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ON STABILITY OF CUBIC FUNCTIONAL EQUATION IN NON-ARCHIMEDEAN FUZZY NORMED SPACES

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ABSTRACT

This paper deals with stability of cubic mappings in non-Archimedean Fuzzy normed spaces by alternative proof which gives better estimation than [8]. Finally, some applications of our results in stability of cubic mapping in non – Archimedean Fuzzy normed space are obtained.

Key words and phrases: *Non-Archimedean fuzzy norm, fuzzy norm, cubic functional equation, valuation.*

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INTRODUCTION

In 1940, Ulam [21] posed the first stability problem while in 1941; Hyers [10] obtained some results regarding stability of linear functional equation. Th. M. Rassias [22] generalized the result of Hyers for additive and linear mappings and also obtained analogous stability problems of quadratic mappings. In 1978 P.M.Gruber [19] stated that stability problems are of particular interest in probability theory and in case of functional equation of different types. A.K. Mirmostafafe and M.S. Moslehian [1] introduced the notion of non- Archimedean Fuzzy norm and also obtained some results regarding stability of Cauchy and Jensen Functional equation in the context of non-archimedean Fuzzy norm space using the approach of Hyper- Ulam-Russias. Since then various authors [1, 3-5, 7, 10-11, 13, 16-17] investigated stability problems regarding Jensen, Cauchy, Quadratic, Cubic with more general domains and ranges. Further, the stability problem of quadratic equation has been investigated by a number of authors and references there in. In addition Alsina [6], Mihet and Radu [11] investigated the stability in the settings of fuzzy, probabilistic and random normed spaces. In [4] Mimostafaee and Moslehian introduced the idea of fuzzy stability of functional equations. The functional equation $f(2x+y)+f(2x-y) = 2f(x+y) - 2f(x-y) + 12f(x)$ is called cubic functional equation since the function $f(x) = cx^3$ is its solution. Every solution of the cubic functional equation is said to be a cubic mapping. The stability problem for cubic functional equation was proved by Jun and Kim [16] for mapping $f: X\rightarrow Y$ where X is a real normed space and Y is Banach Space. In the sequel, we will adopt the usual terminology, notations and convention of the theory of non-archimedean Fuzzy normed space. In this paper we consider the Hyers-Ulam-Rassias stability of the cubic functional equation in non-archimedean fuzzy normed spaces. Finally some applications of the above mentioned results are obtained in non-archimedean Fuzzy normed spaces.

Definition 2.1: Let K be a field. A non-Archimedean absolute value on K is a function (valuation) $|\bullet|: K\rightarrow R$ such that for any $a, b \in K$ we have

(i) $|a| \geq 0$ and equality holds if and only if $a = 0$,
(ii) $|ab| = |a|\,|b|$
(iii) $|a + b| \leq \max\{|a|, |b|\}$

The condition (iii) is called the strict triangle inequally. By (ii), we have $|1| = |-1| = 1$. Thus, by induction, it follows from (iii) that $|n| \leq 1$ for each integer n we always assume in addition that $|\cdot|$ is non trivial, i.e. that there is an $a_0 \in K$ such that $|a_0| \notin \{0, 1\}$

An example of a non-Archimedean valuation is the mapping $|.|$ taking everything but 0 into 1 and $|0| = 0$. This valuation is called trivial.

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Definition 2.2: Let X be a vector space over a scalar field K with a non-Archimedean valuation |.|. A function ||.||: X⟶ [0, ∞) is said to be a non-archimedean norm if it satisfies the following conditions:

(i) $\|x\|=0$ if and only if $x=0$;
(ii) $\|rx\|=|r|\|x\|$ ($r\in K$, $x\in X$);
(iii) the strong triangle inequality $\|x+y\|\leq \max\{\|x\|,\|y\|\}$ ($x, y \in X$)

Then (X, ||.||) is called a non-Archimedean normed space.

Definition 2.3: Let X be a linear space over a non-Archimedean field K. A function $N: X \times R \to [0, 1]$ is said to be a non-Archimedean fuzzy norm on X if for all $x, y\in X$ and all $t\in R$.

