International Journal of Mathematical Archive-7(11), 2016, 60-67
IJMA Available online through www.ijma.info ISSN 2229 – 5046

BETWEEN δ-CLOSED SETS AND δ -SEMI-CLOSED SETS

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(Received On: 27-10-16; Revised & Accepted On: 23-11-16)

ABSTRACT

*In this paper we introduce a new class of sets, namely semi*δ-closed sets, as the complement of semi*δ-open sets. We find characterizations of semi*δ-closed sets. We also define the semi*δ-closure of a subset. Further we investigate fundamental properties of the semi*δ-closure.*

Keywords: *δ-semi-open set, δ-semi-closed set, semi*δ-open set, semi*δ-interior, semi*δ-closed set, semi*δ-closure.*

I. INTRODUCTION

In 1963 Levine [3] introduced the concepts of semi-open sets and semi-continuity in topological spaces. Levine [4] also defined and studied generalized closed sets as a generalization of closed sets. Dunham [2] introduced the concept of generalized closure using Levine's generalized closed sets .N.V.Velico [15] introduced the concept of δ-open sets in 1968. In 1997, Park, Lee and Son [17] introduced the concept of δ-semi-open sets in topological spaces. Pasunkili Pandian [9] defined and studied semi*-pre closed sets and investigated its properties.A.Robert [13] defined and studied semi*α-closed sets. The authors [18] have recently introduced the concept of semi*δ-open sets and investigated its properties. The semi*δ-interior of a subset has also been defined and its properties studied.

In this paper, we define a new class of sets, namely semi*δ-closed sets, as the complement of semi*δ-open sets. We further show that the class of semi*δ-closed sets is placed between the class of δ-closed sets and the class of δ-semi-closed sets. We find characterizations of semi*δ-closed sets. We investigate fundamental properties of semi*δ-closed sets. We also define the semi*δ-closure of a subset. We also study some basic properties of semi*δ-closure.

II. PRELIMINARIES

Throughout this paper (X, τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of the space (X, τ), $Cl(A)$ and $Int(A)$ denote the closure and the interior of A respectively. Also $\mathcal{F}$ denotes the class of all closed sets in the space (X, τ).

Definition 2.1: A subset A of a space X is
- (i) ***generalized closed*** (briefly ***g-closed***) [2] if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X.
- (ii) ***generalized open*** (briefly ***g-open***) [2] if $X\backslash A$ is g-closed in X.

Definition 2.2: If A is a subset of X,
- (i) the ***generalized closure*** [3] of A is defined as the intersection of all g-closed sets in X containing A and is denoted by $Cl^*(A)$.
- (ii) the ***generalized interior*** of A is defined as the union of all g-open subsets of A and is denoted by $Int^*(A)$.

Definition 2.3: A subset A of a topological space (X, τ) is **semi-open** [3](respectively **semi*-open** [12]) if there is an open set U in X such that $U\subseteq A\subseteq Cl(U)$ (respectively $U\subseteq A\subseteq Cl^*(U)$) or equivalently if $A\subseteq Cl(Int(A))$ (respectively $A\subseteq Cl^*(Int(A))$).

Definition 2.4: Definition 2.4. A subset A of a topological space (X, τ) is **pre-open** [5](respectively **pre*-open** [14]) if $A\subseteq Int(Cl(A))$ (respectively $A\subseteq Int^*(Cl(A))$).

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Definition 2.5: A subset *A* of a topological space (*X*, τ) is **α-open** [7] (respectively **α*-open** [10]) if *A*⊆*Int*(*Cl*(*Int*(*A*))), (respectively *A*⊆*Int**(*Cl*(*Int**(*A*))).

Definition 2.6: A subset *A* of a topological space (*X*, τ) is **semi-preopen** [1]= β - open (respectively **semi*-preopen** [9]) if *A*⊆*Cl*(*Int*(*Cl*(*A*))) (respectively *A*⊆*Cl**(*pInt*(*A*))).

Definition 2.7: A subset *A* of a topological space (*X*, τ) is **regular-open** [6] if *A*=*Int*(*Cl*(*A*)).

