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STRONGLY $g\omega\alpha$ -CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

The study of $g\omega\alpha$ -continuous function in topological spaces is continued in this paper, which is used to define and study strongly $g\omega\alpha$ -continuous functions. Further, we obtain basic properties and preservation theorems of strongly $g\omega\alpha$ -continuous functions and relationship with other similar functions.

Keywords and Phrases: *$g\omega\alpha$ -closed sets, $g\omega\alpha$ -continuous functions, strongly $g\omega\alpha$ -continuous functions, strongly $g\omega\alpha^*$ -continuous functions.*

1. INTRODUCTION

Levine [10] introduced the concept of generalized closed sets in topological spaces and class of topological spaces called $T_{\frac{1}{2}}$ -spaces. Stronger forms of continuous functions have been introduced and investigated by several mathematicians. Strongly continuous functions, perfectly continuous functions, completely continuous functions and clopen continuous functions were introduced by Levine [9], Noiri [14], Munshi and Bassan [11] and Reilly and Vamanamurthy [16] respectively. Ganster and Reilly [5] introduced contra continuous functions and almost s-continuous functions. Erdal Ekici [6] introduced and studied a new class of functions called almost contra-pre-continuous functions which generalize classes of regular set-connected [5], contra-pre continuous [7], contra continuous [4], almost s-continuous [13], perfectly continuous functions [14] and prefectly g* pre-continuous functions [15]. In this paper, we define and study the strongly $g\omega\alpha$ -continuous functions and strongly $g\omega\alpha^*$ -continuous functions in topological spaces.

2. PRELIMINARIES

Throughout this paper, (X, τ), (Y, σ) and (Z, η) (or simply X , Y and Z) always mean topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1: *A subset A of a space X is called*

(i) Semiopen set [8] if $A \subset cl(int(A))$.

(ii) α -open set [12] if $A \subset int(cl(int(A)))$.

(iii) Regular open set [17] if $A = int(cl(A))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.2 [1]: *A subset A of X is $\omega\alpha$ -closed if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is ω -open in X .*

Definition 2.3 [2]: *A subset A of X is $g\omega\alpha$ -closed if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is $\omega\alpha$ -open in X . The family of all $g\omega\alpha$ -closed subsets of the space X is denoted by $G\omega\alpha C(X)$.*

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Definition 2.4 [2]: *The intersection of all $g\omega\alpha$ -closed sets containing a set A is called $g\omega\alpha$ -closure of A and is denoted by $g\omega\alpha\text{-}cl(A)$.*

A set A is $g\omega\alpha$ -closed if and only if $g\omega\alpha\text{-}cl(A) = A$.

Definition 2.5 [2]: *The union of all $g\omega\alpha$ -open sets contained in A is called $g\omega\alpha$ -interior of A and is denoted by $g\omega\alpha\text{-}int(A)$.*

A set A is $g\omega\alpha$ -open if and only if $g\omega\alpha\text{-}int(A) = A$.

Definition 2.6 [3]: *A function $f : X \to Y$ is called $g\omega\alpha$ -continuous, if the inverse image of every closed set in Y is $g\omega\alpha$ -closed in X .*

3. STRONGLY $g\omega\alpha$ -CONTINUOUS FUNCTIONS

In this section, the notion of a new class of function called strongly $g\omega\alpha$ -continuous function is introduced and obtained some of their properties. Also, the relationships with existing functions are discussed.

Definition 3.1: *A function $f : X \to Y$ is called strongly $g\omega\alpha$ -continuous, if $f^{-1}(V)$ is closed in X for every $g\omega\alpha$ -closed set V in Y.*

Theorem 3.2: *A function $f : X \to Y$ is strongly $g\omega\alpha$ -continuous if and only if the inverse image of each $g\omega\alpha$ -open set in Y is an open set in X.*

Proof: Let $f : X \to Y$ is strongly $g\omega\alpha$ -continuous and V be $g\omega\alpha$ -open set in Y. Then Y - V is $g\omega\alpha$ -closed set in Y. Since f is strongly $g\omega\alpha$ -continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is closed in X. Therefore $f^{-1}(V)$ is an open in X.

Conversely: Assume $f^{-1}(V)$ is an open set in X for every $g\omega\alpha$ -open set V in Y. Let F be a $g\omega\alpha$ -closed set in Y, then Y - F is a $g\omega\alpha$ -open set in Y. By assumption $f^{-1}(Y - F) = X - f^{-1}(F)$ is an open set in X, which implies that $f^{-1}(F)$ is closed set in X. Therefore f is strongly $g\omega\alpha$ -continuous.

