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DEGREE EQUITABLE CONNECTED CO-TOTAL DOMINATION NUMBER OF GRAPH

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ABSTRACT

Let $G = (V, E)$ be any connected graph. A subset D of V is called a connected dominating set of G if every vertex $v \in V - D$ is adjacent to some vertex in D and $< D >$ is connected. A connected dominating set D is said to be degree equitable connected co-total dominating set if for every vertex $u \in V - D$ there exist a vertex $v \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$ provided $< V - D >$ contains no isolated vertices. The minimum cardinality of degree equitable connected co-total dominating set is called degree equitable connected co-total domination number and it is denoted by $\gamma_{cc}^{e}(G)$. In this paper, we have obtained the $\gamma_{cc}^{e}(G)$ of some standard class of graphs and further established some bounds for $\gamma_{cc}^{e}(G)$.

Keywords: *Domination number; connected domination number; degree equitable connected co-total domination number.*

2010 Mathematics Subject Classification: *05C69.*

1. INTRODUCTION

All graphs considered here are simple, finite, connected and nontrivial. Let $G = (V(G), E(G))$ be a graph, where *V(G)* is the vertex set and *E(G)* be the edge set of G. The vertex $v \in V$ is called a *pendant vertex*, if $deg_G(v) = 1$ and an *isolated vertex* $if\ deg_G(v) = 0$, where $deg_G(x) is\ the\ degree\ of\ a\ vertex\ x \in V(G)$. A vertex which is adjacent to a pendant vertex is called a *support vertex*. We denote $\delta(G)(\Delta(G))$ as the $minimum(maximum) degree\ and$ $p = |V(G)|, q = |E(G)|$ the *order* and *size* of *G* respectively. A *spanning subgraph* is a subgraph containing all the vertices of *G*. A shortest $u - v\ path$ is often called a *geodesic*. The *diameter* $diam(G)$ of a connected graph G is the length of any longest geodesic. The *neighborhood* of a vertex u in *V* is the set *N(u)* consisting of all vertices *v* which are adjacent with *u*.

By a graph, we mean a simple and connected. Any undefined terms in this paper may be found in [4].

A set *D* of vertices in a graph G is a *dominating set* if every vertex not in *D* is adjacent to at least one vertex in *D*. The *domination number* $\gamma(G)$ of G is the minimum cardinality of minimal dominating set of *G* [5].

A dominating set *D* is said to be a *connected dominating* set if <*D*> is connected. The *connected domination number* γ_c of G is the minimum cardinality of a minimal connected dominating set of *G* [6].

A subset D of V is called an *equitable dominating set* if for every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$, where $\deg(u)\ and\ \deg(v)$ denotes the degree of a vertex *u* and *v* respectively. The minimum cardinality of such a vertex *u* and *v* respectively. The minimum cardinality of such a dominating set is denoted by γ^e and is called the *equitable domination number* [7].

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Analogously, we define degree equitable connected co-total domination as follows.

Definition 1: A connected dominating set D is said to be *degree equitable connected co-total dominating set* if for every vertex $u \in V - D$ there exist a vertex $v \in D$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$ provided $< V - D >$ contains no isolated vertices.

The minimum cardinality of degree equitable connected co-total dominating set is called *degree equitable connected co-total domination number* of a graph and it is denoted by $\gamma_{cc}^{e}(G)$.

Example:

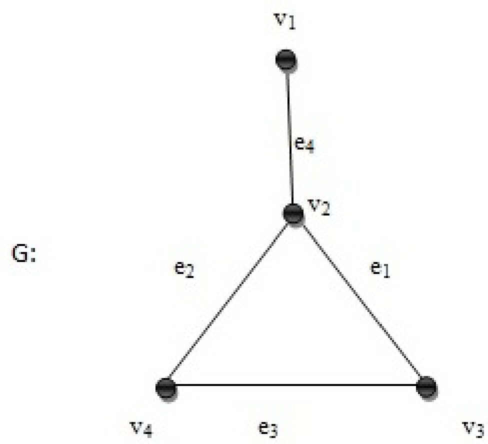

Figure-1

In the above Figure 1, we can see that the degree equitable connected cototal dominating set D is given by

D= {v_1,v_2} V-D={v_3,v_4}

$|deg(u) - deg(v)| \leq 1$

$\gamma_{cc}^{e}(G)$ = *2*

2. PRELIMINARY RESULTS

We need the following auxiliary results which will be helpful in proving our results.

