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ON CONTRA-Ω*gα-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

*The notion of contra continuous functions was introduced by Dontchev.J. In this paper we apply the notion of Ω*gα-open sets in topological space to present and study a new class of functions called contra-Ω*gα-continuous functions as a new generalization of contra continuity. Futhermore, we define contra-Ω*gα-closed graph and investigate the relationship between contra-Ω*gα-continuity and contra-Ω*gα-closed graph.*

Keywords: *Ω*gα –closed, contra-Ω*gα –closed, Ω*gα-compact, Ω*gα-continuous function, contra-Ω*gα-continuous function, contra-Ω*gα-closed graph.*

1. INTRODUCTION

One of the important and basic topics in general topology and several branches of mathematics which have been researched by many authors is the continuity of the functions. In 1996, Dontchev [3] introduced the notions of contra-continuity in topological spaces. He defined a function f: X → Y is contra continuous if the preimage of every open set of Y is closed in X. Recently Ganster and Reilly [6] introduced a new class of functions called regular set connected functions(in 1999). Jafari and Noiri [7] introduced contra-pre-continous functions. J.Mercy and I.Arockiarani [10] introduced Ω*-closed sets and Ωp-closed sets in topological spaces. In this paper we introduce and study a new class of functions called contra-Ω*gα -continuous functions which generalize classes of regular set connected [6] contra continuous [3] and perfectly continuous [13] functions. Moreover, the relationship between contra-Ω*gα -continuity and contra-Ω*gα -closed graphs are also investigated.

2. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) will denote topological spaces. For a subset A of (X, τ), cl(A) and int (A) represent the closure of A and interior of A with respect to τ respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called
(i) a preopen set [9] if $A \subset int(cl(A))$ and a preclosed set if $cl(int(A)) \subset A$,
(ii) an α-open set [12] if $A \subset int(cl(int(A)))$ and a α-closed set if $cl(int(cl(A))) \subset A$,
(iii) a regular open set [14] if $A = int(cl(A))$ and a regular closed set if $A = cl(int(A))$,
(iv) πg-closed set [5] if $cl(A) \subset U$ whenever $A \subset U$ and U is π-open,
(v) πgp-closed set [15] if $pcl(A) \subset U$ whenever $A \subset U$ and U is π-open,
(vi) πgα-closed set [1] if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is π - open,
(vii) Ω*-closed set [10] if $pcl(A) \subset int(U)$ whenever $A \subset U$ and U is preopen.

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Definition 2.2: A function f: (X, τ) → (Y, σ) is called
(i) perfectly-continuous [13] if f^{-1} (V) is clopen in X for every open set V of Y.
(ii) πgα-continuous [1] if for each open set V of Y, f^{-1}(V) is πgα-open in X.
(iii) contra-continuous [3] if f^{-1} (V) is closed in X for every open set V of Y.
(iv) regular set connected [6] if f^{-1} (V) is clopen in X for every V ε RO(Y).
(v) contra- α -continuous [8] if f^{-1} (V) is α- closed in (X, τ) for every open set V of (Y, σ).
(vi) pre-α-closed [12] if f(V) is α-closed in (Y, σ) for every α-closed set V ε (X, τ).
(vii) contra-πg-continuous [5] if f^{-1}(V) is πg-closed in (X, τ) for every open set V of (Y, σ).
(viii) π-continuous [6] if f^{-1} (V) is π-closed in X for every closed set V of Y.
(ix) RO-continuous[1] if f^{-1}(V) is regular open in X for every open set V of Y.
(x) contra- πgα-continuous [1] if f^{-1} (V) is πgα -closed in (X, τ) for each open set V of (Y, σ).

Definition 2.3: A space (X, τ) is called:
(i) locally indiscrete [2] if every open set is closed.
(ii) πgα-space [1] if every πgα-open set is open.
(iii) πgα-$T_{\frac{1}{2}}$ space [1] if every πgα-closed set is α-closed.
(iv) mildly compact [7] if every clopen cover of X has a finite subcover.
(v) πgα-locally indiscrete [1] if every πgα-open set is closed.
(vi) πGα –compact[1] if every πgα-open cover of X has a finite sub cover.