Remark 3.3: *Every strongly $g\omega\alpha$ -continuous function is continuous but converse need not be true in general.*

Example 3.4: *Let X = Y = $\{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, c\}\}$ and $\mu = \{Y, \phi, \{a\}\}$. Define a function $f : X \to Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is continuous but not strongly $g\omega\alpha$ -continuous, since for $g\omega\alpha$ -closed set $\{c\}$ in Y, $f^{-1}(\{c\}) = \{c\}$ is not closed in X.*

Theorem 3.5: *For a function $f : X \to Y$ the followings are equivalent:*

(i) f is strongly $g\omega\alpha$ -continuous.

(ii) For each $x \in X$ and each $g\omega\alpha$ -open set V in Y with $f(x) \in V$, there exists an open set U in X such that $x \in U$ and $f(U) \subset V$.

(iii) $f^{-1}(V) \subset int(f^{-1}(V)$ for each $g\omega\alpha$ -open set V of Y.

(iv) $f^{-1}(F)$ is closed in X for every $g\omega\alpha$ -closed set F of Y.

Proof:

(i) $\Rightarrow$ (ii): Let $x \in X$ and V be a $g\omega\alpha$ -open set in Y containing f(x). By hypothesis, $f^{-1}(V)$ is an open set in X such that $x \in f^{-1}(V)$. Put $U = f^{-1}(V)$, then $x \in U$ and $f(U) = f(f^{-1}(V)) \subset V$. Thus (ii) holds

(ii)$\Rightarrow$(iii): Let V be any $g\omega\alpha$ -open set in Y and $x \in f^{-1}(V)$. by (ii), there exists an open set U in X such that $x \in X$ and $f(U) \subset V$. This implies $x \in U \subset int(U) \subset int(f^{-1}(V))$, which implies $x \in int(f^{-1}(V)$. Therefore, $f^{-1}(V) \subset int(f^{-1}(V))$.

(iii)$\Rightarrow$(iv): Let F be any $g\omega\alpha$ -closed set of Y. Set $V = Y - F$, then V is $g\omega\alpha$ -open in Y. By (iii) $f^{-1}(V) \subset int(f^{-1}(V))$. That is $f^{-1}(Y-F) \subset int(f^{-1}(Y-F))$. This implies $X - f^{-1}(F) \subset X - cl(f^{-1}(F))$. This implies $cl(f^{-1}(F)) \subset f^{-1}(F)$. But $f^{-1}(F) \subset cl(f^{-1}(F))$ is always true. Therefore, $f^{-1}(F) = cl(f^{-1}(F))$. This shows that, $f^{-1}(F)$ is closed in X.

(vi)$\Rightarrow$(i): Let V be any $g\omega\alpha$ -open set of Y. Set $F = Y - V$. Then F is $g\omega\alpha$ -closed set of Y. By (iv), $f^{-1}(F)$ is closed in X. But $f^{-1}(F) = f^{-1}(Y-V) = X - f^{-1}(V)$. This implies $f^{-1}(V)$ is an open set in X. Therefore f is strongly $g\omega\alpha$ -continuous.

Theorem 3.6: *Let $f : X \to Y$ be a function and $\{A_i : i \in I\}$ be an open cover of X. Then f is strongly $g\omega\alpha$ -continuous, if the restricted function $f|_{A_i} : A \to Y$ is strongly $g\omega\alpha$ -continuous for each $i \in I$.*

Proof: Let V be a $g\omega\alpha$ -open set of Y. Since $f|_{A_i}$ is strongly $g\omega\alpha$ -continuous, $(f|_{A_i})^{-1}(V)$ is an open in A_i. Since A_i is an open set in X, $(f|_{A_i})^{-1}(V)$ is open in X for each $i \in I$. Therefore $f^{-1}(V) = X \cap f^{-1}(V) = \bigcup\{A_i \cap f^{-1}(V) : i \in I\} = \bigcup\{(f|_{A_i})^{-1}(V) : i \in I\}$ is open in X. Hence f is strongly $g\omega\alpha$ -continuous.