(N1) $N(x, t) = 0$ for $t \leq 0$;
(N2) $x = 0$ if and only if $N(x, t) = 1$ for all $t > 0$;
(N3) $N(cx, t) = N\left(x, \frac{t}{|c|}\right)$ if $c \neq 0$; $c\in K$.
(NA4) $N(x + y, \max\{s, t\}) \geq \min\{N(x, s), N(y, t)\}$
(N5) $\lim_{t\to\infty} N(x, t) = 1$.

The pair (X, N) is called a non-Archimedean fuzzy normed space. Clearly, if (NA4) holds then so is
(N4) $N(x + y, s + t) \geq \min\{N(x, s), N(y, t)\}$.

A classical vector space over a complex or real field satisfying (N1) – (N5) is called fuzzy normed space. It is easy to see that (NA4) is equivalent to the following condition

(NA4′) $N(x + y, t) \geq \min\{N(x, t), N(y, t)\}$ ($x, y \in X$; $t\in R$).

Example 2.4: Let (X, ||·||) be a non-Archimedean normed linear space. Then

$$N(x,t)=\begin{cases}\dfrac{t}{t+\|x\|}, & t\leq\|x\|, x\in X\\ 0, & t>\|x\|, x\in X\end{cases}$$

is a non-Archimedean fuzzy norm on X.

Example 2.5: Let (X, ||·||) be a non-Archimedean linear space. Then

$$N(x,t)=\begin{cases}0, & t\leq\|x\|\\ 1, & t>\|x\|\end{cases}$$

is a non-Archimedean fuzzy norm on X.

Definition 2.6: Let (X, N) be a non-Archimedean fuzzy normed linear space. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is said to be convergent if there exists $x\in X$ such that

$$\lim_{n\to\infty} N(X_n - X, t) = 1 \text{ for all } t > 0$$

In that case, x is called the limit of the sequence $\{x_n\}$ and we denote it by N-lim $x_n = x$.

A sequence $\{x_n\}$ in X is called Cauchy if for each $\in > 0$ and each $t > 0$. There exists n_0 such that for all $n \geq n_0$ and all $p > 0$ we have $N(x_{n+p} - x_n, t) > 1-\in$. Due to

$$N(x_{n+p} - x_n, t) \geq \min\{N(x_{n+p} - x_{n+p-1}, t), \ldots, N(x_{n+1} - x_n, t)\}$$

The sequence $\{x_n\}$ is Cauchy if for each $\in>0$ and each $t > 0$. There exists n_0 such that for all $n \geq n_0$ we have

$$N(x_{n+1} - x_n, t) > 1-\in$$

It is easy to see that every convergent sequence is a non-Archimedean fuzzy normed space is Cauchy.

If each Cauchy sequence is convergent, then non-Archimedean fuzzy normed space is called a non-Archimdean fuzzy Banach space.

3. In the rest of this paper, unless otherwise explicitly stated,

We will assume that K is a non-archimedean field, X is a vector space over K, (Y, N) is a non-Archimedean fuzzy Banach space over K and (Z, N′) is a (Archimedean or non-Archimedean) fuzzy normed space. The functional equation

$$cf(x, y) = f(2x + y) + f(2x - y) - 2f(x + y) + 2f(x - y) + 12f(x)$$

is said to be the cubic functional equation since cx^3 is its solution. Every solution of the cubic functional equation is said to be cubic mapping.

In this paper, we establish the stability of the cubic functional equations in non-Archimedean fuzzy normed space.

Proposition 3.1[14]: Suppose that a complete generalized metric space (E, d) (i.e one for which d may assume infinite values) and a strictly contractive mapping J: E→E with the Lipschitz Constant $0 < L < 1$ are given. Then, for a given element x∈E, exactly one of the following assertions is true: either
(a) $d(J^n x, J^{n+1} x) = \infty$ for all $n \geq 0$,
or
(b) there exists k such that $d(J^n x, J^{n+1} x) < \infty$ for all $n \geq k$.