Definition 2.8: The ***δ-interior*** [15] of *A* is defined as the union of all regular-open sets of *X* contained in *A*. It is denoted by **δ*Int*(*A*).**

Definition 2.9: A subset *A* of a topological space (*X*, τ) is **δ-open** [11] if *A*=*δInt*(*A*).

Definition 2.10: A subset *A* of a topological space (*X*, τ) is **semi α-open** [6] (respectively **semi* α-open** [13]) if there is a α-open set *U* in *X* such that *U*⊆*A*⊆*Cl*(*U*) (respectively *U*⊆*A*⊆*Cl**(*U*)) or equivalently if *A*⊆*Cl*(α*Int*(*A*)). (respectively *A*⊆*Cl**(α*Int*(*A*))).

Definition 2.11: A subset *A* is **δ-semi-open** [17] if *A*⊆*Cl*(δ*Int*(*A*)).

Definition 2.12: A subset *A* of a topological space (*X*, τ) is called a ***semi*δ-open*** **[**18] if there exists a δ-open set *U* in *X* such that *U*⊆*A*⊆*Cl**(*U*) or equivalently if *A*⊆*Cl**(δ*Int*(*A*)).

Definition 2.13:A subset *A* of a topological space (*X*, τ) is **semi-closed (**respectively **semi*-closed, pre-closed**[5], **pre*-closed** [14], **α-closed** [7], **α*-closed** [10], **semi-pre closed** [1], **semi*-preclosed** [9], **regular-closed**[6], **δ-closed** [11], **semi α-closed**[6], **semi* α-closed**[13] and **δ-semi-closed** [17]) if *Int*(*Cl*(*A*))⊆*A*(respectively *Int**(*Cl*(*A*))⊆*A*, *Cl*(*Int*(*A*)⊆*A*, *Cl**(*Int*(*A*)⊆*A*,*Cl*(*Int*(*Cl*(*A*)))⊆*A* ,*Cl**(*Int*(*Cl**(*A*)))⊆*A*, *Int*(*Cl*(*Int*(*A*))) ⊆*A* ,*Int**(*pCl*(*A*))⊆*A*, *Cl*(*Int*(*A*) =*A*,*δCl*(*A*)=*A*, *Int*(*Cl*(*Int*(*Cl*(*A*)))) ⊆*A*, *Int**(α*Cl*(*A*))⊆*A* and *Int*(δ*Cl*(*A*))⊆*A*).

The class of all semi*δ-open sets in (*X*, τ) is denoted by S*δO(*X*, τ) or simply S*δO(*X*).

The class of all semi-closed (respectively semi*-closed, pre-closed, pre*-closed, α-closed, α*-closed, semi-preclosed, semi*-preclosed, semi α-closed, semi* α-closed, regular-closed, δ-closed and δ-semi-closed) sets in (*X*, τ) is denoted by SC(*X*) (respectively S*C(*X*), PC(*X*). P*C(*X*),αC(X) , α*C(X) SPC(*X*), S*PC(*X*) ,SαC(*X*), S*αC(*X*), RC(*X*), δC(*X*) and δSC(*X*).

Definition 2.14: A topological space *X* is $T_{1/2}$[4] if every g-closed set in *X* is closed.

Theorem 2.15: [2] *Cl** is a Kuratowski closure operator in *X*.

Definition 2.16: [2] If τ* is the topology on *X* defined by the Kuratowski closure operator *Cl**, then (*X*, τ*) is $T_{1/2}$.

Definition 2.17: [16] A space *X* is locally indiscrete if every open set in *X* is closed.

Theorem 2.18: [18] For a subset *A* of a topological space (*X*, τ) the following statements are equivalent:
(i) *A* is semi*δ-open.
(ii) *A*⊆*Cl**(δ*Int*(*A*)).
(iii) *Cl**(δ*Int*(*A*))=*Cl**(*A*).

Theorem 2.19: [18] Every δ-open set is semi*δ-open.

Theorem 2.20: [18**]** In any topological space,
(i) Every semi*δ-open set is δ-semi-open.
(ii) Every semi*δ-open set is semi - open.
(iii) Every semi*δ-open set issemi* - open.
(iv) Every semi*δ-open set is semi*-preopen.
(v) Every semi*δ-open set is semi-preopen.
(vi) Every semi*δ-open set is semi*α-open
(i) Every semi*δ-open set is semiα-open.