3. CONTRA-Ω*gα -CONTINUOUS FUNCTIONS

Definition 3.1: A subset A of a topological space (X, τ) is said to be Ω*g–closed if cl(A) ⊂U whenever A ⊂U and U is Ω*-open.

Definition 3.2: A subset A of a topological space (X, τ) is said to be Ω*gp–closed if pcl(A)⊂U whenever A ⊂U and U is Ω*-open.

Definition 3.3: A subset A of a topological space (X, τ) is said to be Ω*gα–closed if α-cl(A) ⊂U whenever A ⊂U and U is Ω*-open.

Definition 3.4: A function f: (X, τ) → (Y, σ) is called contra - Ω*g -continuous if f^{-1}(V) is Ω*g -closed in (X, τ) for each open set V of (Y, σ).

Definition 3.5: A function f: (X, τ) → (Y, σ) is called contra- Ω*gp -continuous if f^{-1}(V) is Ω*gp -closed in (X, τ) for each open set V of (Y, σ).

Definition 3.6: A function f: (X, τ) → (Y, σ) is called contra- Ω*gα -continuous if f^{-1}(V) is Ω*gα -closed in (X, τ) for each open set V of (Y, σ).

Theorem 3.7:
(i) Every contra-continuous function is contra- Ω*gα –continuous.
(ii) Every contra- α –continuous function is contra- Ω*gα –continuous.
(iii) Every contra-Ω*g-continuous function is contra- Ω*gα –continuous.
(iv) Every contra- Ω*gα –continuous function is contra- Ω*gp –continuous.

Converse of above theorem is not true as the following example shows:

Example 3.3:
(a) Let X = {a, b, c}, τ = {Φ, X, {a}} and σ = {Φ, X, {b}, {c}, {b, c}}. Then the identity function f: (X, τ) → (X, σ) is contra-Ω*gα –continuous, but not contra-continuous.

(b) Let X = {a, b, c}, τ = {Φ, X, {a, b}} and σ = {Φ, X, {a}, {a, b}}. Then the identity function f : (X, τ) → (X, σ) is contra-Ω*gα –continuous, but not contra-α-continuous.

(c) Let X = {a, b, c, d}, τ = {Φ, X, {a}, {a, c}, {a, d}, {a, c, d}} and σ = {Φ, X, {a}, {b}, {a, b}}. Then the identity function f : (X, τ) → (X, σ) is contra-Ω*gα –continuous but not contra-Ω*g-continuous.

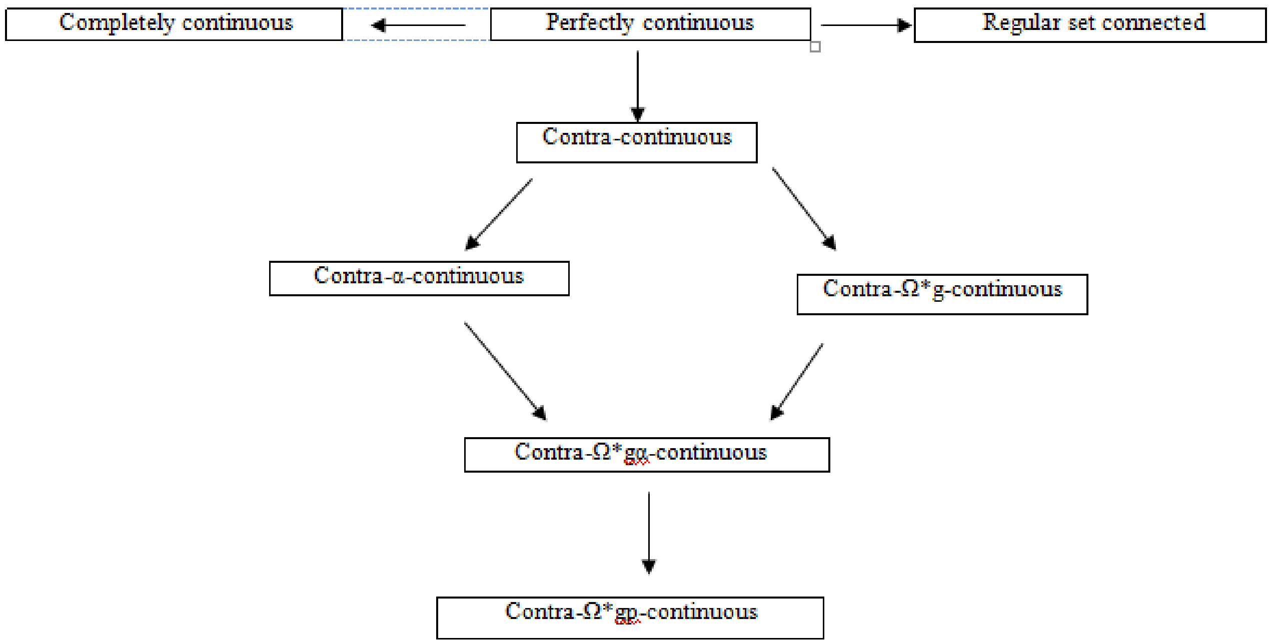
(d) Let X = {a, b, c}, τ = {Φ, X, {a}, {b}, {a, b}, {a, c}} and σ = {Φ, X, {b}, {c}, {b, c}}. Then the identity function f : (X, τ) → (X, σ) is contra-Ω*gp-continuous but not contra-Ω*gα-continuous since {c} Ω*gp-closed but not Ω*gα-closed in (X, τ).

