



FIXED POINT THEOREM FOR FIVE MAPPINGS IN METRIC SPACE

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(Received on: 23-07-11; Accepted on: 07-08-11)

ABSTRACT

In this paper we prove a common fixed point theorem taking five mappings satisfying a weakly commuting condition. Our result generalize the results of Fisher [2], Pathak [9], Rao and Rao [10], Saluja and Dehariya [12].

Key Words: Fixed point, Metric Space, Weakly commuting Mappings.

Subject Classification: 47H10, 54H25.

2. INTRODUCTION & PRELIMINARIES:

There are many generalization of Banach Contraction Principle. Some of them are obtained by adding the terms Kannan[8], Fisher[2], Rhoades[11] generalized this principle in many ways. Jaggi [5], Jaggi and Das [6] extended this principle for rational expression. On the same way Bhardwaj et. al., Yadava et.al. also worked for rational expression. Initially Jungck [7] proved a common fixed point theorem for commuting mappings. This result was extended and generalized in various ways by many authors. On the way Sessa [13] gave the concept of weak commutativity.

In the present paper we prove a fixed point theorem for five mappings taking weakly commuting condition for certain mappings. Our result is motivated by Pathak[9], Rao and Rao[10], Saluja and Dehariya [12].

**Definition 2.1[3, 4]:** Let  $K$  be a non empty subset of a metric space  $(X, d)$ ,  $T : K \rightarrow X$  and  $F : K \rightarrow CB(X)$ .

The pair  $(F,T)$  is said to be weakly commuting if for every  $x, y \in K$  with  $x \in Fy$  and  $Ty \in K$ , we have

$$d(Tx, FTy) = d(Ty, Fy)$$

**Definition 2.2[4]:** Let  $K$  be a non empty subset of a metric space  $(X, d)$ ,  $T : K \rightarrow X$  and  $F : K \rightarrow CB(X)$ . The pair  $(F,T)$  is said to be point-wise  $R$ -weakly commuting on  $K$  if for given  $x \in K$  and  $Tx \in K$ , there exists some  $R=R(x)>0$  such that

$$d(Ty, FTx) \leq R d(Tx, Fx) \quad \text{for each } y \in K \cap Fx.$$

**Definition 2.3:** Let  $R, S, T, U$  and  $V$  are five mappings of a metric space  $(X, d)$ . Then the pair  $(R, S, T, U)$  is called weakly commuting pair of two mappings with respect to  $V$  if

$$(2.3.1) \quad d(SVRSV(x), TUV(x)) \leq d(RSVSV(x), TUV(x))$$

$$(2.3.2) \quad d(RSV(x), UVTV(x)) \leq d(RSV(x), TUVUV(x))$$

$$(2.3.3) \quad d(VRSV(x), TUV(x)) \leq d(RSVV(x), TUVV(x))$$

$$(2.3.4) \quad d(RSV(x), VTUV(x)) \leq d(RSV(x), TUVV(x))$$

3. MAIN RESULT:

**Theorem 3.1:** Let  $R, S, T, U$  and  $V$  are five self mappings of complete metric space  $(X, d)$  satisfying the conditions:

(A) The pair  $(R, S, T, U)$  is weakly commuting pair of two mappings with respect to  $V$ .

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$$(B) \quad d(RSV(x), TUV(y))^2 \leq \alpha \frac{d(x, TUV(y))d(y, RSV(x))}{1 + d(x, TUV(y))d(y, RSV(x)) + d(x, TUV(y))d(x, y)} \\ + \beta d(y, TUV(y))d(x, RSV(x)) + \gamma d(y, TUV(y))d(y, RSV(x)) \\ + \delta d(RSV(x), TUV(y))d(x, y) + \eta d(x, y)^2$$

for all  $x, y \in X, x \neq y$ , where  $\alpha, \beta, \gamma, \delta, \eta \geq 0$  and  $\alpha + \beta + 2\gamma + \delta + \eta < 1$ .

Then R, S, T, U and V have a unique common fixed point in X.