Theorem 2.21: [18] In any topological space, arbitrary union semi*δ-open sets is semi*δ-open.

Theorem 2.22: [18] If *A* is semi*δ-open in *X* and *B* is open in *X*, then *A*∩*B* is semi*δ-open in *X*.

Theorem 2.23: [18] If A is semi*δ-open in X and $B \subseteq X$ is such that $\delta Int(A) \subseteq B \subseteq Cl^*(A)$. Then B is semi*δ-open.

III. SEMI*δ-CLOSED SETS

Definition 3.1: The complement of a semi*δ-open set is called ***semi*δ-closed.*** The class of all semi*δ-closed sets in (X, τ) is denoted by $S^*\delta C(X, \tau)$ or simply $S^*\delta C(X)$

Definition 3.2: A subset A of X is called ***semi*δ-regular*** if it is both semi*δ-open and semi*δ-closed.

Theorem 3.3: For a subset A of a topological space (X, τ), the following statements are equivalent:

(i) A is semi*δ-closed.
(ii) $Int^*(\delta Cl(A)) \subseteq A$.
(iii) $Int^*(\delta Cl(A)) = Int^*(A)$.

Proof: (i)⟹(ii): Suppose A is semi*δ-closed.Then $X \setminus A$ is semi*δ-open.Then by Theorem 2.18 $X \setminus A \subseteq Cl^*(\delta Int(X \setminus A))$. Taking the complements we get, $A \supseteq X \setminus Cl^*(\delta Int(X \setminus A) \Longrightarrow A \supseteq Int^*(\delta Cl(A))$.

(ii)⟹(iii): By assumption, $Int^*(\delta Cl(A)) \subseteq A$. This implies that $Int^*(\delta Cl(A)) \subseteq Int^*(A)$. Since it is true that $A \subseteq \delta Cl(A)$, we have $Int^*(A) \subseteq Int^*(\delta Cl(A))$. Therefore $Int^*(\delta Cl(A)) = Int^*(A)$.

(iii)⟹(i): If $Int^*(\delta Cl(A)) = Int^*(A)$, then taking the complements, we get $X \setminus Int^*(\delta Cl(A)) = X \setminus Int^*(A)$. Hence $Cl^*(\delta Int(X \setminus A)) = Cl^*(X \setminus A)$. Therefore by Theorem 2.18, $X \setminus A$ is semi*δ-open and hence A is semi*δ-closed.

Theorem 3.4: A subset A of a space (X, τ) is semi*δ-closed iff there is a δ-closed set F in (X, τ) such that $Int^*(F) \subseteq A \subseteq F$.

Proof: ***Necessity*:** Suppose A is semi*δ-closed. Then X\A is semi*δ-open. Then by definition 2.12 there exists a δ- open set U in X such that $U \subseteq X \setminus A \subseteq Cl^*(U)$ which implies $X \setminus U \supseteq A \supseteq X \setminus Cl^*(U)$. Note that in any space, $X \setminus Cl^*(U) = Int^*(X \setminus U)$. Therefore $F \supseteq A \supseteq Int^*(F)$ where $F = X \setminus U$ is δ-closed in X.

***Sufficiency*:** Suppose there is a δ-closed set F in (X, τ) such that $Int^*(F) \subseteq A \subseteq F$ which implies $X \setminus Int^*(F) \supseteq X \setminus A \supseteq X \setminus F$. Since $X \setminus Int^*(F) = Cl^*(X \setminus F)$, we have $Cl^*(X \setminus F) \supseteq X \setminus A \supseteq X \setminus F$ where X\F is a δ-open set. Hence by Definition 2.12, X\A is semi*δ-open. Therefore A is semi*δ-closed.

Remark 3.5:

(i) In any topological space (X, τ), ϕ and X are semi*δ-closed sets.
(ii) In a $T_{1/2}$ space, the semi*δ-closed sets and the δ-semi-closed sets coincide. In particular, in the real line with usual topology the semi*δ-closed sets and the δ-semi-closed sets coincide.

