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REMARKS ON STRONGLY (1, 2)*-πgα - CLOSED MAPPINGS

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ABSTRACT

In this paper we investigate the properties of strongly (1, 2)-πgα-closed maps in bitopological spaces.*

Keywords: *(1, 2)*-πgα-closed, (1, 2)*-πgα-continuous, (1, 2)*-πgα-irresolute.*

1. INTRODUCTION

The concept of generalized closed sets was first initiated by Levine [5] in 1970. Malghan [6] introduced generalized closed maps. Lellis Thivagar *et al* [7] studied stronger form of $\alpha\hat{g}$ closed mappings in topological spaces. I. Arockiarani and K. Mohana [2, 3] introduced *(1, 2)*-πgα-continuous* functions and *(1, 2)*-πg*α-closed maps in bitopological spaces. *In this paper we study the properties of strongly (1, 2)*-πgα-closed maps in bitopological spaces.*

2. PRELIMINARIES

Throughout this paper by a space X we mean it is a bitopological space. We recall the following definitions which are useful in the sequel.

Definition: 2.1 [4] A subset S of a bitopological space X is said to be $\tau_{1,2}$-open if S=A$\cup$B where $A\in\tau_1$ and $B\in\tau_2$.A subset S of X is said to be (i) $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$-open. (ii) $\tau_{1,2}$-clopen if S is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed. Let S be a subset of the bitopological space X. Then the $\tau_{1,2}$-interior of S denoted by $\tau_{1,2}$-int(S) is defined by $\cup${G: G$\subseteq$S and G is $\tau_{1,2}$- open} and the $\tau_{1,2}$-closure of S denoted by $\tau_{1,2}$-cl(S) is defined by $\cap${F: S$\subseteq$F and F is $\tau_{1,2}$- closed }. $\tau_{1,2}$-open sets need not form a topology.

Definition: 2.2 A subset A of a bitopological space X is called

(i) $(1,2)^*$-regular open [4] if $A=\tau_{1,2}-\text{int}(\tau_{1,2}-cl(A))$.

(ii) $(1,2)^*$-α -open [4] if $A\subseteq\tau_{1,2}-\text{int}(\tau_{1,2}-cl(\tau_{1,2}-\text{int}(A)))$.

(iii) $(1,2)^*$- πgα - closed [1] if $(1,2)^*-\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is $\tau_{1,2}$-π- open

The complement of the sets mentioned from (i) and (ii) are called their respective closed sets and the complement of the set mentioned in (iii) is called the respective open set.

The class of all (1, 2)*-α-open (resp. (1, 2)*- πgα-open) subsets of a space X is denoted by (1, 2)*-αO(X) ((1, 2)*-πgαO(X)).

Definition: 2.5 Let S be a subset of the bitopological space X. Then

(i) The (1, 2)*-α-interior of S denoted by (1, 2)*-α-int(S) is defined by $\cup$ {G: G$\subseteq$S and G is (1, 2)*-α-open}

(ii) The (1, 2)*-α-closure of S denoted by (1, 2)*-α-cl(S) is defined by $\cap$ {F: S$\subseteq$F and F is (1, 2)*-α-closed}.

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Definition: 2.6 [2] A function $f : X \to Y$ is called

(i) $(1,2)^* - \pi g\alpha$ -irresolute if $f^{-1}(V)$ is $(1,2)^* - \pi g\alpha$ closed in X, for every $(1,2)^* - \pi g\alpha$ closed set V of Y.

(ii) $(1,2)^* - \pi g\alpha$ -continuous if $f^{-1}(V)$ is $(1,2)^* - \pi g\alpha$ closed in X, for every $\sigma_{1,2}$ -closed set V of Y.

Definition 2.7 [3] A map $f : X \to Y$ is called

(i) $(1,2)^* - \pi g\alpha$ -closed if f(V) is $(1,2)^* - \pi g\alpha$ closed in Y, for every $\tau_{1,2}$ -closed set V of X.

(ii) almost $(1,2)^* - \pi g\alpha$ -closed if f(V) is $(1,2)^* - \pi g\alpha$ closed in Y, for every (1,2)*-regular closed set V of X.

(iii) (1, 2)*-pre-α-closed if f(V) is (1, 2)*-α-closed set in Y, for every (1, 2)*-α-closed set V of X.

(iv) (1, 2)*-π-irresolute if $f^{-1}(V)$ is $\tau_{1,2}$ -π-closed in X, for every $\sigma_{1,2}$–π- closed set V of Y.

3. STRONGLY (1, 2)*-πgα –CLOSED MAPS

Definition : 3.1 A map $f : X \to Y$ is called strongly (1,2)*-πgα-closed or (1, 2)*-M-πgα –closed [3] (resp. strongly (1,2)*-πgα-open or (1,2)*-M-πgα-open) if the image of every (1,2)*-πgα-closed (resp. (1,2)*-πgα-open) set in X is (1,2)*-πgα-closed (resp. (1,2)*-πgα-open)set in Y.