**Proof:** Let  $x_0$  be an arbitrary point of X. We define a sequence

$$\{x_n\} \text{ by } x = x_{2n}, y = x_{2n-1} \text{ and } RSV(x_{2n}) = x_{2n+1}, TUV(x_{2n-1}) = x_{2n}$$

$$d(x_{2n}, x_{2n+1})^2 = d(RSV(x_{2n}), TUV(x_{2n-1}))^2 \\ \leq \alpha \frac{d(x_{2n}, TUV(x_{2n-1}))d(x_{2n-1}, RSV(x_{2n}))}{1 + d(x_{2n}, TUV(x_{2n-1}))d(x_{2n-1}, RSV(x_{2n})) + d(x_{2n}, TUV(x_{2n-1}))d(x_{2n}, x_{2n-1})} \\ + \beta d(x_{2n}, RSV(x_{2n}))d(x_{2n-1}, TUV(x_{2n-1})) + \gamma d(x_{2n-1}, TUV(x_{2n-1}))d(x_{2n-1}, RSV(x_{2n})) \\ + \delta d(RSV(x_{2n}), TUV(x_{2n-1}))d(x_{2n-1}, x_{2n}) + \eta d(x_{2n-1}, x_{2n})^2 \\ \leq \alpha \frac{d(x_{2n}, x_{2n})d(x_{2n-1}, x_{2n+1})}{1 + d(x_{2n}, x_{2n})d(x_{2n-1}, x_{2n+1}) + d(x_{2n}, x_{2n})d(x_{2n}, x_{2n-1})} \\ + \beta d(x_{2n}, x_{2n+1})d(x_{2n-1}, x_{2n}) + \gamma d(x_{2n-1}, x_{2n})d(x_{2n-1}, x_{2n+1}) \\ + \delta d(x_{2n+1}, x_{2n})d(x_{2n-1}, x_{2n}) + \eta d(x_{2n-1}, x_{2n})^2$$

By using inequalities, we obtain

$$d(x_{2n}, x_{2n+1})^2 \leq \beta d(x_{2n}, x_{2n+1})d(x_{2n-1}, x_{2n}) + \gamma d(x_{2n-1}, x_{2n})[d(x_{2n-1}, x_{2n}) + d(x_{2n}, x_{2n+1})] \\ + \delta d(x_{2n+1}, x_{2n})d(x_{2n-1}, x_{2n}) + \eta d(x_{2n-1}, x_{2n})^2 \\ \leq \beta d(x_{2n}, x_{2n+1})d(x_{2n-1}, x_{2n}) + \gamma d(x_{2n-1}, x_{2n})^2 + \gamma d(x_{2n-1}, x_{2n})d(x_{2n}, x_{2n+1}) \\ + \delta d(x_{2n+1}, x_{2n})d(x_{2n-1}, x_{2n}) + \eta d(x_{2n-1}, x_{2n})^2 \\ \leq (\beta + \gamma + \delta)d(x_{2n}, x_{2n+1})d(x_{2n-1}, x_{2n}) + (\gamma + \eta)d(x_{2n-1}, x_{2n})^2$$

Since  $AM \geq GM$ , then

$$d(x_{2n}, x_{2n+1})^2 \leq \frac{(\beta + \gamma + \delta)}{2} [d(x_{2n}, x_{2n+1})^2 + d(x_{2n-1}, x_{2n})^2] + (\gamma + \eta)d(x_{2n-1}, x_{2n})^2$$

$$d(x_{2n}, x_{2n+1})^2 \leq \left\{ \frac{\eta + \left(\frac{\beta + 3\gamma + \delta}{2}\right)}{1 - \left(\frac{\beta + \gamma + \delta}{2}\right)} \right\} d(x_{2n-1}, x_{2n})^2$$

$$d(x_{2n}, x_{2n+1})^2 \leq k d(x_{2n-1}, x_{2n})^2$$

$$\text{where } k = \left\{ \frac{\eta + \left(\frac{\beta + 3\gamma + \delta}{2}\right)}{1 - \left(\frac{\beta + \gamma + \delta}{2}\right)} \right\} < 1, \text{ since } \alpha + \beta + 2\gamma + \delta + \eta < 1.$$