Actually, if (b) holds, then the sequence $\{J^n x\}$ is convergent to a fixed point x* of J and
(b_1) x* is the unique fixed point of J in $F = \{y \in E, d(J^*x, x) < \infty\}$;
(b_2) $d(y, x^*) \leq d(y, Jy)/1-L$ for all $y \in F$.

Lemma 3.2: Let (Z, N') be a non-Archimedean Fuzzy normed linear space and $\psi: X \to Z$ be a function. Let $E=\{g: X \to Y;\ g(0)=0\}$ and define $d(g, h)=\inf \{a>0 \mid N(g(x)-h(x), a_1^q t) \geq N'(\psi(x),t)$ for all $x \in X$ and $t>0\}$ ($g, h \in E$). Then d is a generalized complete metric on E.

Proof: Let $g, h, k \in E$, $d(g, h) < a_1$ and $d(h, k) < a_2$, then

$N(g(x)-h(x), a_1^q t) \geq N'(\psi(x), t)$ and

$N(g(x)-k(x), a_2^q t) \geq N'(\psi(x),t)$ for each $x \in X$ and $t > 0$. Therefore,

$$N(g(x)-k(x), \max(a_1^q t, a_2^q t)) \geq \min\{N(g(x)-h(x), a_1^q t), N(g(x)-k(x), a_2^q t)\}$$
$$\geq N'(\psi(x),t)$$

for each $x \in X$ and $t > 0$. Using $\max\left\{a_1^q, a_2^q\right\} \leq a_1^q + a_2^q \leq \left(a_1 + a_2\right)^q$

($a_1, a_2>0$), and by definition of $d(h, k) \leq a_1+a_2$. This proves the triangle inequality for d. Other properties are direct by using definition of non-Archimedean Fuzzy normed linear space.

Theorem 3.3: Let X be linear space, (Z, N′) be a non -Archimedean fuzzy normed space and φ: X × X→ Z be a function let f: X → Y be mapping such that
(3.1) $N(cf(x, y), t) \geq N'(\varphi(x, y), t)$ $(x, y \in X, t > 0)$

If for some $\alpha < 8$
(3.2) $N'(\varphi(2x, 0), t) \geq N'(\alpha \varphi(x, 0), t)$ $(x \in X, t > 0)$
$f(0) = 0$ and $\lim_{n\to\infty} N'(2^{-3n} \phi(2^n x, 2^n y), t)=1$
for all $x, y \in X$ and $t > 0$, then there exists a unique cubic mapping $T : X \to Y$ such that
(3.3) $N(T(x)-f(x), t) \geq N'(\phi(x, 0), 2(8^p-\alpha^p)^q t)$

Proof: Letting y = 0 in (3.1)
(3.4) $N(2f(2x)- 2^4 f(x), t) \geq N'(\phi(x, 0), t)$
Let
$E = \{g : X\to Y;\ g(0) = 0\}$
Define
$H: E \to E$ by $H(g)(x) = 2^{-3} g(2x)$ for each g∈E and x∈X. By Lemma 3.2

$d(g, h) = \inf \{a > 0; N(g(x)-h(x), a^q t)$
$\geq N'(\phi(x, 0), t)$ for all $x \in X$ and $t > 0\}$ for each $g \in E$ and $x \in X$. By Lemma 3.2

Defines a metric on E. Let $d(g, h) < a$, by the definition

$$N(g(x)-h(x),\ a^{q}\ t) \geq N'(\phi(x, 0), t)\ (x \in X, t > 0)$$

By (3.2), for each $x \in X$, $t > 0$

$$N(H(g)(x) - H(h)(x),\ 2^{-3}\ a^{q}\ t) = N(2^{-3} g(2x) - 2^{-3} h(2x),\ 2^{-3}\ a^{q}\ t)$$
$$\geq N(\phi(2x, 0), t)$$
$$\geq N'(\alpha\ \phi(x, 0), t)$$

Hence, by definition,

$$d(H(g), H(h)) \leq \left(\frac{\alpha}{8}\right)^{p} a. \text{ Therefore}$$

$$d(H(g), H(h)) \leq \left(\frac{\alpha}{8}\right)^{p} d(g, h)\ (g, h \in E)$$

This means that H is a contractive mapping with Lipschitz constant

$$L = \left(\frac{\alpha}{8}\right)^{p} < 1$$

Then by (3.4) $d(f, H(f)) \leq \left(\frac{1}{2^{4}}\right)^{p}$.