Definition 3.4: A space (X, τ) is called Ω*gα-locally indiscrete if every Ω*gα-open set is closed.

Example 3.5: Let X = {a, b, c}, τ = {Φ, X, {a}, {b}}. Then X is Ω*gα-locally indiscrete.

Example 3.6: Let X = {a, b, c}, τ = {Φ, X, {a}, {b, c}} and τc = {Φ, X, { b, c}, {a}}.Then X is not Ω*gα-locally indiscrete since {b} is Ω*gα-open but not closed.

Remark 3.12: The following diagram holds.



Thus the class of contra-Ω*gα-continuous function properly contains the class of contra-Ω*g-continuous function and contra-α-continuous function but stronger than contra-Ω*gp-continuous function.

Theorem 3.7: If a function f: (X, τ)→ (Y, σ) is Ω*gα -continuous and (X, τ) is Ω*gα-locally indiscrete then f is contra-continuous.

Proof: Let V be open in (Y, σ). Since f is Ω*gα-continuous, $f^{-1}(V)$ is Ω*gα-open in X. Since X is Ω*gα-locally indiscrete, $f^{-1}(V)$ is closed in X. Hence f is contra-continuous.

Theorem 3.8: If a function f: (X, τ) → (Y, σ) is contra -Ω*gα -continuous and pre-α-closed surjection. If X is Ω*gα-$T_{\frac{1}{2}}$ space, then Y is locally indiscrete.

Proof: Let V be open in (Y, σ). Since f is contra-Ω*gα-continuous, $f^{-1}(V)$ is Ω*gα-closed in (X, τ). Since X is Ω*gα-$T_{\frac{1}{2}}$, $f^{-1}(V)$ is α-closed in X. Since f is pre-α-closed surjection, $f(f^{-1}(V)) = V$ is α-closed in Y.

Now $cl(V) = cl(int(V)) \subset cl(int(cl(V))) \subset V$ shows that V is closed in Y. Therefore Y is locally indiscrete.

Theorem 3.9: If a function f: (X, τ) → (Y, σ) is contra -Ω*gα-continuous and is a Ω*gα-$T_{\frac{1}{2}}$ space, then f: X → Y is contra-α-continuous.

Proof: Straight forward.

Theorem 3.10: If a function f: (X, τ) → (Y, σ) is contra -Ω*gα-continuous and X is a Ω*gα-space, then f: X → Y is contra-continuous.

Proof: Straight forward.

Definition 3.11: A space (X, τ) is called $T_{\Omega*}$gα-space if every Ω*gα-closed set is Ω*g-closed.

Theorem 3.12: If f: X → Y is contra-Ω*gα-continuous and if X is T_{Ω^*}gα-space, then f is contra- Ω*g-continuous.

Proof: Straight forward.