Theorem 3.6: Arbitrary intersection of semi*δ-closed sets is also semi*δ-closed.

Proof: Let $\{A_i\}$ be a collection of semi*δ-closed sets in X. Since each A_i is semi*δ-closed, $X \setminus A_i$ is semi*δ-open. By Theorem 2.21, $X \setminus (\cap A_i) = \cup (X \setminus A_i)$ is semi*δ-open. Hence $\cap A_i$ is semi*δ-closed.

Corollary 3.7: If A is semi*δ-closed and U is semi*δ-open in X, then $A \setminus U$ is semi*δ-closed.

Proof: Since U is semi*δ-open, $X \setminus U$ is semi*δ-closed. Also since $A \setminus U = A \cap (X \setminus U)$, and hence by Theorem 3.6, $A \setminus U$ is semi*δ-closed.

Remark 3.8: Union of two semi*δ-closed sets need not be semi*δ-closed as seen from the following examples.

Example 3.9: Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. In the space (X, τ), the subsets $\{a\}$ and $\{b\}$ are semi*δ-closed but their union $\{a, b\}$ is not semi*δ-closed.

Example 3.10: Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a\},\{b\},\{c\},\{a, b\},\{a, c\}, \{b, c\}, \{a, b, c\}, X\}$. In the space (X, τ), the subsets $\{a, c\}$ and $\{b, c\}$ are semi*δ-closed but their union $\{a, b, c\}$ is not semi*δ-closed.

Theorem 3.11: If A is semi*δ-closed in X and B is closed in X, then $A \cup B$ is semi*δ-closed.

Proof: Since A is semi*δ-closed, $X \setminus A$ is semi*δ-open in X. Also $X \setminus B$ is open. By Theorem 2.22, $(X \setminus A) \cap (X \setminus B) = X \setminus (A \cup B)$ is semi*δ-open in X. Hence $A \cup B$ is semi*δ-closed in X.

Theorem 3.12: Every δ-closed set is semi*δ-closed.

Let *U* be δ-closed in *X*. Then X\U is δ-open. By theorem 2.19, *X*\U is semi*δ-open, Hence *U* is semi*δ-closed.

Remark 3.13: The converse of the above theorem is not true as shown in the following examples.

Example 3.14: In the space (*X*, τ) where *X*={*a, b, c*} and τ={ϕ, {*a*}, {*b*}, {*a, b*}, *X*}, the subsets {*a*} and{*b*} are semi*δ-closed but not δ-closed.

Example 3.15: In the space (*X*, τ) where *X*={*a, b, c, d*} and τ={*ϕ*, {*a*}, {*b*}, {*a, b*},{*a, b, c*}, *X*}, the subsets {*c*}, {*a, c*}and {*b, c*} are semi*δ-closed but not δ-closed.

Theorem 3.16: In any topological space,

(i) Every semi*δ-closed set is δ-semi-closed.
(ii) Every semi*δ-closed set is semi-closed.
(iii) Every semi*δ-closed set is semi*-closed.
(iv) Every semi*δ-closed set is semi*-preclosed.
(v) Every semi*δ-closed set is semi-preclosed.
(vi) Every semi*δ-closed set is semi*α-closed.
(vii) Every semi*δ-closed set is semi-α-closed.

Proof:

(i) Let *A* be a semi*δ-closed set. Then *X**A* is semi*δ-open. By Theorem 2.20(i), *X**A* is δ-semi-open. Hence A is δ-semi-closed.
(ii) Suppose *A* is a semi*δ-closed set. Then *X**A* is semi*δ-open. By Theorem2.20 (ii), *X**A* is semi-open. Hence, A is semi-closed.
(iii) Suppose *A* is a semi*δ-closed set. Then *X**A* is semi*δ-open. By Theorem 2.20(iii), *X**A* is semi*-open. Hence, *A* is semi*-closed.
(iv) Let *A* be a semi*δ-closed set. Then *X**A* is semi*δ-open in X. By Theorem 2.20(iv), *X**A* is semi*-preopen Hence *A* is semi*-preclosed in X.
(v) his statement follows from (iv) and the fact that every semi*-preclosed set is semi-pre closed.
(vi) Let *A* be a semi*δ-closed set. Then *X**A* is semi*δ-open By Theorem 2.20(vi), *X**A* is semi*α-open. Hence, A is semi*α-closed.
(vii) This statement follows from (vi) and the fact that every semi*α-closed set is semiα-closed.