OBSERVATIONS

1) $\gamma_c(G) \leq \gamma_{ccl}(G)$ *and* $\gamma_{cc}^{e}(G) \geq \gamma_{ccl}(G)$.
2) Let D be a $\gamma_{cc}^{e}(G) - set$, then $< D >$ is a tree.
3) Every connected dominating set contains no pendant vertices.
4) Every $\gamma_{cc}^{e} - set$ contains all pendant vertices of G

3. RESULTS

Firstly, we obtain the degree equitable connected co-total domination number of some standard class of graphs. Which are listed in the following propositions.

Proposition 3.1:

i) $\gamma_{cc}^{e}(K_n) = 1$

ii) $\gamma_{cc}^{e}(P_n) = n$

iii) $\gamma_{cc}^{e}(C_n) = n - 2$

iv) $\gamma_{cc}^{e}(K_{1,n-1}) = n$

v) $\gamma_{cc}^{e}(K_{m,n}) = \begin{cases} 2 & \text{if } |m-n| \leq 1 \\ m+n & \text{otherwise} \end{cases}$

Proof:

i). Let G be a complete graph, $G = K_n, n \geq 3$

Let $D = \{v_i\}$ for some $i \in V(G)$ then $< V - D >$ contains no isolated vertices. Further $< D >$ is connected. Hence we need to show that for every $u \in V - D$ and $v \in D$

$$|\deg(u) - \deg(v)| \leq 1$$

Since G is (n-1)-regular graph. Therefore $|\deg(u) - \deg(v)| = 0$.

Hence D is the minimal degree equitable connected co-total dominating set of G.

Hence $\gamma_{cc}^e(G) = |D| = |\{v\}| = 1$. Therefore, $\gamma_{cc}^e(K_n) = 1$.

ii). Let G be a path graph. That is $G = P_n \; for\; some\; n \geq 2$.

Let D be any connected dominating set of G. Since any connected dominating set of a tree containing all support vertices. Therefore D does not contain two pendant vertices of P_n. Further every cototal dominating set contains no isolated vertices in $< V - D >$.

Let $D' = D \;\cup \{u, v\}\, \{where\; u, v\; \in V(G) are\; pendant\; vertices\; in\; G\}$ is the connected cototal dominating set of G.

Since $< V - D > = \emptyset$. Therefore D' is also a degree equitable connected co-total dominating set of *G*. Hence,

$$\begin{aligned} \gamma_{cc}^e(G) &= |D'| \\ &= |D \cup \{u, v\} \\ &= n - 2 + 2 \;= n \end{aligned}$$

Therefore, $\gamma_{cc}^e(P_n) = n$.

iii). Let G be a cycle graph and $G = C_n$

Since *G* is *2-regular* graph. Therefore every minimal connected co-total dominating set will act as a degree equitable connected co-total dominating set of *G* by (i)

Since $\gamma_{cc}^e(G) = n - 2$. Therefore $\gamma_{cc}^e(C_n) = n - 2$.

iv). *Let* $G = K_{1,n-1}$, for some $n \geq 2, if\; n = 2,3\; then\; G = P_2\; and\; P_3$, then by (ii) results holds good.

Now for $n \geq 4$, we can observe that $deg(v) = n - 1$. Where *v* is the central vertex of a star and remaining all other vertices are of degree one. Therefore, $|deg(v) - deg(v_i)| > 1$. Hence $D = \{v\} \cup \{u_i\}_{i=1}^n$ will act as a minimum degree equitable connected co-total dominating set of *G*.

$$\begin{aligned} \text{Hence } \gamma_{cc}^e(G) &= |D| \\ &= \{v\} \cup \{u_i\}_{i=1}^n \\ &= 1 + n - 1 \end{aligned}$$

Therefore $\gamma_{cc}^e\left(K_{1,\; n-1}\right) = n$

v). Let $G = K_{m,n}$, $2 \leq m \leq n$. we consider the following cases.