Theorem 3.7: *If $f : X \to Y$ is strongly $g\omega\alpha$ -continuous, then the restriction function $f|_A : A \to Y$ is strongly $g\omega\alpha$ -continuous.*

Proof: Let V be $g\omega\alpha$ -open set of Y. Since f is strongly $g\omega\alpha$ -continuous, $f^{-1}(V)$ is an open set in X. Since A is open in X, implies $(f|_A)^{-1}(V) = A \cap f^{-1}(V)$ is open in A and hence $f|_A$ is strongly $g\omega\alpha$ -continuous.

Theorem 3.8: *Let Y be $T_{g\omega\alpha}$ -space and $f : X \to Y$ be any function. Then followings are equivalent*

(i) f is strongly $g\omega\alpha$ -continuous function.
(ii) f is continuous.

Proof:
$(i) \Rightarrow (ii)$: Obvious because every open set is $g\omega\alpha$ -open set.

$(ii) \Rightarrow (i)$: Suppose F is $g\omega\alpha$ -closed set in Y and Y is $T_{g\omega\alpha}$ -space. This implies F is closed in Y. Since f is continuous, $f^{-1}(F)$ is closed in X. Hence f is strongly $g\omega\alpha$ -continuous function.

Remark 3.9: *Every strongly $g\omega\alpha$ -continuous function is $g\omega\alpha$ -irresolute. But converse need not be true in general.*

Example 3.10: *Let X = Y = $\{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $\mu = \{Y, \phi, \{a\}\}$. Let function $f : X \to Y$ be an identity function, then f is $g\omega\alpha$ -irresolute but not strongly $g\omega\alpha$ -continuous. Since for $g\omega\alpha$ -closed set $\{c\}$ in Y, $f^{-1}(\{c\}) = \{c\}$ is not closed in X.*

Remark 3.11: *Every strongly continuous function is strongly $g\omega\alpha$ -continuous but not conversely.*

Example 3.12: *Let X = Y = { a, b, c } and* τ = { X, ϕ, { a, b }, { b, c }, { b }} *and* μ = { Y, ϕ, { a }, { a, c }}. *Define a function* $f: X \to Y$ *by* $f(a) = b$, $f(b) = a$ *and* $f(c) = c$ *then* f *is strongly* $g\omega\alpha$ *-continuous but not strongly continuous. Because for* $g\omega\alpha$ *-open set* $\{a\}$ *in Y,* $f^{-1}(\{a\})$ = $\{b\}$ *is open but not closed in X.*

Theorem 3.13: *Let* $f: X \to Y$ *and* $g: Y \to Z$ *be two functions. Then*

(i) If f and g are strongly $g\omega\alpha$ -continuous functions, then $(g \circ f)$ is strongly $g\omega\alpha$ -continuous.
(ii) If f is continuous and g is strongly $g\omega\alpha$ -continuous, then $(g \circ f)$ is strongly $g\omega\alpha$ -continuous.
(iii) If f is $g\omega\alpha$ -continuous and g is strongly $g\omega\alpha$ -continuous, then $(g \circ f)$ is $g\omega\alpha$ -irresolute.
(iv) If f is strongly $g\omega\alpha$ -continuous and g is $g\omega\alpha$ -continuous, then $(g \circ f)$ is continuous.
(v) If f is strongly $g\omega\alpha$ -continuous and g is continuous then $(g \circ f)$ is continuous function.

4. STRONGL $g\omega\alpha$ -CONTINUOUS FUNCTIONS

Definition 4.1: *A function* $f: X \to Y$ *is said to be strongly* $g\omega\alpha^*$ *-continuous, if* $f^{-1}(V)$ *is* α *-closed in X for every* $g\omega\alpha$ *-closed set V in Y.*

Theorem 4.2: *A function* $f: X \to Y$ *is strongly* $g\omega\alpha^*$ *-continuous, if and only if the inverse image of each* $g\omega\alpha$ *-open set in Y is an* α *-open set in X.*

Remark 4.3: *Every strongly* $g\omega\alpha^*$ *-continuous function is* α *-continuous, but converse need not be true in general.*

Example 4.4: *Let X = Y = { a, b, c } and* τ = { X, ϕ, { a }, { b, c }} *and* μ = { Y, ϕ, { a }}. *Then an identity function* $f: X \to Y$ *is* α *-continuous, but not strongly* $g\omega\alpha^*$ *-continuous. Because for* $g\omega\alpha$ *-open set* { a, c } *in Y,* $f^{-1}(\{ a, c \})$ = { a, c } *is not* α *-open in X.*

Theorem 4.5: *Let X be a topological space, Y is* $T_{g\omega\alpha}$ *-space and* $f: X \to Y$ *is any function, then followings are equivalent:*

(i) f is strongly $g\omega\alpha^*$ -continuous function.
(ii) f is α -continuous.