Remark 3.17: The converse of each of the statements in Theorem 3.16 is not true as shown in the following examples.

Example 3.18: In the space (*X*, τ) where *X*={*a, b, c, d*} and τ={*ϕ*, {*a*}, {*b*}, {*a, b*},{*a, b, c*}, *X*}, the subsets {*a*}, {*b*},{*d*}, {*a, d*} and {*b, d*} are semiδ-closed but not semi*δ-closed.

Example 3.19: In the space (*X*, τ) where *X*={*a, b, c, d*} and τ={*ϕ*, {*a*}, *X*}, the subsets{*b*},{*c*}, {*d*},{*b, c*},{*b, d*},{*c, d*} and {*b, c, d*} are semi-closed but not semi*δ-closed.

Example 3.20: In the space (*X*, τ) where *X*= {*a, b, c, d*} and τ={ϕ, {*a, b, c*}, *X*}, the subset {d} is semi*-closed but not semi*δ-closed.

Example 3.21: In the space (*X*, τ) where *X*={*a, b, c, d*} and τ={ϕ, {*a, b*},{*a, b, c*}, *X*}, the subsets {*a*}, {b}, {c}, {d},{*a, c*},{*a, d*}, { *b, c*}, {*b, d*},{c, *d*},{*a, c, d*} and {b, *c, d*} are semi*-preclosed but not semi*δ-closed.

Example 3.22: In the space (*X*, τ) where *X*={*a, b, c, d*} and τ={ϕ, {*a*},{ *b, c, d*}, *X*}, the subsets {b}, {c},{d},{*a, b*}, {a, c},{*a, d*},{*b, c*}, { *b, d*},{c, d},{*a, b, c*},{*a, b, d*} and {*a, c, d*} are semi-preclosed but not semi*δ-closed.

Example 3.23: In the space (*X*, τ) where *X*={*a, b, c, d*} and τ={ϕ, {*a*},{a, *b, c*}, *X*}, the subsets{b}, {*c*}, {d},{b, c},{*b, d*},{c, *d*} and {b, *c, d*} are semi*α-closed but not semi*δ-closed.

Example 3.24: In the space (*X*, τ) where *X*={*a, b, c, d*} and τ={ϕ, {*a*},{*a, b*},{a, *b, c*}, *X*}, the subsets{b}, {c}, {d}, {b, *c*},{ *b, d*},{*c, d*} and {b, *c, d*} are semiα-closed but not semi*δ-closed.

Corollary 3.25: If *A* is semi*δ-closed and *F* is δ-closed in *X*, then *A*∩*F* is semi*δ-closed in *X*.

Proof: Since *F* is δ-closed, by Theorem 3.12 *F* is semi*δ-closed. Then by Theorem 3.6, *A*∩*F* is semi*δ-closed

Theorem 3.26: In any topological space (X,τ), $\delta C(X, \tau)\subseteq S^*\delta C(X, \tau)\subseteq \delta SC(X, \tau)$ .That is the class of semi*δ-closed set is placed between the class of δ-closed sets and the class of δ-semi-closed sets.

Proof: Follows from Theorem 3.12 and Theorem 3.16.

Remark 3.27:

(i) If (X, τ) is a locally indiscrete space,
$\mathcal{F}=\delta C(X,\tau)=S^*\delta C(X,\tau)=\delta SC(X,\tau)=S^*C(X,\tau)=SC(X,\tau)=\alpha C(X,\tau)=S^*CO(X,\tau)=S\alpha C(X,\tau)=RC(X, \tau)$.

(ii) The inclusions in Theorem 3.26 may be strict and equality may also hold. This can be seen from the following examples.