Theorem 3.13: Suppose Ω*GαO(X) is closed under arbitrary union. Then the following are equivalent for a function f: X → Y
(i) f is contra-Ω*gα-continuous
(ii) For every closed subset F of Y, $f^{-1}(F)$ ε Ω*GαO(X)
(iii) For each x ε X and each F ε C(Y, f(x)), there exists U ε Ω*GαO(X, x) such that f(U) ⊂ F.

Proof: (i) ⇔ (ii) and (ii) ⇒ (iii) is obvious (iii) ⇒ (ii). Let F be any closed set of Y and x ε $f^{-1}(F)$. Then f(x) ε F and there exists U_x ε Ω*GαO(X, x) such that $f(U_x) \subset F$.

Therefore we obtain $f^{-1}(F) = \cup_{x \in f^{-1}(F)}$ ε Ω*GαO(X).

Lemma 3.14: If Y is open in X, A ε Ω*GαC(X) ⇒ A ε Ω*GαC(Y).

Proof: Straight forward.

Theorem 3.15: If a function f: X → Y is contra-Ω*gα-continuous and U is open in X, then $\frac{f}{U}$: U → Y is contra- Ω*gα-continuous.

Proof: Let V be closed in Y. Since f: X → Y is contra-Ω*gα-continuous, $f^{-1}(V)$ is Ω*gα-open in X.
$(\frac{f}{U})^{-1}(V) = f^{-1}(V) \cap U$ is Ω*gα-open in X. By lemma 3.14, $(\frac{f}{U})^{-1}(V)$ is Ω*gα-open in U.

Lemma 3.16: If A ⊂ Y⊂ X and if Y is regular open and Ω*gα-closed set in X, then A ε Ω*GαC(Y) ⇒ A ε Ω*GαC(X).

Proof: Straight forward.

Theorem 3.17: Let f: X → Y be a function and {U_i / i ε I } be a cover of X such that U_i ε Ω*GαC(X) and regular open for each i ε I. If $\frac{f}{U_i}$: U_i → Y is contra- Ω*gα-continuous for each i ε I, then f is contra- Ω*gα-continuous.

Proof: Suppose that F is any open set of Y. We have $f^{-1}(F) = \cup f^{-1}(F) \cap U_i = \cup_{i \in I} \left(\frac{f}{U_i}\right)^{-1}(F)$.
Since $\frac{f}{U_i}$ is contra- Ω*gα-continuous for each i ε I, it follows $\cup_{i \in I} \left(\frac{f}{U_i}\right)^{-1}(F)$ ε Ω*GαC(U_i). By lemma 3.16, we have $f^{-1}(F)$ ε Ω*GαC(X) which implies f is contra- Ω*gα-continuous.

Lemma 3.18: The following properties are equivalent for a subset A of a space X.
(i) A is clopen.
(ii) A is Ω*gα-closed and regular open.
(iii) A is Ω*gα-closed and Ω*-open.

Proof: Straight forward.

Theorem 3.19: For a function f: X → Y the following properties are equivalent.
(i) f is perfectly-continuous.
(ii) f is contra- Ω*gα-continuous and RO-continuous.
(iii) f is contra- Ω*gα-continuous and Ω*-continuous.

Proof: Proof follows from lemma 3.18.

Theorem 3.20: Let Y be a regular space. If f: X → Y is contra-Ω*gα-continuous then f is Ω*gα-continuous.

Proof: Let x be an arbitrary point of X and V is an open set of Y containing f(x). Since Y is regular by lemma 3.31[11] there exist W in Y containing f(x) such that cl(W) ⊂ V. Since f is contra- Ω*gα-continuous by theorem 3.13, there exist U ε Ω*GαO(X, x) such that f(U) ⊂ cl(W) ⊂ V. Hence f is Ω*gα-continuous.

4. Ω*Gα -COMPACTNESS AND Ω*Gα-CONNECTEDNESS

Definition 4.1: A space X is said to Ω*Gα -compact if every Ω*gα-open cover of X has a finite sub cover.

Definition 4.2: A space X is said to be strongly-S-closed if every closed cover of X has a finite sub cover. [3]

Definition 4.3: A subset A of a space X is said to be Ω*Gα –compact relative to X if every cover of A by Ω*gα-open sets of X has a finite sub cover.