$$(3.1.1) \quad d(x_{2n}, x_{2n+1}) \leq k^{\frac{1}{2}} d(x_{2n-1}, x_{2n})$$

Similarly on taking  $x = x_{2n+1}$ ,  $y = x_{2n}$  and  $RSV(x_{2n+1}) = x_{2n+2}$ ,  $TUV(x_{2n}) = x_{2n+1}$

in condition (3.1) (B), we obtain

$$\begin{aligned}
 d(x_{2n+1}, x_{2n+2})^2 &= d(RSV(x_{2n+1}), TUV(x_{2n}))^2 \\
 &\leq \alpha \frac{d(x_{2n+1}, TUV(x_{2n}))d(x_{2n}, RSV(x_{2n+1}))}{1 + d(x_{2n+1}, TUV(x_{2n}))d(x_{2n}, RSV(x_{2n+1})) + d(x_{2n+1}, TUV(x_{2n}))d(x_{2n}, x_{2n+1})} \\
 &\quad + \beta d(x_{2n+1}, RSV(x_{2n+1}))d(x_{2n}, TUV(x_{2n})) + \gamma d(x_{2n}, TUV(x_{2n}))d(x_{2n}, RSV(x_{2n+1})) \\
 &\quad + \delta d(RSV(x_{2n+1}), TUV(x_{2n}))d(x_{2n+1}, x_{2n}) + \eta d(x_{2n+1}, x_{2n})^2 \\
 &\leq \alpha \frac{d(x_{2n+1}, x_{2n+1})d(x_{2n}, x_{2n+2})}{1 + d(x_{2n+1}, x_{2n+1})d(x_{2n}, x_{2n+2}) + d(x_{2n+1}, x_{2n+1})d(x_{2n}, x_{2n+1})} \\
 &\quad + \beta d(x_{2n+1}, x_{2n+2})d(x_{2n}, x_{2n+1}) + \gamma d(x_{2n}, x_{2n+1})d(x_{2n}, x_{2n+2}) \\
 &\quad + \delta d(x_{2n+2}, x_{2n+1})d(x_{2n}, x_{2n+1}) + \eta d(x_{2n}, x_{2n+1})^2 \\
 &\leq \beta d(x_{2n+1}, x_{2n+2})d(x_{2n}, x_{2n+1}) + \gamma d(x_{2n}, x_{2n+1})[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})] \\
 &\quad + \delta d(x_{2n+2}, x_{2n+1})d(x_{2n}, x_{2n+1}) + \eta d(x_{2n}, x_{2n+1})^2 \\
 &\leq (\beta + \gamma + \delta)d(x_{2n+2}, x_{2n+1})d(x_{2n+1}, x_{2n}) + (\gamma + \eta)d(x_{2n+1}, x_{2n})^2 \\
 &\leq \left(\frac{\beta + \gamma + \delta}{2}\right)[d(x_{2n+2}, x_{2n+1})^2 + d(x_{2n+1}, x_{2n})^2] + (\gamma + \eta)d(x_{2n+1}, x_{2n})^2 \\
 d(x_{2n+2}, x_{2n+1})^2 &\leq \left\{ \frac{\eta + \left(\frac{\beta + 3\gamma + \delta}{2}\right)}{1 - \left(\frac{\beta + \gamma + \delta}{2}\right)} \right\} d(x_{2n}, x_{2n+1})^2
 \end{aligned}$$

$$\begin{aligned}
 d(x_{2n+2}, x_{2n+1})^2 &\leq k d(x_{2n}, x_{2n+1})^2 \\
 (3.1.2) \quad d(x_{2n+2}, x_{2n+1}) &\leq k^{\frac{1}{2}} d(x_{2n}, x_{2n+1})
 \end{aligned}$$

where  $k = \left\{ \frac{\eta + \left(\frac{\beta + 3\gamma + \delta}{2}\right)}{1 - \left(\frac{\beta + \gamma + \delta}{2}\right)} \right\} < 1$ , since  $\alpha + \beta + 2\gamma + \delta + \eta < 1$ .