Example 3.28: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau =\{\phi, \{a\}, \{b, c, d\}, X\}$,$\delta C(X, \tau)=S^*\delta C(X, \tau)=\delta SC(X, \tau)$.

Example 3.29: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau =\{\phi,\{a\},\{b\},\{c\},\{a, b\},\{a, c\},\{b, c\}, \{a, b, c \}, X\}$, $\delta C(X, \tau) \subsetneq S^*\delta C(X, \tau) = \delta SC(X, \tau)$.

Example 3.30: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\},\{a, b, c\}, X\}$, $\delta C(X, \tau) \subsetneq S^*\delta C(X, \tau) \subsetneq \delta SC(X, \tau)$.

Theorem 3.31: If A is semi*δ-closed in X and B be a subset of X such that $Int^*(A)\subseteq B\subseteq \delta Cl(A)$, then B is semi*δ-closed in X.

Proof: Since A is semi*δ-closed, $X\backslash A$ is semi*δ-open. Now $Int^*(A)\subseteq B\subseteq \delta Cl(A)$ which implies $X\backslash Int^*(A)\supseteq X\backslash B\supseteq X\backslash \delta Cl(A)$. That is, $Cl^*(X\backslash A)\supseteq X\backslash B\supseteq \delta Int(X\backslash A)$. Therefore by Theorem 2.23, $X\backslash B$ is semi*δ-open. Hence B is semi*δ-closed.

Remark 3.32: The concept of semi*δ-closed sets and closed sets are independent as seen from the following example:

Example 3.33: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, the subsets $\{c\}$,$\{a, c\}$and $\{b, c\}$ are semi*δ-closed but not closed and $\{d\}$ is closed but not semi*δ-closed.

Remark 3.34: The concept of semi*δ-closed sets and g-closed sets are independent as seen from the following example:

Example 3.35: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, the subsets $\{c\}$,$\{a, c\}$and $\{b, c\}$ are semi*δ-closed but not g-closed and $\{d\}$,$\{a, d\}$,$\{b, d\}$and$\{a, b, d\}$ are g-closed but not semi*δ-closed.

Remark 3.36: The concept of semi*δ-closed sets and α-closed sets are independent as seen from the following examples:

Example 3.37: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi,\{a, b\}, \{a, b, c\}, X\}$, the subsets $\{c\}$,$\{d\}$and $\{c, d\}$ are α-closed but not semi*δ-closed.

Example 3.38: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau =\{\phi, \{a\}, \{b\},\{c\},\{a, b\},\{a, c\}, \{b, c\}, \{a, b, c\}, X\}$,the subsets $\{a\}$,$\{b\}$, $\{c\}$,$\{a, b\}$,$\{a, c\}$ and $\{b, c\}$ are semi*δ-closed but not α-closed.

Remark 3.39: The concept of semi*δ-closed sets and pre-closed sets are independent as seen from the following examples:

Example 3.40: In the topological space (X, τ) where $X= \{a, b, c\}$ and $\tau=\{\phi,\{a\},\{b\},\{a, b\}, X\}$, the subsets $\{a\}$ and$\{b\}$ are semi*δ-closed but not pre-closed.

Example 3.41: In the topological space (X, τ) where $X= \{a, b, c\}$ and $\tau =\{\phi, \{a, b\}, X\}$, the subsets $\{a\}$,$\{b\}$, $\{c\}$,$\{ a, c\}$and$\{b, c\}$ are pre-closed but not semi*δ-closed.

