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THERMAL DIFFUSION EFFECT OF FREE CONVECTION MASS TRANSFER FLOW PAST A UNIFORMLY ACCELERATED POROUS PLATE WITH HEAT SINK

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ABSTRACT

This paper deals with the combined effect of thermal diffusion and heat absorption on the free convection mass transfer flow of a viscous incompressible fluid past a continuously moving infinite porous plate. Closed form of solution for the velocity field, the temperature field and the concentration field are obtained and discussed graphically for various values of the physical parameters present. In addition, expressions for the viscous drag, Nusselt number and Sherwood number are also derived and finally discussed with the help of tables and graphs.

Keywords: *Free convection, Mass transfer, Heat Sink, Thermal diffusion.*

AMS Subject Classification: *76R10, 80A20.*

Short Title: *Thermal Diffusion effect on free convection mass transfer flow.*

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1. INTRODUCTION:

In heat transfer processes, the temperature distribution in a medium is controlled by three distinct modes of heat transfer. Of them convection is a mode of heat transfer that occurs between two mediums which are in relative motion and if the relative motion is set up by buoyancy effects, resulting from density differences, caused by temperature differences in the medium, the convection heat transfer will be called a free or natural convection heat transfer. Extensive studies have been conducted on this type of heat transfer flow by various authors of them Soundalgekar[11], Martynenko et al.[6], Perdikis and Takhar [7], are worth mentioning. Like heat transfer, mass transfer considerations are also very important in modern engineering designs particularly in Chemical engineering. The theory of mass transfer considered as the mass transition occur due to a species concentration difference in a mixture and is used in many areas like the working system in home humidifier, evaporation of liquid refrigerant etc, biological applications like oxygenation of blood, food and drug assimilation, respiration mechanism and in many more fields. Researchers like Raptis and Kafoussias [9], Rahman and Sattar [8], Yih [12] are some of the notabale contributers in combined heat and mass transfer phenomena.

In the above mentioned studies the effect of heat absorption and thermal diffusion effects got little attention by the researchers. Chamkha[3] studied the effect of heat generation/absorption in MHD flow of uniformly stretched vertical permeable surface in presence of chemical reaction. It is found that a mass flux can be generated by temperature gradient also, known as Soret effect or thermal-diffusion effect. The Soret effect for instance has been utilized for isotope separation and in mixtures between gases with very light molecular weight like (H_2, He). In particular, researchers like Durusunkaya and Worek [4], Kafousias and Williams [5], Afify[1], investigated Soret effect on MHD flow. Recently, Renuka et al. [10] have studied the Soret effect on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with variable suction. Of late Ahmed and Sengupta [2] have investigated the Soret and Dufour effects in steady MHD three – dimensional flow past an infinite porous plate in presence of thermal radiation.

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The objective of the present study is to investigate the effects of thermal diffusion and heat absorption on two-dimensional free convection mass transfer flow past a uniformly accelerated infinite vertical porous plate, where the effects of thermal radiation and chemical reactions are neglected.

2. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM:

We consider a steady laminar flow of a viscous incompressible fluid past an infinite vertical porous flat plate. Introduce a co-ordinate system ($\bar{x}$, $\bar{y}$) with $\bar{x}$ -axis along the length of the plate in the upward vertical direction and $\bar{y}$ -axis normal to the plate towards the fluid region. The plate is subjected to a constant suction parallel to $\bar{y}$ -axis. The viscous dissipations of energy are assumed to be negligible for the study. Since, the plate is infinite in length all the fluid property except possibly the pressure remain constant along the $\bar{x}$ -direction. The fluid property variation with temperature is limited to density variation only and the influence of variation of density with temperature is restricted to the body force term only, in accordance with the Boussinesq approximation. Under boundary layer and Boussinesq approximations, the equations governing the steady laminar two-dimensional free convective flow with medium concentration in presence of Soret and heat absorption effects reduce to:

Continuity Equation:

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

Momentum Equation:

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(\bar{T} - \bar{T}_\infty) + g\beta^*(\bar{C} - \bar{C}_\infty) \tag{2}$$

