International Journal of Mathematical Archive-7(10), 2016, 185-191
Available online through www.ijma.info ISSN 2229 – 5046

SOME THEOREMS ON $\alpha - \Psi$ QUASI CONTRACTIVE ON QUASI PARTIAL METRIC SPACE

Dr. U. KARUPPIAH[1], A. MARY PRIYA DHARSINI*[2]

[1]Department of Mathematics, St. Joseph's College (Autonomous), Trichy-620 002, India.

[2]Department of Mathematics, Holy Cross College (Autonomous), Trichy-620 002, India.

(Received On: 11-09-16; Revised & Accepted On: 20-10-16)

ABSTRACT

In this paper, we consider $\alpha - \Psi$ contractive mappings in the setting of quasi partial metric spaces and verify the existence of a fixed point on such spaces. Also, we present some examples of obtained results.

Keywords: *Quasi-Partial Metric space; Fixed point; α - admissible; Contractive mapping.*

Mathematics Subject Classification: *Primary 47h10, Secondary 54h25.*

1. INTRODUCTION

One of the most interesting expansions of distance function was reported by Matthews [1] by introducing the notion of a partial metric in which self-distance need not be zero. Matthews [1] successfully characterised the distinguished result, Banach contraction mapping, in the sitting of partial metric spaces.

Many authors have generalised some fixed point theorems on quasi-partial metric spaces. Recently Erdal Karpinar *et al*. [11] presented $\alpha - (\Psi, \emptyset)$ contractive mappings on quasi-partial metric space and investigated the existence and uniqueness of certain operators in the context of quasi-partial metric space.

A fixed point theorem is proved in setting of such spaces and a example is given to verify the effectiveness of the main results.

2. PRELIMINARIES

Definition 1: A quasi metric on a non-empty set X is a function d: X × X → [0, +∞) such that for all x, y, z ∈ X:
1. $d(x, y) = 0 \Leftrightarrow x = y$
2. $d(x, y) \le d(x, z) + d(z, y)$

A quasi-metric space is a pair (X, d) such that X is a non-empty set and d is a quasi-metric on X.

Definition 2: A partial metric on a non-empty set X is a function p: X × X → [0, +∞) such that for all x, y, z ∈ X:
1. $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,
2. $p(x, x) \le p(x, y)$,
3. $p(x, y) = p(y, x)$,
4. $p(x, y) \le p(x, z) + p(z; y) - p(z, z)$.

A partial metric space is a pair (X, p) such that X is a non-empty set and p is a partial metric on X.

Definition 3 [5]: A quasi-partial metric space on a non-empty set X is a function q: X × X → [0, +∞) such that for all x, y, z ∈ X:
1. if $q(x, x) = q(x, y) = q(y, y)$ then $x = y$ (equality)
2. $q(x, x) \le q(x, y)$ (small self-distances)
3. $q(x, x) \le q(y, x)$ (small self-distances)
4. $q(x, z) + q(y, y) \le q(x, y) + q(y, z)$ (triangle inequality)

Corresponding Author: A. Mary Priya Dharsini[2]*
[2]Department of Mathematics, Holy Cross College (Autonomous), Trichy-620 002, India.

A quasi partial metric space is a pair (X, q) such that X is a non-empty set and q is a partial metric on X. If q(x, y) = q(y; x) for all x, y ∈ X, then (X, q) becomes a partial metric space.