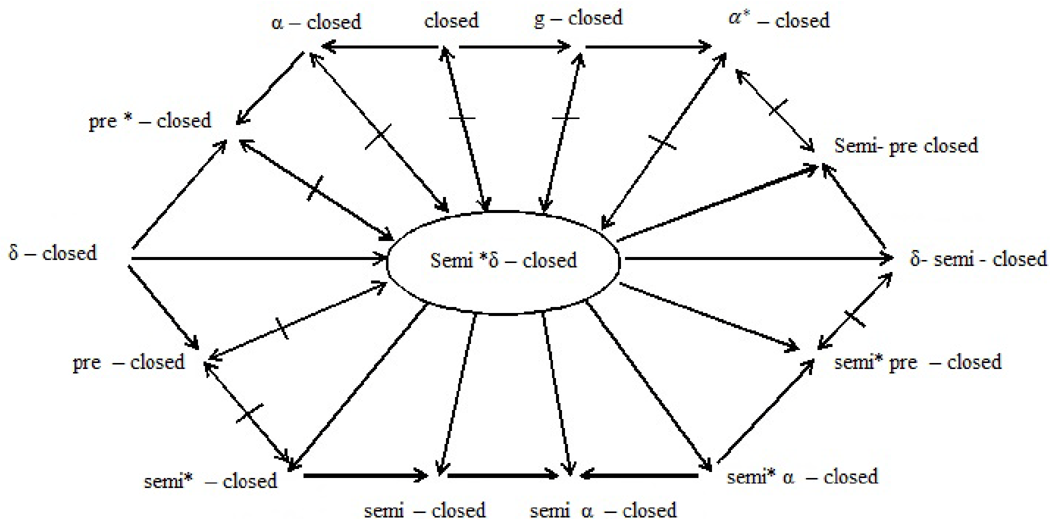
Remark 3.42: The concept of semi*δ-closed sets and α*-closed sets are independent as seen from the following examples:

Example 3.43: In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, the subsets $\{a, c\}$ and $\{b, c\}$ are semi*δ-closed but not α*-closed and $\{d\}$,$\{ a, d\}$,$\{b, d\}$and $\{a, b, d\}$ are α*-closed but not semi*δ-closed.

Remark 3.44: The concept of semi*δ-closed sets and pre*-closed sets are independent as seen from the following examples:

Example 3.45: In the topological space (*X*, τ) where *X*={*a*, *b*, *c*, *d*} and τ={ϕ, {*a*}, {*b*}, {*a*, *b*}, {*a*, *b*, *c*}, *X*}, the subsets {*a*, *c*}and{b, *c*} are semi*δ-closed but not pre*-closed and {*d*},{ *a*, *d*},{b, *d*} and {a, *b*, *d*} are pre*-closed but not semi*δ-closed.

From the above discussions we have the following diagram:



IV. SEMI*δ-CLOSURE OF A SET

Definition 4.1: If *A* is a subset of a topological space *X*, the ***semi*δ-closure*** of *A* is defined as the intersection of all semi*δ-closed sets in *X* containing *A*. It is denoted by *s*δCl*(*A*).

Theorem 4.2: If *A* is any subset of a topological space (*X*, τ), then
(i) *s*δCl*(*A*) is the smallest semi*δ-closed set in X containing *A*.
(ii) *A* is semi*δ-closed if and only if s**δCl*(*A*)=*A*.

Proof:
(i) Since *s*δCl*(*A*) is the intersection of all semi*δ-closed supersets of *A*, by Theorem 3.6, it is semi*δ-closed and is contained in every semi*δ-closed set containing *A* and hence it is the smallest semi*δ-closed set in *X* containing *A*.
(ii) If *A* is semi*δ-closed, then s*δ*Cl*(*A*) = *A* is obvious from definition 4.1. Conversely, let *s*δCl*(*A*)=*A*. By (i) *s*δCl*(*A*) is semi*δ-closed and hence *A* is semi*δ-closed.

Theorem 4.3: (Properties of Semi*δ-Closure)
In any topological space (*X*, τ) the following statements hold:
(i) $s^*\delta Cl(\phi)=\phi$.
(ii) $s^*\delta Cl(X)=X$.

If *A* and *B* are subsets of *X*,
(iii) $A\subseteq s^*\delta Cl(A)$.
(iv) $A\subseteq B \Longrightarrow s^*\delta Cl(A)\subseteq s^*\delta Int(B)$.
(v) $s^*\delta Cl(s^*\delta Cl(A))=s^*\delta Cl(A)$.
(vi) $A \subseteq \delta sCl(A)\subseteq s^*\delta Cl(A) \subseteq \delta Cl(A$
(vii) $s^*\delta Cl(A\cup B)\supseteq s^*\delta Cl(A)\cup s^*\delta Cl(B)$.
(viii) $s^*\delta Cl(A\cap B)\subseteq s^*\delta Cl(A) \cap s^*\delta Cl(B)$.