Energy Equation:

$$\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho c_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{Q_0}{\rho c_p}(\bar{T}_\infty - \bar{T}) \tag{3}$$

Species Continuity Equation:

$$\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} = D_M \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + D_T \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \tag{4}$$

The relevant boundary conditions are:

$$\bar{y}=0:\ \bar{u} = \bar{U},\ \bar{v} = -V_0,\ (V_0>0), \bar{T} = \bar{T}_w,\ \bar{C} = \bar{C}_w$$

$$\bar{y} \to \infty:\ \bar{u} \to 0,\ \bar{v} \to -V_0, \bar{T} \to \bar{T}_\infty,\ \bar{C} \to \bar{C}_\infty$$

We introduce the following non-dimensional quantities as:

$$y = \frac{V_0 \bar{y}}{\nu},\ u = \frac{\bar{u}}{V_0},\ U_0 = \frac{\bar{U}}{V_0},\ v = \frac{\bar{v}}{V_0},$$

$$\theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}, \Pr = \frac{\nu c_p}{k}, Sc = \frac{\nu}{D_M},$$

$$Sr = \frac{D_T(\bar{T}_w - \bar{T}_\infty)}{\nu(\bar{c}_w - \bar{c}_\infty)}, G_r = \frac{g\beta\nu(\bar{T}_w - \bar{T}_\infty)}{V_0^3}\ G_m = \frac{g\beta^*\nu(\bar{C}_w - \bar{C}_\infty)}{V_0^3},\ Q = \frac{Q_0 \nu^2}{k V_0^2}, (Q_0 > 0)$$

The non-dimensional forms of equations are:

$$\frac{dv}{dy} = 0 \tag{5}$$

$$\frac{d^2u}{dy^2} + \frac{du}{dy} = -G_r\theta - G_m\phi \tag{6}$$

$$\frac{d^2\theta}{dy^2} + \Pr\frac{d\theta}{dy} - Q\theta = 0 \tag{7}$$

$$\frac{d^2\phi}{dy^2} + Sc\frac{d\phi}{dy} = -ScSr\frac{d^2\theta}{dy^2} \tag{8}$$

The corresponding non-dimensional boundary conditions are:

y=0 : $u = U_0$, $v = -1$, $\theta = 1$, $\phi = 1$

$y \to \infty$: $u \to 0$, $v \to -1$, $\theta \to 0$, $\phi \to 0$.

The solutions of (6), (7), and (8) are,

$$u(y) = A_5 e^{-Scy} + A_7 e^{-A_1 y} - A_8 e^{-y} + U_0 e^{-y} \tag{9}$$

$$\theta(y) = e^{-A_1 y} \tag{10}$$

$$\phi(y) = A_3 e^{-Scy} + A_2 e^{-A_1 y} \tag{11}$$

Where, $A_1 = \dfrac{\Pr + \sqrt{\Pr^2 + 4Q}}{2}$,

$A_2 = \dfrac{ScSrA_1}{Sc - A_1}$, $(Sc \neq A_1)$,

$A_3 = 1 - A_2$, $A_4 = \dfrac{Gr}{A_1(1-A_1)}$, $(A_1 \neq 1)$,

$A_5 = \dfrac{GmA_3}{Sc(1-Sc)}$, $A_6 = \dfrac{GmA_2}{A_1(1-A_1)}$,

$(Sc \neq 0, Sc \neq 1)$,

$A_7 = A_4 + A_6$, $A_8 = A_5 + A_7$.

Skin Friction at the plate:

The non-dimensional skin friction at the plate is:

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = A_8 - A_1A_7 - A_5Sc - U_0 \tag{12}$$

Rate of Heat Transfer Co-efficient:

The non-dimensional heat transfer co-efficient at the plate is:

$$Nu = \frac{A_1}{\Pr} \tag{13}$$

Rate of Mass Transfer Co-efficient:

The non-dimensional mass transfer co-efficient at the plate is:

$$Sh = \frac{1}{Sc}(A_1 A_2 + A_3 Sc) \tag{14}$$

3. RESULTS AND DISCUSSION:

During the course of discussion of the effects of various parameters on the flow field, the following considerations are made:

(1) The value of the Prandtl number Pr are taken as .71 and 7, which corresponds physically to air and water.