Proof:
$(i) \Rightarrow (ii)$: Obvious because every open set is $g\omega\alpha$ -open set.

$(ii) \Rightarrow (i)$: Suppose F is $g\omega\alpha$ -closed in Y and Y is $T_{g\omega\alpha}$ -space.This implies F is closed in Y. Since f is α -continuous $f^{-1}(F)$ is α -closed in X. Hence f is strongly $g\omega\alpha^*$ -continuous function.

Remark 4.6: *Every strongly* $g\omega\alpha^*$ *-continuous function is* $g\omega\alpha$ *-irresolute, but converse need not be true in general.*

Example 4.7: *Let X = Y = { a, b, c } and* τ = { X, ϕ, { a }, { b, c }} *and* μ = { Y, ϕ, { a }, { a, c }}. *Then an identity function* $f: X \to Y$ *is* $g\omega\alpha$ *-irresolute but not strongly* $g\omega\alpha^*$ *-continuous, since* { a, b } *is* $g\omega\alpha$ *-open set in Y, but* $f^{-1}(\{a,b\}) = \{a,b\}$ *is not* α *-open in X.*

Theorem 4.8: *The followings are equivalent for the function* $f: X \to Y$ *:*

(i) f is strongly $g\omega\alpha^*$ -continuous.
(ii) For each $x \in X$ and each $g\omega\alpha$ -open set V in Y with $f(x) \in V$, there exist an α -open set U in X such that $x \in U$ and $f(U) \subset V$.

(iii) $f^{-1}(V) \subset \alpha - int(f^{-1}(V)$ for each $g\omega\alpha$ -open set V of Y.

(iv) $f^{-1}(F)$ is α -closed in X for every $g\omega\alpha$ -closed set F of Y.

Proof. Proof is obvious.

Definition 4.9: *A function $f : X \to Y$ is said to be perfectly $g\omega\alpha$ -continuous, if $f^{-1}(V)$ is clopen in X for every $g\omega\alpha$ -open set V in Y.*

Theorem 4.10: *A function $f : X \to Y$ is perfectly $g\omega\alpha$ -continuous, if and only if the inverse image of every $g\omega\alpha$ -closed set in Y is clopen in X.*

Proof: Similar to the proof of theorem 3.2.

Remark 4.11: *Every perfectly $g\omega\alpha$ -continuous function is continuous function. But converse need not be true in general.*

Example 4.12: *Let X = Y = $\{$ a, b, c $\}$ and τ = $\{$ X, ϕ, $\{$ a $\}$, $\{$ b, c $\}\}$ and μ = $\{$ Y, ϕ, $\{$ a $\}\}$. Then an identity function $f : X \to Y$ is continuous, but not perfectly $g\omega\alpha$ -continuous. Because for $g\omega\alpha$ -open set $\{a,c\}$ in Y, $f^{-1}(\{a,c\}) = \{a,c\}$ is not clopen in X.*

Remark 4.13: *Every perfectly $g\omega\alpha$ -continuous function is strongly $g\omega\alpha$ -continuous function. But converse need not be true in general.*

Example 4.14: *Let X = Y = $\{$ a, b, c $\}$ and τ = $\{$ X, ϕ, $\{$ a $\}$, $\{$ a, b $\}$, $\{$ a, c $\}\}$ and μ = $\{$ Y, ϕ, $\{$ a $\}\}$. Then an identity function $f : X \to Y$ is strongly $g\omega\alpha$ -continuous, but not perfectly $g\omega\alpha$ -continuous. Because for $g\omega\alpha$ -open set $\{a,b\}$ in Y, $f^{-1}(\{a,b\}) = \{a,b\}$ is not clopen in X.*

Remark 4.15: *Every perfectly $g\omega\alpha$ -continuous function is perfectly continuous function, But not conversely.*

Example 4.16: *Let X = Y = $\{$ a, b, c $\}$ and τ = $\{$ X, ϕ, $\{$ a $\}$, $\{$ b $\}$, $\{$ a, b $\}$, $\{$ a, c $\}\}$ and μ = $\{$ Y, ϕ,$\{$ a $\}\}$. Define a function $f : X \to Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$ Then f is perfectly continuous, but not perfectly $g\omega\alpha$ -continuous, because for $g\omega\alpha$ -open set $\{$ a, c $\}$ in Y, $f^{-1}(\{a,c\}) = \{b,c\}$ is closed but not open in X.*

Remark 4.17: *The converse of the above remark 4.15 is true if Y is $T_{g\omega\alpha}$ -space.*

Proof: Let G be a $g\omega\alpha$ -open in Y. Since Y is $T_{g\omega\alpha}$ -space, G is an open set in Y. Since f is perfectly continuous, $f^{-1}(G)$ is clopen in X. Therefore f is perfectly $g\omega\alpha$ -continuous.