Proof: (i), (ii), (iii) and (iv) follow from Definition 4.1. By Theorem 4.2(i), $s^*\delta Cl(A)$ is semi*δ-closed and by Theorem 4.3(ii), $s^*\delta Cl(s^*\delta Cl(A))=s^*\delta Cl(A)$. Thus (v) is proved. The statements (vi) follows from Theorem 3.12and Theorem 3.16(i). Since $A\subseteq A\cup B$, from statement (iv) we have $s^*\delta Cl(A)\subseteq s^*\delta Cl(A\cup B)$. Similarly, $s^*\delta Cl(B)\subseteq s^*\delta Cl(A\cup B)$. This proves (vii). The proof for (viii) is similar.

Remark 4.4: In (vi) of Theorem 4.3,each of the inclusions may be strict and equality may also hold. This can be seen from the following examples:

Example 4.5: In the space (X, τ) where X={a, b, c, d} and τ={ϕ,{a},{b},{a, b}, {a, b, c}, X}.
$\mathcal{F}$={ϕ,{d},{c, d},{a, c, d}, {b, c, d}, X}.

Let A= {a, c, d}. Then δ*sCl*(A) =*s*δCl*(A)= *δCl*(A)={a, c, d}=A.

Let B={a, c}.Then δ*sCl*(B)= *s*δCl*(B)= {a, c}; *δCl*(B)={a, c, d};

Here B=δ*sCl*(B)= *s*δCl*(B) ⊊ *δCl*(B)

Let C= { a, d}.Then δ*sCl*(C)={ a, d}; *s*δCl*(C)= *δCl*(C)={a, c, d}

Here C= δ*sCl*(C) ⊊ *s*δCl*(C) = δ*Cl*(C).

Let D= {a, b}.Then δ*sCl*(D)= *s*δCl*(D)= *δCl*(D)= X.

Here D⊊δ*sCl*(D)=*s*δCl*(D)= *δCl*(D).

Let E={b}.Then δ*sCl*(E)={b}; *s*δCl*(E)={b, c}; *δCl*(E)={b, c, d};

Here E = δ*sCl*(E) ⊊ *s*δCl*(E) ⊊ *δCl*(E).

Remark 4.6: The inclusions in (vii) and (viii) of Theorem 4.3 may be strict and equality may also hold. This can be seen from the following examples.

Example 4.7: Consider the space (X, τ) in Example 4.5

Let A= {a, c} and B={c, d} then A∪B={a, c, d}; *s*δCl*(A) ={a, c}; *s*δCl*(B)={c, d}; *s*δCl*(A∪B)={a, c, d};

Here *s*δCl*(A∪B)= *s*δCl*(A) ∪*s*δCl*(B)

Let C= {a, c} and D ={b, c} then C∩D={c}; *s*δCl*(C)={a, c}; *s*δCl*(D)={b, c}; *s*δCl*(C∩D)={c};

Here *s*δCl*(C∩D)= *s*δCl*(C) ∩ *s*δCl*(D)

Let E= {a, b} and F ={c, d} then E∩F = ϕ; *s*δCl*(E) = X; *s*δCl*(F)={c, d}; *s*δCl*(E∩F)=ϕ; *s*δCl*(E) ∩*s*δCl*(F)={c, d}

Here *s*δCl*(E∩F)⊊*s*δCl*(E) ∩*s*δCl*(F)

Let G={a} and H={ b} then G∪H={a, b};
*s*δCl*(G)={a, c}; *s*δCl*(H)= { b, c}; *s*δCl*(G∪H)= X ; *s*δCl*(G) ∪*s*δCl*(H)={a, b, c};

Here s**δCl*(G) ∪*s*δCl*(H)⊊*s*δCl*(G∪H).

ACKNOWLEDGMENT

The first author is thankful to University Grants Commission, New Delhi, for sponsoring this work under grants of Major Research Project-MRP- MAJOR -MATH-2013-30929. F. No. 43-433/2014(SR) Dt. 11.09.2015.

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Source of support: University Grants Commission, New Delhi, India, Conflict of interest: None Declared