Hence by proposition 3.1, H has a unique fixed point in the set $\{g \in E; d(f, g) < \infty\}$, T: X → Y defined by (3.5)

$$T(x) = N\text{-}\lim_{n\to\infty} H^{n}(f)(x)$$
$$= \lim_{n\to\infty} 2^{-3n} f(2^{n} x)$$

and

$$d(f, T) = \frac{d(f, H(f))}{1-L} = \frac{16^{-p}}{1-8^{-p}\alpha^{p}} = \frac{1}{2^{p}(8^{p}-\alpha^{p})}$$

It means (3.3) holds.

By (3.5) $cT(x, y) = \lim_{n\to\infty} 2^{-3n} c(f(2^{n} x, 2^{n} y))$

Replacing x, y by 2^{n} x, 2^{n} y in (3.2)

$$N(2^{-3n} c(f(2^{n} x, 2^{n} y), t)) \geq N'(2^{-3n}\ \phi(2^{n} x, 2^{n} y), t)$$

By our assumption

$$\lim_{n\to\infty} N'(2^{-3n}\phi(2^{n} x, 2^{n} y), t) = 1$$

it follows that

$$N(T(x, y), t) = 1 \text{ for all } x, y \in X, t > 0$$

Hence by (N2) T satisfies cubic i.e. T is cubic function. To prove the uniqueness assertion, let us assume that there exists a cubic function T': X→Y which satisfies (3.3). Thus T' is a fixed point of H.

However, by Proposition 3.1, H has only one fixed point.

Hence T ≅ T'.

Theorem 3.4: Let (Z, N′) be non-Archimedean fuzzy normed space and φ: X × X → Z be a function. Let f: X → Y be mapping such that

$$N(c(f(x, y)), t) \geq N'(\varphi(x, y), t)\ (x, y \in X, t > 0)$$

If for some $\alpha > 8$

$N'(\varphi(x/2), 0), t) \geq N'(\varphi(x, 0), \alpha t)$ (x, y ∈ X, t > 0)

f(0) = 0 and $\lim_{n\to\infty} N'(2^{3n}\phi\ (2^{-n} x, 2^{-n}y), t)=1$

for all x, y in X and t > 0, then there exists a unique cubic mapping T : X $\rightarrow$ Y such that

$N(T(x)-f(x), t) \geq N'(\phi\ (x, 0), 2(\alpha^p\ 2^p\ )^q\ t)$ (x, y ∈ X, t > 0).

Proof: After a simple modification in the above proof, we obtain the required result.

Theorem 3.5: Let for each x ∈ X, the functions s → f(sx) and s → φ(sx, 0) be continuous, then for each x ∈ X, then the function s→T(sx) is continuous and $T(sx) = s^3\ T(x)$ for each x ∈ X and s ∈ R.

Proof: fix x ∈ X, s_0 ∈ R, t > 0 and 0 < β < 1 take n large enough so that

(3.6) $$\left(N'(\phi s_0 x,0),(8^p-\alpha^p)^q\left(\frac{8}{\alpha}\right)^n\left(\frac{t}{3^q}\right)\right) > \beta$$

Since, $\lim_{n\to\infty}(8^p-\alpha^p)^q(8/\alpha)^n(t/3^q)=\infty$. Then we have

$$N\left(2^{-3n}f(2^n s_0 x)-T(s_0x),\frac{t}{3^q}\right)=N\left(f(2^n s_0 x)-T(2^n s_0 x),\frac{2^{3n}t}{3^q}\right)$$
$$\geq N'\left(\phi(s_0x,0),2(8^p-\alpha^p)^q\left(\frac{8}{\alpha}\right)^n\left(\frac{t}{3^q}\right)\right)>\beta$$