Case-(i): $If \mid m - n \mid \leq 1$.

Since $K_{m\,,n}$ is a *bi-regular* graph, if $\mid m - n \mid \leq 1$, the every minimum connected co-total dominating set act as degree equitable connected co-total dominating set.

By (ii), we have

$$\gamma_{cc}^e(G) = 2$$

Therefore $\gamma_{cc}^e(K_{m,n}) = 2$

Case-(ii): $|m - n| > 1$

Suppose $\mid m - n \mid > 1$ and we know that $K_{m,n}$ is *bi-regular* graph therefore every vertex of $K_{m,n}$ belongs to the degree equitable connected co-total dominating set of *G*.
i.e $all\; v \in V(K_{m,n}) \in D$, Since $V\left(K_{m,n}\right) = m + n$

Therefore $m + n = |D|$

Hence $\gamma_{cc}^{e}(K_{m,n}) = D = m + n$
$\gamma_{cc}^{e}(K_{m,n}) = m + n$
This completes the proof.

Theorem 3.1: For any graph *G*, $1 \leq \gamma_{cc}^{e}(G) \leq n$ equality lower bound attains if and only if $\Delta(G) \geq 2$ and equality of upper bound attains if and only if *G* contains a vertex $v \in D$. Such that $|deg(v) - deg(v_i)| > 1$ and $V - D = \emptyset$.

Proof: We first consider the equality of lower bond.

The lower bound follows from definition of γ_{cc}^{e}-set. For equality, Suppose $\Delta(G) = n - 1$and $\Delta(G) \geq 2$ then we can observe that for some $u \in D$ there exist a vertex $v \in V - D$ such that $|degu - degv| \leq 1$. Further $< D >$ is connected.

Therefore *D* is a γ_{cc}^{e}-set. Hence $\gamma_{cc}^{e}(G) = 1$.

Conversely, suppose $\gamma_{cc}^{e}(G) = 1$ and G does not satisfy the hypothesis of the theorem, then for every $u \in D$ there exist a vertex $v \in V - D$, such that $|deg(v) - deg(v_i)| > 1$. Hence $D = \{u, v\}$ will form the minimal degree equitable connected co-total dominating set of G. This is a contradiction to our assumption.

Now, for upper bound, by Proposition (i) it is obvious that, $\gamma_{cc}^{e}(G) \leq n$.

Now for equality of the upper bound. Suppose *G* satisfies the hypothesis of the theorem then the equality follows directly.

Conversely, suppose $\gamma_{cc}^{e}(G) = n$ and *G* does not satisfy the hypothesis of the theorem, then there exist a vertex $v \in V - D$ such that there exist a vertex $u \in D$ and $|\, deg(u) - deg(v) \,| \leq 1.$ Hence $|V - D| = 1$ which is the contradiction to our assumption.

Theorem 3.2: For any regular graph, $\gamma_{cc}^{e}(G) = \gamma_{cc}(G)$.

Proof: suppose *G* is the regular graph. Then every vertex as the same degree *r*. Let *D* be a minimum connected co-total dominating set of *G*, then $|D| = \gamma_{ccl}(G)$. Let $u \in V - D$ then as *D* is a connected co-total dominating set, there exist a vertex $v \in D$ and $uv \in E(G)$. Also $degu = degv = r.\ Therefore\ |\deg(u) - \deg(v)| = 0 < 1$. Hence *D* is degree equitable connected co-total dominating set of *G*, so that $\gamma_{cc}^{e}(G) \leq |D| \leq \gamma_{ccl}(G)$. But $\gamma_{ccl}(G) \leq \gamma_{cc}^{e}(G)$.
Hence $\gamma_{ccl}(G) \leq \gamma_{cc}^{e}(G)$.