(2) The value of the Schmidt number Sc is chosen at 0.22, 0.66, 0.96, which represents hydrogen, water vapour and carbon dioxide at 25^0C and 1 atm.

(3) The values of the Grashof number G_r, modified Grashof number G_m, plate velocity U_0 are taken to be fixed at 4, 1, 1 respectively.

(4) Finally the values of the Soret number Sr, heat sink parameter Q, are chosen arbitrarily. Under the above assumptions, results are shown in figures 1-3 and in tables 1-2.

Table 1 represents numerically how the skin-friction at the plate has been affected by the presence of Schmidt number Sc against soret number Sr. It reflects that the skin-friction τ increases (positively) as Sc increases upto 0.708 and as sc ≥ 0.709, τ decreases and becomes negative.

Table 2 represents numerically the effect of the parameter Sc on the rate of mass transfer co –efficient quantified by the Sherwood number Sh against Sr. It has seen that when Sc ≤ 0.708, Sh decreases and becomes negative and for Sc ≥ 0.709, Sh increases gradually against Sr.

The effect of Soret number Sr on the flow velocity u against y is shown in figure 1. It is seen that as the effect of mass buoyancy force near to the plate increases in presence of Sr, the flow velocity increases sharply due to increase in values of Sr and decreases steadily far away from the plate.

Figure 2 depicts the changes of the non-dimensional temperature Θ due to changes of values of heat absorption parameter Q. As the thickness of the thermal boundary layer gets reduce by the heat absorption parameter, the value of non-dimensional temperature Θ is thus reducing by the heat absorption effect.

The effect of the Soret parameter Sr on the species concentration ϕ is shown graphically in figure 3. Due to Soret effect the thickness of the concentration boundary layer raises, thereby increases the value of ϕ .

Table 1. Numerical values of non-dimensional skin-friction τ for different values of Schmidt number Sc against arbitrary values of soret number Sr and for fixed values of Pr =0.71, G_r=4.0, G_m=1.0, U_0=1.0, Q = 1.0

Sr	Sc= 0.22	Sc=0.708	Sc=0.709
0.0	6.370	3.237	3.235
0.5	6.983	2474.917	-187.190
1.0	7.595	4946.597	-377.615

1.5	8.208	7418.277	-568.04
2.0	8.821	9889.957	-758.466
2.5	9.434	12361.637	-948.891
3.0	10.046	14833.317	-1139.316
3.5	10.659	17304.997	-1329.741
4.0	11.272	19776.677	-1520.166
4.5	11.884	22248.357	-1710.591
5.0	12.497	24720.037	-1901.017

Table 2. Numerical values of non-dimensional rate of mass transfer coefficient Sh for different values of Schmidt number Sc against arbitrary values of Soret number Sr and for fixed values of Pr =0.71, G_r=4.0, G_m=1.0, U_0=1.0, Q = 1.0.

Sr	Sc= 0.22	Sc=0.708	Sc=0.709
0.0	1.000	1.000	1.000
0.5	0.132	-3499.253	270.669
1.0	-0.735	-6999.506	540.339
1.5	-1.603	-10499.759	810.008
2.0	-2.471	-14000.012	1079.677
2.5	-3.338	-17500.265	1349.346
3.0	-4.206	-21000.517	1619.016
3.5	-5.074	-24500.770	1888.685
4.0	-5.941	-28001.023	2158.354
4.5	-6.809	-31501.276	2428.024
5.0	-7.677	-35001.529	2697.693