Definition 4 [5]: Let (X, q) be a quasi-partial metric space. Then,
(i) a sequence $\{x_n\} \subset$ X converges to x ∈ X if and only if

$$q(x, x) = \lim_{n\to+\infty} q(x, x_n) = \lim_{n\to+\infty} q(x_n, x);$$

(ii) a sequence $\{x_n\} \subset$ X is called a Cauchy sequence if and only if

$\lim_{n,m\to+\infty} q(x_n, x_m)$ *and* $\lim_{n,m\to+\infty} q(x_m, x_n)$ exist (and are finite);

(iii) the quasi-partial metric space is said to be complete if every Cauchy sequence $\{x_n\} \subset$ X converges, with respect to τ_q, to a point x ∈ X such that

$$q(x, x) = \lim_{n,m\to+\infty} q(x_n, x_m) \text{ and } \lim_{n,m\to+\infty} q(x_m, x_n)$$

Definition 5 [5]: Let (X; q) be a quasi-partial metric space. Then

1. a sequence $\{x_n\}$ in X is called a left Cauchy sequence if and only if for every $\epsilon > 0$ there exists a positive integer N = N(ϵ) such that
   $q(x_n, x_m) < \epsilon$ for all n > m > N;
2. a sequence $\{x_n\}$ in X is called a left Cauchy sequence if and only if for every $\epsilon > 0$ there exists a positive integer N = N(ϵ) such that
   $q(x_n, x_m) < \epsilon$ for all m > n > N;
3. the quasi-partial metric space is said to be left complete if every left Cauchy sequence $\{x_n\}$ in X is convergent.
4. the quasi-partial metric space is said to be right complete if every right Cauchy sequence $\{x_n\}$ in X is convergent.

Definition 6 [6]: Let T be self-mapping on X and α: X × X → [0, +∞) be a function. We say that T is an α - admissible mapping if T is

$x, y \in X$, $\alpha (x, y) \geq 1$) $\Rightarrow \alpha(Tx, Ty) \geq 1$

Definition 7 [7]: Let T be self-mapping on X and α : X × X → [0, +∞) be a function. We say that T is a triangular α - admissible mapping if T is α - admissible and

$x, y, z \in X$, $\alpha(x, z) \geq 1$ *and* $\alpha(z, y) \geq 1) \Rightarrow \alpha(x, y) \geq 1$

Definition 8 [8]: Let T: X × X be a self-mapping and α : X × X → [0, +∞) be a function. Then T is said to be α - orbital admissible if

$\alpha(x, Tx) \geq 1 \Rightarrow (Tx; T^2 x) \geq 1$

Definition 9 [8]: Let T: X × X be a self-mapping and α: X × X → [0, +∞) be a function. Then T is said to be right- α - orbital-admissible if

$\alpha(x, Tx) \geq 1 \Rightarrow (Tx; T^2 x) \geq 1$ and be left- α - orbital-admissible if $\alpha(Tx, x) \geq 1 \Rightarrow (Tx; T^2 x) \geq 1$

Note that a mapping T is α-orbital admissible if it is both right-α-orbital admissible and left-α-orbital admissible.

Definition 10 [8]: Let T: X × X be a self-mapping and α: X × X → [0, ∞) be a function. Then T is said to be triangular α-orbital admissible if T is α-orbital admissible and

$\alpha(x, y) \geq 1$ *and* $\alpha(y, Ty) \geq 1 \Rightarrow \alpha(x, Ty) \geq 1$

Definition 11 [8]: Let T: X × X be a self-mapping and α: X × X → [0,∞) be a function. Then T is said to be triangular α-orbital admissible if T is right-α-orbital admissible and

$\alpha(x, y) \geq 1$ *and* $\alpha(y, Ty) \geq 1 \Rightarrow \alpha(x, Ty) \geq 1$

and be triangular left-α-orbital admissible if T is a α-orbital admissible and

$\alpha(Tx, x) \geq 1$ *and* $\alpha(x, y) \geq 1 \Rightarrow \alpha(Tx, y) \geq 1$

Definition 12 [8]: Let (X, d) be a α-metric space, X is said α- regular if for every sequence $\{x_n\}$ in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$ there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k.

Lemma 1 [5]: Let (X, q) be a quasi-partial metric space. Let (X, p_q) be the corresponding partial metric space and let (X, d_{p_q}) be the corresponding metric space. The following statements are equivalent.

1. The sequence $\{x_n\}$ is Cauchy in (X, q).
2. The sequence $\{x_n\}$ is Cauchy in (X, p_q).
3. The sequence $\{x_n\}$ is Cauchy in (X, d_{p_q}).