Theorem 3.3: For any graph *G*, $\gamma(G) = \gamma_{cc}^{e}(G)$ $equality\ holds\ if\ G = K_n\ where\ n \geq 3$.

Proof: Let *G* be any connected graph of order *n*. Since for any connected graph *G*, $\gamma(G) \leq \gamma_c(G)$ and $\gamma_c(G) \leq \gamma_{ccl}(G)$ by Observation (A) we know that $\gamma_{ccl}(G) \leq \gamma_{cc}^{e}(G)$

Hence $\gamma(G) = \gamma_{cc}^{e}(G)$ for equality, suppose $G = K_n, n \leq 3$ then $\gamma(G) = 1$ and by Proposition (i) we know that $\gamma_{cc}^{e}(G) = 1$.

Hence the result.

Theorem 3.4: For any tree *T*, $\gamma_c(T) < \gamma_{cc}^{e}(T)$

Proof: Let *T* be any nontrivial tree. Let *D* be a connected dominating set of *G*. By Observation (3), every connected dominating set contains an pendant vertices of *G*. Further, by definition of $\gamma_{cc}^{e}(G)$ - set, $< V - D >$ contains no isolated vertices. Therefore $\gamma_{cc}^{e}(G) \geq \gamma_c(G) + pendant\ vertices$.

Hence $\gamma_c(T) < \gamma_{cc}^{e}(T)$.

Theorem 3.5: For any graph *G*, $\gamma_{cc}^{e}(G) = 1\ if\ and\ only\ if\ \delta(G) \geq 2\ and\ \gamma_c = 1$.

Proof: Let *G* be any graph of order at least *3*. We consider as a following cases.

Case-I: Suppose $\delta(G) = 1\ and\ \gamma_{cc}^{e}(G) = 1$

Let v be a vertex of minimum degree in G, i.e $deg(v) = \delta(G)$. By Observation (1) $v \in D$ where D is the minimal degree equitable connected cototal dominating set of G. Further, there exist a vertex $u \in V - D$ such that which is not dominated by v. Hence dominate u we need at least one in open neighborhood of u, say $w \in N(u)$. Thus $w \in D$ which implies that $|D| \geq 2$ which is a contradiction to our assumption.

Case-II: Suppose $\delta(G) = 2\ and\ \gamma_{cc}^{e}(G) = 1$.

Let $u \in D$ then every vertex in $V - D$. $|\deg(u) - \deg(v)| \leq 1\ and < V - D >$ contains no isolated vertices. Further the subgraph induced by D is connected. Hence D is a degree equitable connected cototal dominating set of G.

Similar argument holds good for a graph with $\delta(G) \geq 2$, converse is obvious.

Theorem 3.6: For any graph G, $\gamma_{cc}^{e}(G) = 2\ if\ and\ only\ if\ \gamma_{ccl}(G) = 2$ and for every $v \in D$ there exist $|\ deg(u) - deg(v)| \leq 1$

Proof: Let G be a any connected graph, such that there exist at least two adjacent vertices of degree $n - 2\ and\ < V - D >$ as no isolated vertices, where D is minimal connected cototal dominating set of G. If for every vertex $v \in D$ there exist a vertex $u \in V - D$. Such that $|\ deg(u) - deg(v)| \leq 1$ then D will be a degree equitable connected cototal dominating set of G, therefore by proposition A, $\gamma_{cc}^{e}(G) = 2$.
Converse is easy to follow.

Theorem 3.7: For any graph G, $\gamma_{cc}^{e}(G) \leq \gamma_{cc}^{e}(H)$ where H is any spanning subgraph of G.

Proof: Let G be any connected graph of order n and size m. Let D be any $\gamma_{cc}^{e} - set$ of G and let H be any spanning subgraph of G. Let D' be $\gamma_{cc}^{e} - set$ of H.

Suppose H is any tree T then $|D'| = n$ and note that $|D| < n$ which implies$|D| \leq D'$. Hence $\gamma_{cc}^{e}(G) \leq \gamma_{cc}^{e}(H)$

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