Hence by (3.1.1) and (3.1.2), we have

$$d(x_{2n+2}, x_{2n+1}) \leq k^{\frac{1}{2}} d(x_{2n}, x_{2n+1}) \leq \left(k^{\frac{1}{2}}\right)^2 d(x_{2n-1}, x_{2n})$$

In general,

$$d(x_{2n+2}, x_{2n+1}) \leq \left(k^{\frac{1}{2}}\right)^{2n+1} d(x_0, x_1) \rightarrow 0 \text{ as } n \rightarrow \infty.$$

Thus  $\{x_n\}$  is a D-Cauchy sequence in X. Since X is complete, there exists a point  $p \in X$ , such that  $\lim_{n \rightarrow \infty} x_n = p$ .

Now, first we shall prove that p is a common fixed point of RSV and TUV.

On taking  $x = p$ ,  $y = x_{2n+1}$  in condition (3.1B), we obtain

$$\begin{aligned}
 d(RSV(p), x_{2n+2})^2 &= d(RSV(p), TUV(x_{2n+1}))^2 \\
 &\leq \alpha \frac{d(p, TUV(x_{2n+1}))d(x_{2n+1}, RSV(p))}{1 + d(p, TUV(x_{2n+1}))d(x_{2n+1}, RSV(p)) + d(p, TUV(x_{2n+1}))d(p, x_{2n+1})} \\
 &\quad + \beta d(p, RSV(p))d(x_{2n+1}, TUV(x_{2n+1})) + \gamma d(x_{2n+1}, TUV(x_{2n+1}))d(x_{2n+1}, RSV(p)) \\
 &\quad + \delta d(RSV(p), TUV(x_{2n+1}))d(p, x_{2n+1}) + \eta d(p, x_{2n+1})^2 \\
 &\leq \alpha \frac{d(p, x_{2n+2})d(x_{2n+1}, RSV(p))}{1 + d(p, x_{2n+2})d(x_{2n+1}, RSV(p)) + d(p, x_{2n+2})d(p, x_{2n+1})} \\
 &\quad + \beta d(p, RSV(p))d(x_{2n+1}, x_{2n+2}) + \gamma d(x_{2n+1}, x_{2n+2})d(x_{2n+1}, RSV(p)) \\
 &\quad + \delta d(RSV(p), x_{2n+2})d(x_{2n+1}, p) + \eta d(x_{2n+1}, p)^2
 \end{aligned}$$

making limit  $n \rightarrow \infty$ , we have

$$\begin{aligned}
 d(RSV(p), p)^2 &\leq \alpha \frac{d(p, p)d(p, RSV(p))}{1 + d(p, p)d(p, RSV(p)) + d(p, p)d(p, p)} \\
 &\quad + \beta d(p, RSV(p))d(p, p) + \gamma d(p, p)d(p, RSV(p)) \\
 &\quad + \delta d(RSV(p), p)d(p, p) + \eta d(p, p)^2
 \end{aligned}$$

$$d(RSV(p), p)^2 \leq 0$$

$$\Rightarrow d(RSV(p), p) \leq 0$$

which is possible only when  $RSV(p) = p$ .

Similarly  $TUV(p) = p$ .

Now, we shall prove that  $p$  is the common fixed point of  $RSV$  and  $TUV$ . Let  $q$  be another common fixed Point of  $RSV$  and  $TUV$ , then by condition (3.1)(B), we have

$$\begin{aligned}
 d(p, q)^2 &= d(RSV(p), TUV(q))^2 \\
 &\leq \alpha \frac{d(p, TUV(q))d(q, RSV(p))}{1 + d(p, TUV(q))d(q, RSV(p)) + d(p, TUV(q))d(p, q)} \\
 &\quad + \beta d(p, RSV(p))d(q, TUV(q)) + \gamma d(q, TUV(q))d(q, RSV(p)) \\
 &\quad + \delta d(RSV(p), TUV(q))d(p, q) + \eta d(p, q)^2 \\
 &\leq \alpha \frac{d(p, q)d(q, p)}{1 + d(p, q)d(q, p) + d(p, q)d(q, p)} \\
 &\quad + \beta d(p, p)d(q, q) + \gamma d(q, q)d(q, p) \\
 &\quad + \delta d(p, q)d(p, q) + \eta d(p, q)^2
 \end{aligned}$$