Lemma 2 [5]: Let (X, q) be a quasi-partial metric space. Let (X, p_q) be the corresponding partial metric space and let (X, d_{p_q}) be the corresponding metric space. The following statements are equivalent:

1. (X, q) is complete.
2. (X, p_q) is complete.
3. (X, d_{p_q}) is complete.

Moreover,

$$\lim_{n\to\infty} d_{p_q}(x, x_n) = 0 \Leftrightarrow p_q(x, x) = \lim_{n\to\infty} p_q(x, x_n) = \lim_{n,m\to\infty} p_q(x_n, x_m) \quad (1)$$

$$\Leftrightarrow q(x, x) = \lim_{n,m\to\infty} q(x, x_n) = \lim_{n,m\to\infty} q(x_n, x_m) \quad (2)$$

$$= \lim_{n\to\infty} q(x_n, x) = \lim_{n\to\infty} q(x_m, x_n) \quad (3)$$

In this paper, we shall handle definition 5 in the following way.

Lemma 3 [8]: Let T: X × X be a triangular α-orbital admissible mapping. Assume that there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for each n ∈ N_0. Then we have $\alpha(x_n, x_m) \geq 1$ for all m, n ∈ N with n < m.

Lemma 4 [11]: Let T: X × X be a triangular α- orbital admissible mapping. Assume that there exists $x_0 \in X$ such that $\alpha(Tx_0, x_0)$. Define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$, for each n ∈ N_0. Then we have $\alpha(x_n, x_m) \geq 1$ for all m, n ∈ N with n < m.

Definition 13 [11]: Let ^ be set of functions. Φ: [0, +∞) ⇒ [0; +1) such that $\Phi^{-1}(0) = 0; \Psi = \{\Psi \in \wedge$ is continuous, non- decreasing } and Φ = {φ ∈: φ is lower semi-continuous}. Let (X, q) be a quasi-partial metric space. We consider the following expressions:

$$M(x; y) = \max \{q(x, y), q(x, Tx), q(y, Ty)\} \quad (4)$$

$$N(x; y) = \min\{ \alpha_m^q(x, Tx), \alpha_m^q(y, Ty), \alpha_m^q(x, Ty), \alpha_m^q(y, Tx)\} \quad \text{for all } x, y \in X. \quad (5)$$

Definition 14 [11]: Let (X; q) be a quasi-partial metric space. Where X is a non-empty set. we say that X is said to be α-left-regular if for every sequence $\{x_n\}$ in X such that $\alpha(x_{n+1}, x_n) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x, x_{n(k)}) \geq 1$ for all k. Analogously, a quasi-partial metric space X is said to be an α-right-regular if for every sequence $\{x_n\}$ in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence$\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k. We say that X is regular if it is both α-left-regular and α-right-regular.

Theorem 1 [11]: Let (X, q) be a complete quasi partial metric space.

Let T: X → X be a self-mapping. Assuming that there exists $\psi \in \Psi$, $\varphi \in \Phi$, L ≥ 0 and a function α: X × X → [0, ∞) such that for all x, y ∈ X,

$$\alpha(x, y)\, \psi((Tx; Ty)) \leq \psi(M(x, y)) - \varphi(M(x, y)) + LN(x, y). \quad (6)$$

Also suppose that the following assertions hold:

(i) T is triangular α-orbitable admissible.
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$ and $\alpha(Tx_0, x_0) \geq 1$.
(iii) T is continuous or X is α-regular.
Then T has a fixed point u ∈ X and q(u, u)=0.

3 MAIN RESULTS

Theorem 2: Let (X, q) be a complete quasi partial metric space. Let T: X →X be a self-mapping .Assuming that there exists $\psi \in \Psi$ and a function α: X × X → [0,∞) such that for all x, y ∈ X,

$$\alpha(x, y) q(Tx; Ty) \leq \psi(M(x; y)) \quad (7)$$

Also suppose that the following assertions hold:

(i) T is triangular α-orbitable admissible.
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$ and $\alpha(Tx_0, x_0) \geq 1$.
(iii) T is continuous or X is α-regular.
Then T has a fixed point u ∈ X and q(u, u)=0.