Theorem 4.18: *Every perfectly $g\omega\alpha$ -continuous function in finite T_1 -space is strongly continuous.*

Proof: Obvious, because every finite T_1 -space is discrete space. Therefore every subset of X is open and hence $g\omega\alpha$ -open. Since f is perfectly $g\omega\alpha$ -continuous function, $f^{-1}(A)$ is clopen for every subset of Y. Therefore f is strongly continuous.

Theorem 4.19: *Let X be a discrete topological space, Y be any topological space and $f : X \to Y$ be a function. Then the followings are equivalent:*

(i) f is perfectly $g\omega\alpha$ -continuous.

(ii) f is strongly $g\omega\alpha$ -continuous.

Proof:
$(i) \Rightarrow (ii)$: Follows from every clopen set is open.

$(ii) \Rightarrow (i)$: Let V be $g\omega\alpha$ -open in Y. By hypothesis, $f^{-1}(V)$ is open in X. Since X is discrete space, $f^{-1}(V)$ is also closed set in X. Therefore f is perfectly $g\omega\alpha$ -continuous.

Theorem 4.20: *A function $f : X \to Y$ is perfectly $g\omega\alpha$ -continuous if the graph function $g : X \times X \to Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, is perfectly $g\omega\alpha$ -continuous.*

Proof. Let V be any $g\omega\alpha$ -open set of Y. Then $X \times V$ is $g\omega\alpha$ -open set of $X \times Y$. Since g is perfectly $g\omega\alpha$ -continuous, $f^{-1}(V) = g^{-1}(X \times V)$ is clopen in X. Therefore f is perfectly $g\omega\alpha$ -continuous.

Theorem 4.21: *If $f : X \to Y$ is perfectly $g\omega\alpha$ -continuous, then the restricted function $f|_A : A \to Y$ is perfectly $g\omega\alpha$ -continuous for any subset A of X.*

Proof: Let V be a $g\omega\alpha$ -open set of Y. Since f is perfectly $g\omega\alpha$ -continuous, $f^{-1}(V)$ is clopen set in X. Then $(f|_A)^{-1}(V) = A \cap f^{-1}(V)$ is clopen in A and hence $f|_A$ is perfectly $g\omega\alpha$ -continuous.

Theorem 4.22: *Let $f : X \to Y$ and $g : Y \to Z$ be two functions.*

(i) If f and g are perfectly $g\omega\alpha$ -continuous functions, then $(g \circ f)$ is perfectly $g\omega\alpha$ -continuous function.
(ii) If f is perfectly $g\omega\alpha$ -continuous function and g is $g\omega\alpha$ -irresolute, then $(g \circ f)$ is perfectly $g\omega\alpha$ -continuous function.
(iii) If f is perfectly continuous function and g is strongly continuous, then $(g \circ f)$ is perfectly $g\omega\alpha$ -continuous function.
(iv) If f is perfectly $g\omega\alpha$ -continuous function and g is $g\omega\alpha$ -continuous, then $(g \circ f)$ is perfectly $g\omega\alpha$ -continuous function.
(v) If f is perfectly $g\omega\alpha$ -continuous function and g is $g\omega\alpha^*$ -continuous, then $(g \circ f)$ is totally α -continuous function.
(vi) If f is $g\omega\alpha$ -continuous function and g is strongly continuous, then $(g \circ f)$ is $g\omega\alpha$ -continuous function.
(vii) If f is $g\omega\alpha$ -irresolute function and g is perfectly $g\omega\alpha$ -continuous, then $(g \circ f)$ is $g\omega\alpha$ -irresolute function.