Thus s $\rightarrow f(2^n$ sx) and s $\rightarrow \phi$ (sx, 0) are continuous at s_0, we can find some $\delta > 0$ such that, $0<|s-s_0|<\delta$

$$\Rightarrow \begin{cases} N(f(2^n sx)-f(2^n s_0x),\frac{2^{3n}t}{3^q})>\beta \\ N'\left(\phi(sx,0)-\phi(s_0x,0),\left(\frac{8}{\alpha}\right)^n\left(\frac{t}{3^q}\right)\right)>\beta \end{cases}$$

Let $|s-s_0|<\delta$ then (3.7)

$$N(T(sx)-T(s_0x),t)\geq \min\begin{Bmatrix} N\left(T(sx)-2^{-3n}f(2^n s_0x),\frac{t}{3^q}\right), \\ N\left(2^{-3n}f(2^n sx)-2^{-3n}f(2^n s_0x),\frac{t}{3^q}\right), \\ N\left(2^{-3n}f(2^n s_0x)-T(s_0x),\frac{t}{3^q}\right) \end{Bmatrix}$$

$$\Rightarrow\quad N\left(T(sx)-2^{-3n}f(2^n sx),\frac{t}{3^q}\right)\geq N'\left(\phi(sx,0),2(8^p-\alpha^p)^q\left(\frac{8}{\alpha}\right)^n\frac{t}{3^q}\right),$$

$$\geq \min\begin{Bmatrix} N'\left(\phi(sx,0)-\phi(s_0x,0),(8^p-\alpha^p)^q\left(\frac{8}{\alpha}\right)^n\frac{t}{3^q}\right), \\ N'\left(\phi(s_0x,0),(8^p-\alpha^p)^q\left(\frac{8}{\alpha}\right)^n\frac{t}{3^q}\right), \end{Bmatrix}$$

$$>\beta$$

The first term of (3.7) is bigger than β. Therefore, for every choice $s_0 \in R$, $x \in X$ and $t > 0$, we can find some $\delta > 0$ such that $N(T(sx, s_0 x)) > \beta$ for every $s \in R$ with $|s - s_0| < \delta$. Thus $s \to T(sx)$ is continuous.

By induction on m, one can easily prove that $T(mx) = m^3\ T(x)$ for every natural number m. It follows that

$$T\left(\frac{m}{k}x\right) = m^3\, T\left(\frac{x}{k}\right) = \left(\frac{m}{k}\right)^3 T(x)\ (m, k \in N)$$

Hence for every rational number r, $T(rx) = r^3\, T(x)$. Let s be a real number, then there exists a sequence $\{r_m\}$ of rational numbers such that $r_m \to s$. Thus by continuity of T(sx) for every $x \in X$.

$$T(sx) = \lim_{m\to\infty} T(r_m x) = \lim_{m\to\infty} r_m^3\, T(x) = s^3\ T(x)$$

4. Applications of stability of cubic mappings in a non-Archimedean fuzzy normed space.

Theorem 4.1: Let $\varphi: X \times Y \to [0, \infty)$ be a mapping such that either
(i) for some $\alpha \neq 8$, $\varphi(2x, 0) \leq \alpha\, \varphi(x, 0)$ for all $x \in X$ and for each $x, y \in X$.
$\lim_{n\to\infty} 2^{-3n}\, \phi\, (2^n x, 2^n y) = 0$ or
(ii) for some $\alpha > 8$, $\alpha\, \phi\, (x, 0) \leq \alpha\, \phi\, (2x, 0)$ for all $x \in X$ and for each $x, y \in X$, $\lim_{n\to\infty} 2^{3n}\, \phi\, (2^{-n} x, 2^{-n} y) = 0$.

Let $f: X \to Y$ satisfy the inequality $\|cf(x, y)\| \leq \varphi(x, y)$ for each $x, y \in X$ and $f(0) = 0$, then there exists a unique cubic mapping $T: X \to Y$ such that

$$\|f(x)- T(x)\| \leq \frac{\phi(x,0)}{2\left|\alpha^p - 8^p\right|^q} \tag{4.1}$$

And, if mappings $s \to f(st)$ and $s - \varphi(sx, 0)$ are continuous, then $s \to T(sx)$ is continuous and $T(sx) = s^3\, T(x)$ for each $s \in R$ and $x \in X$.