Definition 4.4: A subset A of a space X is said to be strongly-S-closed if the subspace A is strongly-S-closed. [3]

Theorem 4.5: If f: X → Y is contra-Ω*gα-continuous and K is Ω*gα-compact relative to X, then f(K) is strongly-S-closed in Y.

Proof: Let {Hα | α ε Δ} be any cover of f(K) by closed sets of the subspace f(K). For each α ε Δ, there exists a closed set Kα of Y such that Hα = Kα ∩ f(k). For each x ε K, there exists α(x) ε Δ such that f(x) ε Kα (x) and by theorem 3.13, there exists Ux ε Ω*GαO(X, x) such that $f(U_X) \subset K\alpha(x)$. Since the family $\{U_X$ / x ε K}is a cover of K by Ω*gα-open sets of X, there exist a finite subset K_o of K such that K⊂ ∪{Ux / x ε K_o}. Therefore, we obtain f(K) ⊂ ∪ {f(Ux) / x ε K_0} ⊂ ∪ {Kα(x) / x ε K_o}. Thus f(K) = ∪{Hα(x) / x ε K_o} and hence f(K) is strongly-S-closed.

Corollary 4.6: If f: X → Y is contra-Ω*gα-continuous surjection and X is Ω*gα-compact, then Y is strongly-S-closed.

Theorem 4.7: If f: X → Y is contra-Ω*gα-continuous, Ω*-continuous surjection and X is mildly compact, then Y is compact.

Proof: Let $\{V_\alpha | \alpha \in \Delta\}$be an open cover of Y. Since f is contra-Ω*gα-continuous, Ω*-continuous by theorem 3.19 $\{f^{-1}(V_\alpha) \mid \alpha \in \Delta\}$ is a clopen cover of X and there exist a finite subset ∇_o of ∇ such that $X = \cup \{f^{-1}(V_\alpha) \mid \alpha \in \nabla_o\}$and hence Y is compact.

Definition 4.8: A topological space (X, τ) is said to be Ω*-connected provided that X is disjoint union of two nonempty Ω*gα-open sets.

Theorem 4.9: Let (X, τ) be Ω*gα-connected and (Y, σ) be T_1. If f: X → Y is contra -Ω*gα-continuous, then f is constant.

Proof: Assume Y is non-empty. Since Y is T1 -space and f is contra-Ω*gα-continuous, $\Omega = \{f^{-1}(y) / y \in Y\}$ is a disjoint Ω*gα-open partition of X. If |Ω|≥2, then X can be written as the disjoint union of Ω*gα-open sets, which is a contradiction. Therefore |Ω| = 1 and hence f is constant.

Theorem 4.10: If f: X → Y is contra-Ω*gα-continuous, Ω*-continuous surjection and X is connected. Then Y has an indiscrete topology.

Proof: Suppose that there exists a proper open set V of Y. Since f is contra-Ω*gα- continuous, Ω*-continuous, $f^{-1}(V)$ is Ω*gα-closed and Ω*-open in X. By Lemma 3.18, $f^{-1}(V)$ is a proper clopen set in X which is a contradiction to the fact that X is connected. Therefore Y has an indiscrete topology.

Theorem 4.11: If f: X →Y is contra -Ω*gα-continuous, Ω*-continuous surjection and X is connected, then Y is connected.

Proof: Suppose Y is not connected. There exist non empty disjoint open sets V_1 and V_2 such that $Y = V_1 \cup V_2$. Therefore, V_1 and V_2 are clopen in Y. Since f is contra-Ω*gα-continuous and Ω*-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are Ω*gα-closed and Ω*-open in X and hence clopen by Lemma 3.18. Also, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are non empty disjoint sets in X such that $X = f^{-1}(V_1) \cup f^{-1}(V_2)$ which shows that X is not connected. Hence Y is connected.

Theorem 4.12: If f: X → Y is contra-Ω*gα-continuous, surjection and X is Ω*Gα-connected, then Y is connected.

Proof: Suppose Y is not connected. There exists non empty disjoint open set V_1 and V_2 are clopen in Y. Since f is contra-Ω*gα-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are Ω*gα-closed and Ω*-open in X such that $X = f^{-1}(V_1) \cup f^{-1}(V_2)$ which shows that X is not Ω*Gα-connected. Hence Y is connected.