$$d(p, q)^2 \leq (\alpha + \delta + \eta)d(p, q)^2$$

which is possible only when  $p = q$ , since  $\alpha + \beta + 2\gamma + \delta + \eta < 1$ .

Now, we shall show that  $SV$  and  $UV$  have a common fixed point.

On taking  $x=SV(p)$ ,  $y=p$  in condition (3.1)(B) and applying definition (2.3.1), we have

$$\begin{aligned}
 d(SV(p), p)^2 &= d(SVRSV(p), TUV(p))^2 \leq d(RSVSV(p), TUV(p))^2 \\
 &\leq \alpha \frac{d(SV(p), TUV(p))d(p, RSVSV(p))}{1 + d(SV(p), TUV(p))d(p, RSVSV(p)) + d(SV(p), TUV(p))d(SV(p), p)} \\
 &\quad + \beta d(SV(p), RSVSV(p))d(p, TUV(p)) + \gamma d(p, TUV(p))d(p, RSVSV(p)) \\
 &\quad + \delta d(RSVSV(p), TUV(p))d(SV(p), p) + \eta d(SV(p), p)^2 \\
 &\leq \alpha \frac{d(SV(p), p)d(p, SV(p))}{1 + d(SV(p), p)d(p, SV(p)) + d(SV(p), p)d(SV(p), p)} \\
 &\quad + \beta d(SV(p), SV(p))d(p, p) + \gamma d(p, p)d(p, SV(p)) \\
 &\quad + \delta d(SV(p), p)d(SV(p), p) + \eta d(SV(p), p)^2 \\
 &\leq \alpha \frac{d(SV(p), p)^2}{1 + 2d(SV(p), p)^2} + \delta d(SV(p), p)^2 + \eta d(SV(p), p)^2 \\
 &\leq (\alpha + \delta + \eta)d(SV(p), p)^2
 \end{aligned}$$

which is possible only when  $p = SV(p)$ , since  $\alpha + \beta + 2\gamma + \delta + \eta < 1$ .

Similarly, on taking  $x=p, y=UV(p)$ , and applying definition (2.3.2), we have

$$\begin{aligned}
 d(p, UV(p))^2 &= d(RSV(p), UVTUV(p))^2 \leq d(RSV(p), TUVUV(p))^2 \\
 &\leq \alpha \frac{d(p, TUVUV(p))d(UV(p), RSV(p))}{1 + d(p, TUVUV(p))d(UV(p), RSV(p)) + d(p, TUVUV(p))d(p, UV(p))} \\
 &\quad + \beta d(p, RSV(p))d(UV(p), TUVUV(p)) \\
 &\quad + \gamma d(UV(p), TUVUV(p))d(UV(p), RSV(p)) \\
 &\quad + \delta d(RSV(p), TUVUV(p))d(p, UV(p)) + \eta d(p, UV(p))^2 \\
 &\leq \alpha \frac{d(UV(p), p)d(p, UV(p))}{1 + d(UV(p), p)d(p, UV(p)) + d(UV(p), p)d(UV(p), p)} \\
 &\quad + \beta d(p, p)d(UV(p), UV(p)) + \gamma d(UV(p), UV(p))d(UV(p), p) \\
 &\quad + \delta d(UV(p), p)d(UV(p), p) + \eta d(UV(p), p)^2 \\
 &\leq \alpha \frac{d(UV(p), p)^2}{1 + 2d(UV(p), p)^2} + \delta d(UV(p), p)^2 + \eta d(UV(p), p)^2 \\
 &\leq (\alpha + \delta + \eta)d(UV(p), p)^2
 \end{aligned}$$

which is possible only when  $p = UV(p)$ , since  $\alpha + \beta + 2\gamma + \delta + \eta < 1$ .