Theorem: 3.2 If $f : X \to Y$ is (1, 2)*-πgα-closed map and $g : Y \to Z$ is strongly (1,2)*-πgα-closed map then $g \circ f : X \to Z$ is (1,2)*-πgα-closed map.

Proof: The proof is obvious.

Theorem: 3.3 If $f : X \to Y$ is (1, 2)*-closed map and $g : Y \to Z$ is strongly (1, 2)*-πgα-closed map then $g \circ f : X \to Z$ is (1, 2)*-πgα-closed map.

Proof: Straight forward.

Remark: 3.4 If $f : X \to Y$ is strongly (1, 2)*-πgα-closed map and $g : Y \to Z$ is (1, 2)*-closed map then also the composite map $g \circ f$ may not be strongly (1, 2)*-πgα-closed map. The following example shows this result.

Example: 3.5 Let X = {a, b, c}=Y, Z = {a, b, c, d}, τ_1 = {ϕ, X, {a}}, τ_2 = {ϕ, X, {b}}, σ_1={ϕ, Y, {b}}, σ_2={ϕ, Y, {b, c}}, η_1 ={ϕ, Z, {a}, {c, d}, {a, c, d}, {d}, {a, d}}, η_2 ={ϕ, Z, {a, c}} and $f : X \to Y$ be an identity map. $g : Y \to Z$ be a map defined by f{a}={a}, f{b}={b}, f{c }=f{d} = {c}. Then f is strongly (1, 2)*-πgα-closed map and g is (1, 2)*-closed map, but g o f is not strongly (1, 2)*-πgα-closed map.

Proposition: 3.6 Every strongly (1, 2)*-πgα-closed map is almost (1, 2)*-πgα-closed map but not conversely.

Example: 3.7 Let X = {a, b, c, d}=Y, τ_1 = {ϕ, X, {a, b, d}}, τ_2 = {ϕ, X, {c}, {b, d}, {b, c, d}}, σ_1={ϕ, Y, {d}, {a, d}}, σ_2={ϕ, Y, {a}, {c, d}, {a, c, d}} and $f : X \to Y$ be an identity maps. Then f is almost (1, 2)*-πgα-closed map but not strongly (1, 2)*-πgα-closed map.

Theorem: 3.8 The composite mapping of two strongly (1, 2)*-πgα-closed maps is strongly (1, 2)*-πgα-closed map.

Proof: The proof follows from definitions.

Remark: 3. 9 The concept of strongly (1, 2)*-πgα-closed map is independent from the concept of (1,2)*-πgα-irresolute map as shown in the following example.

Example: 3.10 Let X = {a, b, c}=Y, τ_1 = {ϕ, X, { b}}, τ_2 = {ϕ, X, {a}, {a, b}}, σ_1={ϕ, Y, {a}}, σ_2={ϕ, Y, {a, b}} respectively. Let $f : X \to Y$ be an identity maps. Then {a}, {b}, {a, b} are (1, 2)*-πgα-closed sets of Y, but f^{-1} ({a}, {b}, {a, b}) = {a}, {b}, {a, b} are not (1, 2)*-πgα-closed set of X. This implies that f is not an (1, 2)*-πgα-irresolute. However, f is a strongly (1, 2)*-πgα-closed map.

Example: 3.11 Let X = {a, b, c}=Y, τ_1 = {ϕ, X, { a}}, τ_2 = {ϕ, X}, σ_1={ϕ, Y, {b}}, σ_2={ϕ, Y, {a}} and $f : X \to Y$ be an identity maps. Then f is (1, 2)*-πgα-irresolute, but not strongly (1, 2)*-πgα-closed map.

Theorem: 3.12 A map $f : X \to Y$ is strongly (1, 2)*-πgα-closed if and only if for each subset B of Y and for each (1, 2)*-πgα-open set U of X containing $f^{-1}(B)$, there exists an (1, 2)*-πgα-open set V of Y, such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof: Necessity: Let B be any subset of Y and U be an (1, 2)*-πgα-open set of X containing $f^{-1}(B)$. Put $V = Y - f(X - U)$. Then V is (1, 2)*-πgα-open set in Y containing B such that $f^{-1}(V) \subset U$.

Sufficiency: Let F be any (1, 2)*-πgα-closed subset of X. Then $f^{-1}(Y - f(F)) \subset X - F$. Put $B = Y - f(F)$. Also, X-F is (1, 2)*-πgα-open set in X. There exists an (1, 2)*-πgα-open set V of Y such that $B \subset V$ and $f^{-1}(V) \subset X - F$. Hence f(F) is (1, 2)*-πgα-closed set in Y.

Proposition: 3.13 If $f : X \to Y$ is (1, 2)* –π-irresolute and (1, 2)*-pre-α-closed then f is a strongly (1, 2)*-πgα-closed map.