Theorem 4.13: If f: X →Y is contra-Ω*gα-continuous, Ω*gα-continuous surjection and X is Ω*gα-connected, then Y has an indiscrete topology.

Proof: Similar as theorem 4.10.

5. CONTRA-Ω*Gα -CLOSED GRAPHS AND APPLICATIONS

In this section we define contra-**Ω***gα -closed graph and investigate the relationship between contra -**Ω***gα –continuous function and contra-**Ω***gα -closed graph.

Definition 5.1: The graph G(f) of a function f: X→ Y is said to be contra -**Ω***gα -closed if for (x , y) ϵ X ⊂ Y- G(f), there exist U ϵ **Ω***GαO(X , x) and Vϵ C(Y, y) such that (U ∪ V) ∩ G(f) = Φ.

Theorem 5.2: If f: X → Y is contra-**Ω***gα–continuous and Y is Urysohn, then G(f) is contra-**Ω***gα –closed in X × Y.

Proof: Let (x, y)εX⊂Y- G(f), then y≠ f(x) and there exist open sets V, W such that f(x)∈V, y∈W and cl(V) ∩cl(W) = Φ. Since f is contra -**Ω***gα-continuous, there exist U ε **Ω***GαO(X, x), such that f(U) ⊂ cl(V). Therefore f(U) ∩ cl(W) = Φ. This shows that G(f) is contra -**Ω***gα-closed.

Theorem 5.2: If f: X → Y is **Ω***gα –continuous and Y is T_1 then G(f) is contra -**Ω***gα-closed in X × Y.

Proof: Let (x, y) ∈ X × Y-G(f), then f(x)≠ y. Since Y is T_1 and f(x)≠ y there exist an open set V of Y such that f(x) ε V, y ⊂ V. Therefore f(U) ∩ (Y-V) = Φ and Y-V ϵ C(Y, y). This implies G(f) is contra-**Ω***gα-closed in X × Y.

REFERENCES

1. I. Arockirani, K. Balachandran and C. Janaki, On contra -πgα-continuous functions, *Kochi J. Math*. 3(2008), 201-209.
2. J. Dontchev, The characterization of spaces and maps via semi-preopen sets, Indian. J. Pure Appl Math, 25(1994),
939- 947.
3. J. Dontchev, contra-continuous functions and strongly-S-closed spaces, Internat. J. Math. Sci. 19 (2) (1996), 303-310.
4. J. Dontchev and T. Noiri, Contra-semi continuous functions, Math. Pannon. 10(2)159-168.(1999),
5. J. Dontchev and T. Noiri, Quasi-normal spaces and πg-closed sets, Acta Math Hungar, 89 (3) (2000), 211-219.
6. M. Ganster and I. Reilly, More on almost s-continuity, Indian J. Math 41, 139- 146.
7. S. Jafari and T. Noiri, Contra-α-continuous functions between topological spaces, Iran. Int. J. Sci. 2 (2) (2001), 153-167.
8. S. Jafari and T. Noiri, On contra-precontinuous functions, Bull. Malays. Math. Sci Soc. 25 (2) (2002), 115-128.
9. A. S. Mashour, M. E. Abd El-Monsef and S. N. El-Deeb, On pre-continuous and weak pre-continuous mappings, Proc. Math and Phys. Soc. Egypt, 53(1982), 47-53.
10. J. Mercy and I. Arockiarani, On Ω*-closed sets and Ωp-closed sets in topological spaces, Acta Ciencia Indica, Vol. XXXIII M. No. 2(2007), 509-515.
11. J. R. Munkres, Topology; A First Course, Prentice-Hall Inc, 1975.
12. O.Njastad, On some classes of nearly open sets, Pacific J. Math 15 (1965), 961-970.
13. T. Noiri, Super continuity and some strong forms of continuity, Indian J. Pure.Appl Math 15 (3) (1984), 241-250.
14. N. Palaniappan and K. C. Rao, Regular generalized closed sets, Kyungpook Math. J, 33(1993), 211-219.
15. J. H Park, On πgp-closed sets in topological spaces, Indian J. Pure Appl. Math to appear.

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