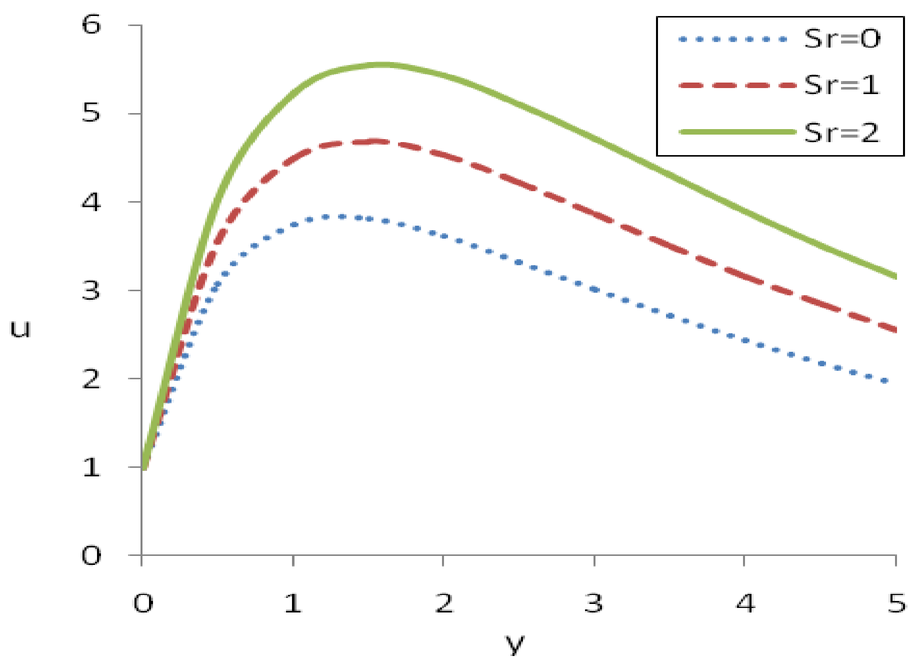

Figure 1: Velocity u versus y for Pr =0.71, Sc= 0.22, G_r =4.0, G_m =1.0, U_0=1.0, Q = 1.0.

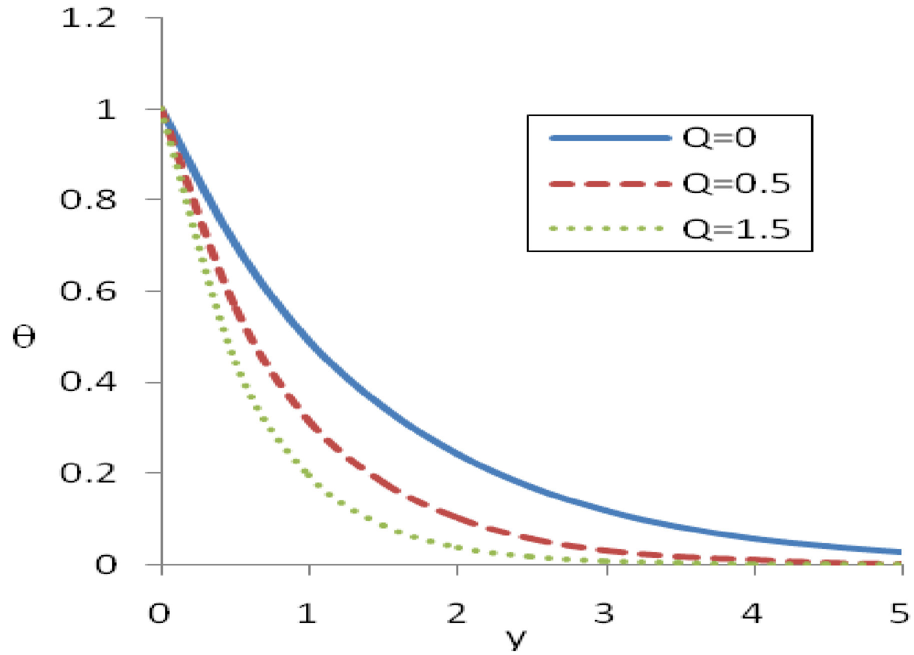

Figure 2: Temperature θ versus y for Pr =0.71, Sc= 0.22, G_r =4.0, G_m =5.0, U_0=1.0, Sr = 0.5.

