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FLOW OF A VISCO-ELASTIC RIVLIN-ERICKSEN TYPE FLUID THROUGH A LONG UNIFORM RECTANGULAR DUCT WHEN AN IMPULSIVE PRESSURE GRADIENT ACTING AT THE CENTRAL PART OF A SECTION

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ABSTRACT

In the present paper the unsteady flow of the visco-elastic Rivlin-Ericksen type fluid through a long uniform rectangular duct, when an impulsive pressure gradient acting at the central part of a section has been studied. The analytical expressions for velocity profile and the flux have been determined by the application of Laplace integral transform. Some particular cases have also been discussed in detail.

Keywords: *Viscosity, Unsteady flow, Visco-elastic fluid, Flux.*

INTRODUCTION

Ghosh (1968) discussed the viscous fluid through a long rectangular channel under the influence of an impulsive pressure gradient acting at the central part of a section. Roy, Sen and Lahiri (1990) investigated the problem of unsteady flow of Rivlin-Ericksen visco-elastic fluid through a rectangular duct due to an impulsive pressure gradient acting at the central part of a section. Many researchers such as: Chauhan and Lal (2001); Sengupta and Kundu (2001); Sengupta and Mukherjee (2001); Sengupta and Basak (2002); Sengupta and Paul (2004); Rahman and Alam Sarkar (2004); Sengupta and Banerjee (2005); Singh and Agarwal (2006); Chauhan and Yadav (2007) and Johri and Ashutosh (2007) etc. have studied the magnetohydrodynamic flow of visco-elastic fluids through channels of various type of cross-section.Alle *et. al* (2011) have studied unsteady flow of a dusty visco-elastic fluid through a inclined channel. Banyal (2012) studied a characterization of Rivlin-Ericksen visco-elastic fluid in the presence of a magnetic field and rotation. Arya, Kumar and Singh (2013) have discussed the unsteady flow of visco-elastic Rivlin-Ericksen fluid through porous medium in a long uniform rectangular channel.Venkateswarlu and Narayana (2015) studied MHD visco-elastic fluid flow over a continuously moving vertical surface with chemical reaction.

The aim of the present paper is to discuss the unsteady flow of a visco-elastic Rivlin-Ericksen type fluid through a long uniform rectangular duct when an impulsive pressure gradient acting at the central part of a section. The analytical expressions for velocity and the flux have been determined by the application of Laplace integral transform. Some particular cases have also been discussed in detail.

BASIC THEORY

For slow motion, the rheological equations for Rivlin-Ericksen visco-elastic fluid are:

$$\tau_{ij} = -p\delta_{ij} + \tau_{ij}' \tag{1}$$

$$\tau_{ij}' = 2\mu\left(1 + \mu_1 \frac{\partial}{\partial t}\right) e_{ij} \tag{2}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

where τ_{ij} is the stresss tensor,$\tau_{ij}'$the deviatoric stress tensor, e_{ij} the rate of strain tensor,μ the coefficient of viscosity, μ_1 the coefficient of visco-elasticity, p the fluid pressure, u_i the velocity components of the fluid and δ_{ij} is the kronecker delta.

The fundamental Navier-stokes equation of motion is

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu^*\nabla\vec{q} + \vec{F} \tag{4}$$

where

$$\tau_{ij}' = 2\mu^* e_{ij}$$

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i.e. $\qquad \mu^* = \mu\left(1 + \mu_1 \frac{\partial}{\partial t}\right)$ and $\nu^* = \frac{\mu^*}{\rho}$

Thus the Navier – Stokes equation of motion for Rivlin – Ericksen fluid is

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu\left(1 + \mu_1 \frac{\partial}{\partial t}\right)\nabla^2\vec{q} + \vec{F} \tag{5}$$

Where $\vec{q}$ is the velocity vector, $\vec{F}$ the body force vector, t the time, ρ the density.

FORMULATION OF THE PROBLEM

Considering the rectangular Cartesian coordinate system (x, y, z), z - axis is taken towards the direction of motion of the fluid. Let u, v, w be the components of the velocity of fluid along positive direction of x-axis, y-axis and z-axis respectively. Since duct is long and motion of the fluid is in the direction of z – axis only, therefore u = 0, v = 0 and w = W (x, y, t).

The equation of motion of Rivlin-Ericksen visco-elastic type fluid through rectangular duct is:

$$\frac{\partial W}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(1 + \mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) \tag{6}$$

Introducing the following non-dimensional quantities:

$$x' = \frac{x}{a},\quad y' = \frac{y}{a},\quad z' = \frac{z}{a},\quad W' = \frac{a}{\nu}W,$$
$$t' = \frac{\nu}{a^2}t,\quad p' = \frac{a^2}{\rho\nu^2}p,\quad \mu_1' = \frac{\nu}{a^2}\mu_1$$

In equation (6), there is found (after dropping the dashes):

$$\frac{\partial W}{\partial t} = -\frac{\partial p}{\partial z} + \left(1 + \mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) \tag{7}$$

Let $\qquad -\frac{\partial p}{\partial z} = f(x,y)\delta t$

δt is the Dirac delta function, be an impulsive pressure gradient acting at the central part of a section such that

$$\left.\begin{aligned} f(x,y) &= p \quad \text{for}\begin{cases}\frac{1}{2} - l_1 \le x \le \frac{1}{2} + l_1\\ \frac{c}{2} - l_2 \le y \le \frac{c}{2} + l_2\end{cases}\\ &= 0 \quad \text{for}\begin{cases}0 \le x < \frac{1}{2}, \frac{1}{2} + l_1 < x \le 1\\ 0 \le y < \frac{c}{2}, \frac{c}{2} + l_2 < y \le c\end{cases}\\ &= 0 \quad \text{for} \quad t \le 0\end{aligned}\right\} \tag{8}$$

Where c (= b/a) < 1, a non-dimensional quantity.