Proof: We construct a sequence f in X in the following way:

$x_n = Tx_{n-1}$ for all n ∈ N.

If $q(x_{n_0}, x_{n_0+1}) = 0$ for some $n_0 \geq 0$, then we have $x_{n_0} = x_{n_0+1} = Tx_{n_0}$, (ie) x_{n_0} is the fixed point of T. Consequently, we suppose that $q(x_{n_0}, x_{n_0+1}) > 0$ for all $n \in N_0$.

By (ii), we have $\alpha(x_0, Tx_0) \geq 1$ and $\alpha(x_0, Tx_0) \geq 1$ on account of (i), we derive that

$$\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \geq 1 \Rightarrow \alpha(x_1, x_2) = \alpha(Tx_0, Tx_1) \geq 1,$$
$$\alpha(x_1, x_0) = \alpha(Tx_0, x_0) \geq 1 \Rightarrow \alpha(x_2, x_1) = \alpha(Tx_1, Tx_0) \geq 1,$$

Recursively, we obtain that

$$\alpha(x_n, x_{n+1}) \geq 1 \text{ and } \alpha(x_{n+1}, x_n) \geq 1 \text{ for all } n \in N_0 \tag{7}$$

Regarding (6) and (7), we find that

$$\begin{aligned} q(x_n, x_{n+1}) &= q(Tx_{n-1}, x_n) \\ &\leq \alpha(x_{n-1}, x_n)\, q(Tx_{n-1}, Tx_n) \\ &\leq \psi(M(x_{n-1}, x_n)) \end{aligned} \tag{8}$$

where

$$\begin{aligned} M(x_{n-1}, x_n) &= \max\{q(x_{n-1}, x_n),\ q(x_{n-1}, Tx_{n-1}), q(x_n, Tx_n)\} \\ &= \max\{q(x_{n-1}, x_n),\ q(x_{n-1}, x_n), q(x_n, x_{n+1})\} \\ &= \max\{q(x_{n-1}, x_n), q(x_n, x_{n+1})\}. \end{aligned} \tag{9}$$

Thus we conclude from (8) that

$$q(x_n, x_{n+1}) \leq \psi(\max\{q(x_{n-1}, x_n), q(x_n, x_{n+1})\}) \tag{10}$$

By taking (9) into account

If for some n we have $\max\{q(x_{n-1}, x_n), q(x_n, x_{n+1}) = q(x_n, x_{n+1})$, then (10) yields that

$$q(x_n, x_{n+1}) \leq \psi(q(x_n, x_{n+1}))$$

Hence, equation (8) turns into $q(x_n, x_{n+1}) \leq \psi(q(x_{n-1}, x_n))$ for all $n \in N$.

Due to the property of the auxiliary function, we have

$$q(x_n, x_{n+1}) \leq q(x_{n-1}, x_n) \text{ for all } n \in N. \tag{11}$$

Eventually, we observe that the sequence $\{q(x_n, x_{n+1})\}$ is non-increasing. So there exists $\delta > 0$ such that $\lim_{n\to\infty} q(x_n, x_{n+1}) = \delta$. If $\delta > 0$, taking lim sup $n \to +\infty$ in inequality (10), by keeping (9) in the mind, we obtain that

$$\lim_{n\to+\infty} sup\, q(x_n, x_{n+1}) \leq \lim_{n\to+\infty} sup\, \psi(q(x_{n-1}, x_n))$$

By continuity of ψ, we obtain $\delta \leq \psi(\delta)$, which is a contradiction. So,

$$\lim_{n\to\infty} q(x_n, x_{n+1}) = 0 \tag{12}$$

Analogously, we derive that

$$\lim_{n\to\infty} q(x_{n+1}, x_n) = \delta \tag{13}$$

Now, we shall show that $\{x_n\}$ is a Cauchy sequence in the quasi-partial metric space (X, q), that is, the sequence $\{x_n\}$ is left-Cauchy and right Cauchy.