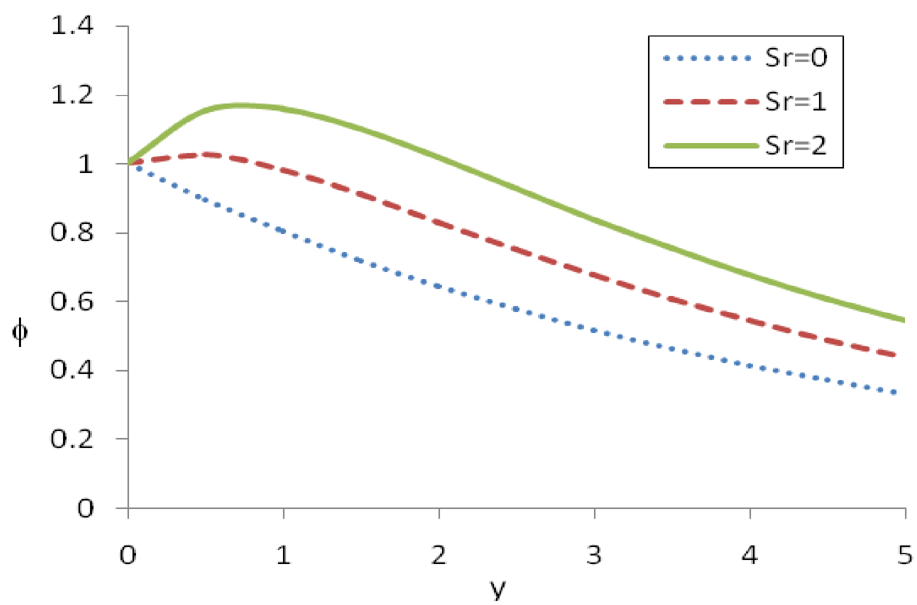

Figure 3: Concentration ϕ versus y for Pr =0.71, Sc= 0.22, G_r =4.0, G_m =1.0, U_0=1.0, Q=1.0

4. CONCLUSIONS:

In this paper a discussion on free convection mass transfer flow past a uniformly accelerated porous plate in presence of heat absorption and thermal diffusion is presented. From the present study the following conclusions can be drawn:

(1) The velocity profile increases with an increase in Soret number and decreases steadily far away from the plate.

(2) The temperature profile decreases due to an increase in value of heat sink parameter.

(3) The concentration profile increases as the Soret number increases and gradually decreases far away from the plate.

(4) The skin-friction increases as Sc increases up to Sc ≤ 0.708 and thereafter decreases when Sc ≥ 0.709.

(5) The rate of mass transfer gets decrease when Sc ≤ 0.708 and after that increases gradually when Sc ≥ 0.709.

LIST OF SYMBOLS:

$\bar{C}$ Species concentration
C_p Specific heat at constant pressure
$\bar{C}_w$ Species concentration at the plate
$\bar{C}_\infty$ Species concentration in the free stream
D_M Co-efficient of mass diffusion
D_T Co-efficient of thermal diffusion
g Acceleration due to gravity
G_m Grashof number for mass transfer
G_r Grashof number for heat transfer
k Thermal conductivity
Nu Nusselt number
Pr Prandtl number
Q Heat sink parameter(non-dimensional)
Q_0 Heat sink parameter(dimensional)
Re Reynolds number
Sc Schmidt number
Sh Non-dimensional Sherwood number
Sr Soret number
$\bar{T}$ Fluid temperature
$\bar{T}_w$ Temperature at the plate
$\bar{T}_\infty$ Temperature in the free stream
$\bar{U}$ Free Stream velocity (dimensional)
U_0 Free Stream velocity (non-imensional)
V_0 Suction velocity
$(\bar{u}, \bar{v})$ Velocity components (dimensional)
(u, v) Velocity components (non- dimensional)
$(\bar{x}, \bar{y})$ Cartesian Co-ordinate (dimensional)
(x, y) Cartesian Co-ordinate (non-dimensional)

Greek Symbols:

β Co-efficient of volume expansion for thermal expansion
β^* Co-efficient of volume expansion for mass expansion
ρ Density of the fluid
θ Non dimensional temperature
ν Kinematic Co-efficient of viscosity
ϕ Non dimensional Species Concentration
τ Non- dimensional skin friction

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