Proof: Let A be an (1, 2)*-πgα-closed set in X. Let V be any $\tau_{1,2}$ –π-open set in Y containing f(A). Then $A \subset f^{-1}(V)$. Since f is $\tau_{1,2}$ –π-irresolute, $f^{-1}(V)$ is $\tau_{1,2}$ –π-open set in X. Since A is (1, 2)*-πgα-closed set in X, $(1,2)^* - \alpha cl(A) \subset f^{-1}(V)$ and hence $f(A) \subset f((1,2)^* - \alpha cl(A)) \subset V$ .Since f is (1, 2)*-pre-α-closed and $(1,2)^* - \alpha cl(A)$ is (1,2)*-α-closed in X, $f((1,2)^* - \alpha cl(A))$ is (1,2)*-α-closed in Y and hence $(1,2)^* - \alpha cl(f(A)) \subset (1,2)^* - \alpha cl(f((1,2)^* - \alpha cl(A))) \subset V$. This shows that f(A) is (1, 2)*-πgα-closed set in Y. Hence f is strongly (1, 2)*-πgα-closed map.

Theorem: 3.14 Let f: X → Y and g : Y → Z be two maps.

(i) If f is (1, 2)*-closed map and g is strongly (1, 2)*-πgα-closed then g o f is (1, 2)*-πgα-closed map.

(ii) If f is (1, 2)*-closed map and g is (1,2)* –π-irresolute and (1, 2)*-pre-α-closed then g o f is (1, 2)*-πgα-closed map.

(iii) If g o f is strongly (1, 2)*-πgα-closed and f is a (1, 2)*-continuous surjection then g is (1, 2)*-πgα-closed map.

Proof:
(i) By Proposition 2. 13[3], the proof is obvious.
(ii) By Proposition 3.13, g is strongly (1, 2)*-πgα-closed map. Hence by (i), gof is (1, 2)*-πgα-closed map.
(iii) Straight forward.

Definition: 3.15
A subset A of a bitopological space X is called $\tau_{1,2}$-π-closed space if every $\tau_{1,2}$-closed set is $\tau_{1,2}$- π-closed set.

Theorem: 3.16 If f: X → Y is (1, 2)*-continuous strongly (1, 2)*-πgα-open bijective map and if X is a (1,2)*-normal space and Y is $\sigma_{1,2}$-π-closed space, then Y is (1, 2)*-normal.

Proof: Let A nd B be disjoint $\sigma_{1,2}$-closed sets of Y. Since f is (1, 2)*-continuous Bijective, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\tau_{1,2}$-closed sets of X. Since X is (1, 2)*-normal, thereexist disjoint $\tau_{1,2}$-open sets G and H of X, such that $G \supset f^{-1}(A)$ and $H \supset f^{-1}(B)$. Every $\tau_{1,2}$-open set is (1, 2)*-πgα-open and hence G and H are disjoint (1, 2)*-πgα-open sets of X.

Since f is strongly (1, 2)*-πgα-open bijective, f (G) and f (H) are disjoint (1, 2)*-πgα-open sets of Y containing A and B respectively. Since Y is $\sigma_{1,2}$-π-closed space, A and B are $\sigma_{1,2}$-π-closed sets in Y.

Then we have $(1,2)^* - \alpha \operatorname{int}(f(G)) \supset A$ and $(1,2)^* - \alpha \operatorname{int}(f(H)) \supset B$

and $(1,2)^* - \alpha \operatorname{int}(f(G)) \cap (1,2)^* - \alpha \operatorname{int}(f(H)) \subset f(G) \cap f(H) = \phi$.

Therefore, there exist disjoint (1, 2)*-α-open sets $(1,2)*-\alpha\,\text{int}(f(G))$ say U and $(1,2)*-\alpha\,\text{int}(f(H))$ say V of Y containing A and B respectively. U and V are (1, 2)*-α-open sets imply

$U \subset \sigma_{1,2}-\text{int}(\sigma_{1,2}-cl(\sigma_{1,2}-\text{int}(U)))$ and $V \subset \sigma_{1,2}-\text{int}(\sigma_{1,2}-cl(\sigma_{1,2}-\text{int}(V)))$.

Since $\sigma_{1,2}-\text{int}(\sigma_{1,2}-cl(\sigma_{1,2}-\text{int}(U))) \cap \sigma_{1,2}-\text{int}(\sigma_{1,2}-cl(\sigma_{1,2}-\text{int}(V))) = \phi,$

$A \subset (1,2)*-\alpha\,\text{int}(f(G)) = U \subset \sigma_{1,2}-\text{int}(\sigma_{1,2}-cl(\sigma_{1,2}-\text{int}(U)))$ and
$B \subset (1,2)*-\alpha\,\text{int}(f(H)) = V \subset \sigma_{1,2}-\text{int}(\sigma_{1,2}-cl(\sigma_{1,2}-\text{int}(V)))$ .Hence, Y is (1, 2)*-normal.

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