Suppose that x_n is not a left-Cauchy sequence in (X, q). Then there is $\varepsilon > 0$ such that for each integer k there exists integers n(k) > m(k) > k such that

$$q(x_{n(k)}, x_{m(k)}) \geq \varepsilon \tag{14}$$

Further, corresponding to m(k), we can choose n(k) so that it is the smallest integer with n(k) > m(k) satisfying (14), consequently, we have

$$q(x_{n(k)-1}, x_{m(k)}) < \varepsilon \tag{15}$$

Due to the triangle inequality, we have

$$\begin{aligned} \varepsilon &\leq q(x_{n(k)}, x_{m(k)}) \\ &\leq q(x_{n(k)}, x_{n(k)-1}) + q(x_{n(k)-1}, x_{m(k)}) - q(x_{n(k)-1}, x_{n(k)-1}) \\ &< q(x_{n(k)}, x_{n(k)-1}) + \varepsilon \end{aligned} \tag{16}$$

Letting $k \to \infty$ and taking (12) into account, we get that

$$\lim_{k\to\infty} q(x_{n(k)}, x_{m(k)}) = \varepsilon \tag{17}$$

On the other hand, again by the triangle inequality, we find that

$$\begin{aligned} q(x_{n(k)}, x_{m(k)}) &\leq q(x_{n(k)}, x_{n(k)-1}) + q(x_{n(k)-1}, x_{m(k)-1}) - q(x_{m(k)-1}, x_{m(k)}) \\ &\qquad - q(x_{n(k)-1}, x_{n(k)-1}) - q(x_{m(k)-1}, x_{m(k)-1}) \\ &\leq q(x_{n(k)}, x_{n(k)-1}) + q(x_{n(k)-1}, x_{m(k)-1}) - q(x_{m(k)-1}, x_{m(k)}) \end{aligned} \tag{18}$$

And

$$\begin{aligned} q(x_{n(k)-1}, x_{m(k)-1}) &\leq q(x_{n(k)-1}, x_{n(k)}) + q(x_{n(k)}, x_{m(k)}) \\ &\qquad + q(x_{m(k)}, x_{m(k)-1}) - q(x_{n(k)}, x_{n(k)}) - q(x_{m(k)}, x_{m(k)}) \\ &\leq q(x_{n(k)-1}, x_{n(k)}) + q(x_{n(k)}, x_{m(k)}) + q(x_{m(k)}, x_{m(k)-1}) \end{aligned} \tag{19}$$

Letting k → ∞ and taking (12), (13), (17), (18), (19) into account, we derive that

$$\lim_{k\to\infty} q(x_{n(k)-1}, x_{m(k)-1}) = \varepsilon \tag{20}$$

$$\begin{aligned} q(x_{n(k)-1}, x_{m(k)}) &\leq q(x_{n(k)-1}, x_{n(k)}) + q(x_{n(k)}, x_{m(k)}) - q(x_{n(k)}, x_{n(k)}) \\ &\leq q(x_{n(k)-1}, x_{n(k)}) + q(x_{n(k)}, x_{m(k)}) \end{aligned} \tag{21}$$

Letting k → ∞ and taking (12), (16), (18), (21) into account, we derive that

$$\lim_{k\to\infty} q(x_{n(k)-1}, x_{m(k)}) = \varepsilon \tag{22}$$

Since T is triangular, α- orbital admissible, from lemma 3 and lemma 4, we derive that

$$\alpha(x_n, x_{n+1}) \geq 1 \text{ and } \alpha(x_m, x_n) \geq 1 \text{ for all } n > m \in N_0 \tag{23}$$