To find the solution of equation (7), it is defined as

$$f(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \tag{9}$$

Multiplying both sides of equation (9) by $\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c}$ and then integrating, it is found

$$B_{mn}\int_0^c\int_0^1 \sin^2\frac{m\pi x}{1}\sin^2\frac{n\pi y}{c}\,dxdy = P\int_{\frac{c}{2}-l_2}^{\frac{c}{2}+l_2}\int_{\frac{1}{2}-l_1}^{\frac{1}{2}+l_1} \sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c}\,dxdy$$

$$\Rightarrow B_{mn} = \frac{16P}{mn\pi^2}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c} \tag{10}$$

Now let it be assumed that the velocity of fluid be

$$W = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}(t)\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \tag{11}$$

Substituting equations (9) and (11) in equation (7), there is obtained

$$\begin{aligned}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\partial}{\partial t}\{A_{mn}(t)\}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c}\\ &+ \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}(t)\left\{\left(-\frac{m^2\pi^2}{1}\right) + \left(-\frac{n^2\pi^2}{c^2}\right)\right\}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c}\\ &+ \mu_1\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\partial}{\partial t}\{A_{mn}(t)\}\left\{\left(-\frac{m^2\pi^2}{1}\right) + \left(-\frac{n^2\pi^2}{c^2}\right)\right\}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c}\end{aligned} \tag{12}$$

Taking Laplace transform of equation (12), there is obtained

$$s\,\bar{A}_{mn}(s) = \frac{B_{mn}}{s} - \bar{A}_{mn}(s)C_{mn} - \mu_1 s\bar{A}_{mn}(s)C_{mn}$$

or $$\{(1+\mu_1 C_{mn})s + C_{mn}\}\bar{A}_{mn}(s) = \frac{B_{mn}}{s}$$

or $$\bar{A}_{mn}(s) = \frac{B_{mn}}{s\{(1+\mu_1 C_{mn})s + C_{mn}\}}$$

or $$\bar{A}_{mn}(s) = \frac{B_{mn}}{(1+\mu_1 C_{mn})}\frac{1}{s\left\{s+\left(\frac{C_{mn}}{1+\mu_1 C_{mn}}\right)\right\}}$$

or $$\bar{A}_{mn}(s) = \frac{B_{mn}}{C_{mn}}\left[\frac{1}{s} - \frac{1}{\left\{s+\left(\frac{C_{mn}}{1+\mu_1 C_{mn}}\right)\right\}}\right] \tag{13}$$

where $\bar{A}_{mn}(s) = \int_0^\infty A_{mn}(t)e^{-st}dt$ (by definition of Laplace transform)

and $$C_{mn} = \frac{m^2\pi^2}{1} + \frac{n^2\pi^2}{c^2} = \left(m^2 + \frac{n^2}{c^2}\right)\pi^2$$

Now applying inverse Laplace transform of equation (13)

$$L^{-1}\{\bar{A}_{mn}(s)\} = A_{mn}(t)$$
$$= \frac{B_{mn}}{C_{mn}}\left[L^{-1}\left(\frac{1}{s}\right) - L^{-1}\frac{1}{\left\{s+\left(\frac{C_{mn}}{1+\mu_1 C_{mn}}\right)\right\}}\right]$$
$$= \frac{B_{mn}}{C_{mn}}\left[1 - e^{-\left(\frac{C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right] \tag{14}$$

From equations (11) and (14) with the help of equation (10), velocity of Rivlin – Ericksen visco – elastic fluid is obtained

$$W = \frac{16P}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\left[1-e^{-\left(\frac{C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right]}{mn\,C_{mn}}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \tag{15}$$

The flux Q across any section at any instant is given by

$$Q = \int_0^1\int_0^c W\,dx\,dy$$
$$= \frac{64cP}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\left[1-e^{-\left(\frac{C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right]}{m^2n^2C_{mn}}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c} \tag{16}$$

when m and n both are odd numbers.

PARTICULAR CASES

Case-I: If coefficient of visco-elasticity becomes zero i.e. $\mu_1 \to 0$

Then velocity of viscous fluid is given from equation (15)

$$W = \frac{16P}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{[1-e^{-C_{mn}t}]}{mn\,C_{mn}}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \tag{17}$$

and from equation (16) flux is

$$Q = \frac{64cP}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{[1-e^{-C_{mn}t}]}{m^2n^2C_{mn}}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c} \tag{18}$$

Case-II: If $\mu_1 \to 0$

Then velocity of purely viscous fluid is given from equation (15)

$$W = \frac{16P}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{[1-e^{-C_{mn}t}]}{mn\,C_{mn}}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \tag{19}$$

and flux from equation (16) is

$$Q = \frac{64cP}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{[1-e^{-C_{mn}t}]}{m^2n^2C_{mn}}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c} \tag{20}$$

which are the same results obtained by Ghosh (1968) with slight change of notations.

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