Proof: Consider the non-archimedean fuzzy norm define in example 2.4. If (i) holds, then hypotheses of theorem 3.3 are satisfied. In either case, by above theorem 3.5,we can find a cubic mapping T: X→ Y such that (4.1) holds. If the mapping $s \to f(st)$ and $s - \varphi(sx, 0)$ are continuous, then by Theorem(3.3) and (3.5), we get the result.

Corollary 4.2: Let for some $\in > 0$, $f: X \to Y$ satisfy the inequality $\|cf(x, y)\| \leq \in$; $x, y \in X$. Then there is a unique continuous cubic mapping $T: X \to Y$ s. t.

$$\|f(x) - f(0) - T(x)\| \leq \frac{2^q \varepsilon}{2\left|1 - 2^{3p}\right|^q} \text{ and}$$

$T(sx) = s^3\ T(x)$ for each $s \in R$ and $x \in X$.

Proof: For each $x, y \in X$, we have

$$\|c(f- f(0))\,(x, y)\|^p \leq \|cf(x, y)\|^p + \|c\, f(0, 0)\|^p$$
$$\leq \in^p + \in^p = 2 \in^p .$$

Hence $\|c(f- f(0))\,(x, y)\| < 2^q\ (x, y \in X)$.

By Theorem 3.3 for $\varphi(x, y) = 2^q \in$ for each $x, y \in X$ and $\alpha = 1$, we get deserved conclusion.

The following correspondence between a family of non-archimedean norms on a space and a non-archimedean fuzzy normed on the space, under some additional properties is presented.

Theorem 4.3: Let (X, N) be non-Archimedean fuzzy normed linear space such that
(1) $N(x, t) > 0$ for each $t > 0 \Rightarrow x = 0$
(2) $N(x, \cdot)$ is continuous function of R and strictly increasing on $\{t\ ;\ 0 < N(x, t) < 1\}$ for each non zero $x \in X$.

Let $\|x\|\alpha = \inf \{t;\ N(x, t) \geq \alpha\}$, $\alpha \in (0, 1)$ and $N_1: X \times R \to [0, 1]$ is defined by

$$N_1(x, t) = \begin{cases} \sup\{\alpha \in (0,1);\ \|x\|_\alpha < t\}, & (x,t) \neq (0,0), \\ 0 & ,(x,t) = (0,0) \end{cases}$$

Then
(i) $\{\|\cdot\|_\alpha\}$ α∈(0, 1) is an increasing family of non-Archimedean norms on the linear space X.
(ii) $N = N_1$

Moreover, (X, N) is complete if and only if (X, $\|\cdot\|_\alpha$) is complete for each α ∈ (0, 1).

Theorem 4.5: Let condition of theorem 3.3 for p = 1 hold let N and N' satisfy conditions (i) and (ii) of above theorem. If $\{\|\bullet\|_\alpha\}\alpha \in (0,1)$ and $\{\|\bullet\|'_\alpha\}\alpha \in (0,1)$ are the increasing non-Archimedean norms corresponding to N and N', respectively. Then there is unique cubic mapping T: X $\rightarrow$ Y such that for each $\alpha \in (0,1)$,

$$\|f(x)-T(x)\|\alpha \le \frac{\|\phi(x,0)\|'_\alpha}{2(8-\alpha)}$$

Proof: By theorem 3.3, there is unique cubic mapping T: X → Y such that

$$N(f(x) - T(x), t) \ge N'\left(\frac{\phi(x,0)}{2(8-\alpha)}, t\right)$$

By definition of $\|\bullet\|_\alpha$ and $\|\bullet\|'_\alpha$

$$\|f(x)-T(x)\|\alpha \le \frac{\|\phi(x,0)\|'_\alpha}{2(8-\alpha)}$$

If T': X $\rightarrow$ Y satisfies

$$\|f(x)-T(x)\|\alpha \le \frac{\|\phi(x,0)\|'_\alpha}{2(8-\alpha)}$$

for each α∈(0, 1), then by the definition and properly (ii) of above theorem.

$$N(f(x)-T'(x)\ \alpha \le), t) \ge N'\left(\frac{\phi(x,0)}{2(8-\alpha)}, t\right).$$

Hence T = T′.