Regarding (6) and (23), we find that

$$\begin{aligned} q(x_{n(k)}, x_{m(k)}) &= q(x_{n(k)-1}, x_{m(k)-1}) \\ &\leq \alpha(x_{n(k)-1}, x_{m(k)-1}) q(Tx_{n(k)-1}, Tx_{m(k)-1}) \\ &\leq \psi(M(x_{n(k)-1}, x_{m(k)-1})) \end{aligned} \tag{24}$$

Where

$$M(x_{n(k)-1}, x_{m(k)-1}) = \max\{q(x_{n(k)-1}, x_{m(k)-1}),\ q(x_{n(k)-1}, x_{n(k)}), q(x_{m(k)}, x_{m(k)-1})\} \tag{25}$$

We get, $$\lim_{k\to\infty} q(x_{n(k)-1}, x_{m(k)-1}) = \varepsilon \tag{26}$$

From the above observation, letting k → ∞ in (26), we obtain $\varepsilon \leq \psi(\varepsilon)$ which is the contradiction. Thus $\{x_n\}$ is a left-Cauchy sequence in the metric space (X, q). Analogously, we derive that $\{x_n\}$ is a right-Cauchy sequence in the metric space (X, q). Since (X, q) is complete, then from lemma 2, (X, d_{p_q}) is a complete metric space. Therefore the sequence $\{x_n\}$ converges to a point u ∈ X in (X, d_{p_q}).

(i.e), $\lim_{n\to\infty} d_{p_q}(x_{n,}u) = 0$.

$\lim_{n\to\infty} d_{p_q}(x_{n,}u) = 0$.

Again from lemma 2

$$p_q(u,u) = \lim_{n\to\infty} p_q(x_n, u) = \lim_{n\to\infty} p_q(x_n, x_n)$$

On the other hand, by (12) and the condition (QPM2) from definition 3, $\lim_{n\to\infty} q(x_n, x_n)$ (27)

So, it follows that

$$\begin{aligned} q(u,u) &= \lim_{n\to\infty} \frac{1}{2}[q(x_n, u) + (u, x_n)] \\ &= \lim_{n\to\infty} \tfrac{1}{2} q(x_n, x_n) = 0 \end{aligned} \tag{28}$$

Now for proving fixed point of T, first we suppose that T is continuous, then we have

$$Tu = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Tx_{n+1} = u$$

So u is a fixed point of T. As the last step, suppose that X is α-regular.

Hence it is α - right regular, then there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_n, u) \geq 1$ for all k. Now, we show that q(u, Tu) = 0. Assume that this is not true, from (1), we obtain

$$\begin{aligned} \psi(q(x_{n(k)+1}, Tu) &= \psi(Tx_{n(k)}, Tu) \\ &\leq \alpha(x_{n(k)}, u)\psi(Tx_{n(k)}, Tu) \\ &\leq \psi\ (M\ (x_{n(k)}, u)) \end{aligned}$$

Where

$$M\left(x_{n(k)},u\right)=\max\{q\left(x_{n(k)},u\right),q\left(x_{n(k)},Tx_{n(k)}\right),q(u,Tu)\} \tag{29}$$

$$=\max\{q\left(x_{n(k)},u\right),q\left(x_{n(k)},x_{n(k)+1}\right),q(u,Tu)\} \tag{30}$$

It is obvious that

$$\lim_{k\to+\infty} q\left(x_{n(k)},Tu\right)=q(u,Tu).$$

Therefore by using (12) and (28), we deduce that

$$\lim_{k\to+\infty} M\left(x_{n(k)},u\right)\le \max\{0,0,q(u,Tu)\}.$$
$$=q(u,Tu)$$

Because (12), (13) and (27) give

$$\lim_{k\to+\infty} d_m^q\left(x_{n(k)},Tx_{n(k)}\right)=0.$$

Now by using the property of ψ and taking the upper limit as lim n → ∞

We obtain ψ ((u, Tu)) ≤ ψ(q(u, Tu)), that is q(u, Tu) = 0 and so Tu = u.

