x+y) \leq \max((\mu_{A_i} \circ f)(x), (\mu_{A_i} \circ f)(y))$  for all  $i$ . And  $(\mu_{A_i} \circ f)(xy) = \mu_{A_i}(f(y)f(x)) \leq \max(\mu_{A_i}(f(x)), \mu_{A_i}(f(y))) \leq \max((\mu_{A_i} \circ f)(x), (\mu_{A_i} \circ f)(y))$ , which implies that  $(\mu_{A_i} \circ f)(xy) \leq \max((\mu_{A_i} \circ f)(x), (\mu_{A_i} \circ f)(y))$  for all  $i$ . Therefore  $A \circ f$  is an anti multi fuzzy subhemiring of a hemiring  $R$ .

## REFERENCES

1. Akram. M and K.H.Dar, On anti fuzzy left h- ideals in hemirings, International Mathematical Forum, 2(46); 2295 – 2304, 2007.
2. Anthony.J.M. and H Sherwood, fuzzy groups Redefined, Journal of mathematical analysis and applications, 69; 124 -130, 1979.
3. Asok Kumer Ray, On product of fuzzy subgroups, fuzzy sets and systems, 105; 181-183, 1999.
4. Biswas. R, Fuzzy subgroups and anti-fuzzy subgroups, fuzzy sets and systems, 35; 121-124, 1990.
5. Choudhury.F.P. and Chakraborty.A.B. and Khare.S.S., A note on fuzzy subgroups and fuzzy homomorphism, Journal of mathematical analysis and applications, 131, 537 -553, 1988.
6. Lee, K.M.(2004), Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets and bipolar valued fuzzy sets. J. Multi fuzzy Logic Intelligent Systems, 14 (2): 125-129.
7. Mustafa Akgul, some properties of fuzzy groups, Journal of mathematical analysis and applications, 133, 93-100, 1988.
8. Palaniappan. N & K. Arjunan, The homomorphism, anti homomorphism of a fuzzy and an anti-fuzzy ideals of a ring, Varahmihir Journal of Mathematical Sciences, 6(1); 181-006, 2008.
9. Palaniappan. N & K.Arjunan, Some properties of intuitionistic fuzzy subgroups, Acta Ciencia Indica, Vol.XXXIII (2); 321-328, 2007.
10. Rajesh Kumar, fuzzy Algebra, University of Delhi Publication Division, Volume 1, 1993.
11. Vasantha kandasamy.W.B, Smarandache fuzzy algebra, American research press, Rehoboth, 2003.
12. Xueling MA. Jianming ZHAN, On fuzzy h - Ideals of hemirings, Journal of Systems science & Complexity, 20; 470 – 478, 2007.
13. Zadeh. L. A, Fuzzy sets, Information and control, 8; 338-353, 1965.

**Source of support: Nil, Conflict of interest: None Declared**

**[Copy right © 2016. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]**