REFERENCES

1. A. K. Mirmostafaee, Approximately additive mappings in non-archimedean normed spaces, Bull. Korean Math. Soc. 46(2009)387-400.
2. A.C.M.van Rooij, Non-archimedean Functional Analysis, Marcel Dekker, New York, 1978.
3. A.K.Mimostafaee and M.S.Moslehian, Stability of additive mappings in non- Archimedean fuzzy normed spaces, Fuzzy Sets and Systems 0114(2008)00506-X.
4. A.K.Mimostafaee, M.Mirzavaziri and M.S.Moslehian, Fuzzy stability of the Jensen functional equation, Fuzzy Sets and Systems 159(2008)730-738.
5. A.K.Mimostafaee, M.S.Moslehian, Fuzzy versions of Hyers-Ulam-Rassias theorem, Fuzzy sets and systems 159(2008),720-729.
6. C. Alsina, On the stability of a functional equation arising in probabilistic normed spaces, General Inequalities, 5(1987), 263-271.
7. C. Park, Fixed points and Hyers Ulam-Rassians stability of Cauchy- Jensen functional equations in Banach algebras, Fixed Point Theory Appl.2007, art. ID50175, 15pp.
8. C.Renu and Sushma, "A fixed point approach to Ulam stability problem for cubic and quartic mapping in non-archimedean Fuzzy Normed spaces," Proceeding of the world congress on Engineering 2010 vol VIII WCE 2010, June 30- July 2, 2010, London, U.K.
9. D. Deses, On the representation of non-archimedean objects, Topology Appl. 153(2005), 774-785.
10. D.H.Hyers, On the stability of the linear functional equation, Proc. Nat. Acad. Sci. U.S.A., 27(1941), 222-224.
11. D.Mihet and V.Radu, On the stability of additive Cauchy functional equation in random normed spaces. J.Math. Anal. Applied, 343 (2008), 567-572.
12. D.Mihet, Fuzzy ψ - contractive mapping in non- Archimedean fuzzy metric spaces, Fuzzy Set and Systems, 159(2008), 739-744.

13. E.Baktash, Y.J.Cho, R.Saadati, and S.M.Vaezpour, On stability of cubic mappings and quadratic mappings in random normed spaces, Journal of inequalities and Applications, 10(2008), 902187, 11 pages.
14. J.B.Diaz, B.Margolis, A fixed point theorem of the alternative for the contractions on generalized Complete metric space, Bull. Amer. Math. Soc. 74 (1968), 305-309.
15. J.Z.Xiao, X.H.Zhu, Fuzzy normed spaces of operators and its completeness, Fuzzy Sets and Systems 133(2003), 389-399.
16. K.W.Jun and H.M.Kim, Stability problem for Jensen-type functional equations of cubic mappings, Acta Math. Sin. (Engl. Ser.), 22 (2006), 1781-1788.
17. M.S.Moslehian and T.M.Rassias, Stability of functional equations in non-archimedean spaces, Appl. Anal. Discrete Math. 1(2007)325-334.
18. P.J.Nylkos, On some non- Archimedean spaces of Alexandrof and Urysohn, Topology Appl. 91(1999), 1-23.
19. P.M.Gruber, Stability of Isometries, Trans. Amer. Math. Soc., U.S.A, 245 (1978), 263-277.
20. P.W.Cholewa, Remarks on the stability of functional equations, Aequationes Math., 27(1984)76-86.
21. S.M.Ulam, A collection of Mathematical Problems, Interscience tracts in Pure and Applied Mathematics, U.S.A 16(1960).
22. T.M.Rassias, On stability of the linear mapping in Banach spaces, Proc. Amer. Math. Soc. 72(1978), 297-300.

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