Now we conclude that T has a fixed point u ∈ X and q(u, u) = 0.

Example: Let X = [0, ∞) and q(x, y) =|x-y| + x for all x, y ∈ X. Then (X, q) is a complete quasi-partial metric space. Consider T: X →X defined by

$$Tx=\frac{1}{3}$$

Take ψ (t) = $\frac{2t}{3}$ for all t ≥ 0. Note that $\psi \in \psi$. Take x ≤ y, then

$$\alpha(x,y)q(Tx,Ty)=\alpha(x,y)(|Tx-Ty|+Tx)=\left(\left|\frac{x}{3}-\frac{y}{3}\right|+\frac{x}{3}\right)$$
$$=\frac{y}{3}$$
$$=\psi\left(\frac{y}{2}\right) \text{ since } M(x,y)=\left(\frac{y}{2}\right)$$
$$\le\psi\,(M(x,y))$$

Now let y < x then

$$\alpha(x,y)q(Tx,Ty)=\alpha(x,y)(|Tx-Ty|+Tx)=\frac{3}{2}\left(\left|\frac{x}{3}-\frac{y}{3}\right|+\frac{x}{3}\right)$$
$$=\frac{2x-y}{2}$$

We have two possibilities for M(x, y)

Case (i): If M(x, y) =$\frac{3(2x-y)}{4}$, then

$$\alpha(x,y)q(Tx,Ty)=\frac{2x-y}{2}=\psi\left(\frac{3(2x-y)}{2}\right)\le\psi\left(M(x,y)\right)$$

Case (ii): If M(x, y) = $\frac{x}{2}$, then

$$\alpha(x,y)q(Tx,Ty)=\frac{9}{4}x-\frac{2}{9}y=\frac{2}{3}\left(\frac{2}{3}x-\frac{y}{3}\right)\le\psi\left(M(x,y)\right)$$

Moreover, T is triangular α-orbital, α(0, T0) ≥1 and α(T0, 0) ≥ 1 . Thus by applying theorem 2, n has a fixed point, which is u = 0.

4. REFERENCES

1. Matthews SG, Partial metric topology.In Proceedings of the 8th Summer Conference on General Topology and Applications. Annals of the New York Academy of Sciences, Vol.728.pp.183-197(1994).
2. Abbas M, llic D, Common fixed points of generalized almost non expansive mappings. Filomat 24(3), 11-18 (2010).
3. Abedeljawad T, Karapinar E, Tas K, Existence and uniqueness of common fixed point on partial metric spaces.Appl.Math.Lett.24, 1984-1899(2011).
4. Aydi H, Karpinar E, Shatanawi W, Coupled fixed point results for (ψ, Φ) - weakly Contractive condition in ordered partial metric spaces. Comput. Math.Appl.62, 4449-4460(2011).

5. Karapinar E, Erhan IM, Ozturk A, Fixed point theorems on quasi- partial metric spaces. Math. Comput. Model.57, 2442-2448(2013)
6. Samet B, Vetro C, Vetro P, α - ψ - Contractive type mappings. Non-linear Anal. 75, 2154-2165 (2012).
7. Karapinar E, Kuman P, Salimi P, On α - ψ - Meir - Keeler contractive mappings. Fixed point theory Appl.2013, 94(2013).
8. Popescu O, some new fixed point theorems for α- Geraghty Contraction type maps in metric spaces, Fixed point theory Appl. 2014, 190(2014).
9. Karapinar E. Ciric types non unique fixed point theorems on partial metric spaces. J. Non-linear Sci.Appl.5, 74-83 (2012).
10. Karapinar E, Shobkolaei N, Sedghi S, Vaezpour SM, A common fixed point theorems for cyclic operators on partial metric spaces. Filomat 26, 407-414(2012).
11. Erdal Karapinar, Leila Gholizadeh, Hamed H Alsulami and Maha Noorwali, α-(ψ,φ) contractive mappings on quasi-partial metric spaces (2015).

Source of support: Nil, Conflict of interest: None Declared