Thus

$$(3.1.3) \quad SV(p) = UV(p) = p.$$

Finally, we shall prove that  $p$  is a common fixed point of  $R, S, T, U$  and  $V$ .

On taking  $x=V(p), y=p$  and applying definition (2.3.3), we have

$$\begin{aligned}
 d(V(p), p)^2 &= d(VRSV(p), TUV(p))^2 \leq d(RSVV(p), TUVV(p))^2 \\
 &\leq \alpha \frac{d(V(p), TUVV(p))d(p, RSVV(p))}{1 + d(V(p), TUVV(p))d(p, RSVV(p)) + d(V(p), TUVV(p))d(V(p), p)} \\
 &\quad + \beta d(V(p), RSVV(p))d(p, TUVV(p)) + \gamma d(p, TUVV(p))d(p, RSVV(p)) \\
 &\quad + \delta d(RSVV(p), TUVV(p))d(V(p), p) + \eta d(V(p), p)^2 \\
 &\leq \alpha \frac{d(V(p), p)d(p, V(p))}{1 + d(V(p), p)d(p, V(p)) + d(V(p), p)d(V(p), p)} \\
 &\quad + \beta d(p, p)d(V(p), V(p)) + \gamma d(p, p)d(V(p), p) \\
 &\quad + \delta d(V(p), p)d(V(p), p) + \eta d(V(p), p)^2 \\
 &\leq \alpha \frac{d(V(p), p)^2}{1 + 2d(V(p), p)} + \delta d(V(p), p)^2 + \eta d(V(p), p)^2 \\
 &\leq (\alpha + \delta + \eta) d(V(p), p)^2
 \end{aligned}$$

which is possible only when  $p = V(p)$ , since  $\alpha + \beta + 2\gamma + \delta + \eta < 1$ .

Similarly, on taking  $x=p, y=U(p)$  and applying definition (2.3.4), we have  $p = U(p)$ .

Thus

$$(3.1.4) \quad U(p) = V(p) = p.$$

Hence by (3.1.3) and (3.1.4), we have

$$SV(p) = UV(p) = U(p) = V(p) = S(p) = p.$$

Also by the uniqueness of  $p$ ,

$$RSV(p) = TUV(p) = R(p) = T(p) = p.$$

Therefore,  $R(p) = S(p) = T(p) = U(p) = V(p) = p$ .

Hence  $p$  is a common fixed point of  $R, S, T, U$  and  $V$ .

**Uniqueness:** Let  $w$  be another common fixed point of  $R, S, T, U$  and  $V$ . Then by condition (3.1) (B), we have

$$\begin{aligned}
 d(p, w)^2 &= d(RSV(p), TUV(w))^2 \\
 &\leq \alpha \frac{d(p, TUV(w))d(w, RSV(p))}{1 + d(p, TUV(w))d(w, RSV(p)) + d(p, TUV(w))d(p, w)} \\
 &\quad + \beta d(p, RSV(p))d(w, TUV(w)) + \gamma d(w, TUV(w))d(w, RSV(p)) \\
 &\quad + \delta d(RSV(p), TUV(w))d(p, w) + \eta d(p, w)^2 \\
 &\leq \alpha \frac{d(p, w)d(w, p)}{1 + d(p, w)d(w, p) + d(p, w)d(w, p)} \\
 &\quad + \beta d(p, p)d(w, w) + \gamma d(w, w)d(w, p) \\
 &\quad + \delta d(p, w)d(p, w) + \eta d(p, w)^2
 \end{aligned}$$

$$\begin{aligned} &\leq \alpha \frac{d(p, w)^2}{1 + 2d(p, w)^2} + \delta d(p, w)^2 + \eta d(p, w)^2 \\ &\leq (\alpha + \delta + \eta) d(p, w)^2 \end{aligned}$$

which is possible only when  $p = w$ , since  $\alpha + \beta + 2\gamma + \delta + \eta < 1$ .

This completes the proof of the theorem.

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