Proof:

(i) Suppose F is a $g\omega\alpha$ -closed set in Z. Since g is perfectly $g\omega\alpha$ -continuous function $g^{-1}(F)$ is clopen in Y. Now f is perfectly $g\omega\alpha$ -continuous function and every closed set is $g\omega\alpha$ -closed set, implies $g^{-1}(F)$ is $g\omega\alpha$ -closed set in Y and $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is clopen in X. Therefore $(g \circ f)$ is perfectly $g\omega\alpha$ -continuous.

(ii) Suppose F is a $g\omega\alpha$ -closed set in Z. Since g is $g\omega\alpha$ -irresolute, $g^{-1}(F)$ is $g\omega\alpha$ -closed set in Y. Now f is perfectly $g\omega\alpha$ -continuous function, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is clopen in X. Therefore $(g \circ f)$ is perfectly $g\omega\alpha$ -continuous.

(iii) Suppose F is a $g\omega\alpha$ -closed set in Z. Since g is strongly continuous, $g^{-1}(F)$ is clopen and hence $g\omega\alpha$ -open set in Y. Now f is perfectly $g\omega\alpha$ -continuous function, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is clopen in X. Therefore $(g \circ f)$ is perfectly $g\omega\alpha$ -continuous.

(iv) Suppose F is an open set in Z. Since g is $g\omega\alpha$ -continuous, $g^{-1}(F)$ is $g\omega\alpha$ -open set in Y. Now f is perfectly $g\omega\alpha$ -continuous function, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is clopen in X. Therefore $(g \circ f)$ is perfectly $g\omega\alpha$ -continuous.

(v) Suppose F is an α -open set in Z. Since g is $g\omega\alpha^*$ -continuous, $g^{-1}(F)$ is $g\omega\alpha$ -open set in Y. Now f is perfectly $g\omega\alpha$ -continuous function, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is clopen in X. Therefore $(g \circ f)$ is totally α -continuous.

(vi) Let G be an open set in Z. Since g is strongly continuous, $g^{-1}(G)$ is clopen in Y and hence open in Y.

(vii) Since f is $g\omega\alpha$ -continuous function, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is $g\omega\alpha$ -open in X. Hence $(g \circ f)$ is $g\omega\alpha$ -continuous.

(viii) Let G be a $g\omega\alpha$ -open set in Z. Since g is perfectly $g\omega\alpha$ -continuous, $g^{-1}(G)$ is clopen and hence it is $g\omega\alpha$ -open in Y. Again since f is $g\omega\alpha$ -irresolute, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is $g\omega\alpha$ -open in X. Therefore $(g \circ f)$ is $g\omega\alpha$ -irresolute.

Definition 4.23: *A function $f : X \to Y$ is called completely $g\omega\alpha$ -continuous, if the inverse image of every $g\omega\alpha$ -open set in Y is regular open in X.*

Theorem 4.24: *A function $f : X \to Y$ is completely $g\omega\alpha$ -continuous, if and only if the inverse image of every $g\omega\alpha$ -closed set in Y is regular closed in X.*

Proof: Similar to the proof of theorem 3.2.

Remark 4.25: *Every completely $g\omega\alpha$ -continuous function is continuous, but converse need not be true in general*

Example 4.26: *Let X = Y = $\{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}, \{b, c\}, \{b\}\}$ and $\mu = \{Y, \phi, \{a\}\}$. Define a function $f : X \to Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is continuous but not completely $g\omega\alpha$ -continuous, since for the $g\omega\alpha$ -open set $\{a, c\}$ in Y, $f^{-1}(\{a, c\}) = \{b, c\}$ is not regular open in X.*

Remark 4.27: *Every completely $g\omega\alpha$ -continuous function is completely continuous. But converse need not be true in general.*

Example 4.28: *Let X = Y = $\{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $\mu = \{Y, \phi, \{a\}\}$. Then an identity function $f : X \to Y$ is completely continuous, but not completely $g\omega\alpha$ -continuous, since for the $g\omega\alpha$ -open set $\{a, c\}$ in Y, $f^{-1}(\{a, c\}) = \{a, c\}$ is not regular open in X.*

Remark 4.29: *Every completely $g\omega\alpha$ -continuous function is strongly $g\omega\alpha$ -continuous. But converse need not be true in general.*

Example 4.30: *Let X = Y = $\{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}, \{b, c\}, \{b\}\}$ and $\mu = \{Y, \phi, \{a\}, \{a, c\}\}$. Define a function $f : X \to Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is strongly $g\omega\alpha$ -continuous, but not completely $g\omega\alpha$ -continuous, since for the $g\omega\alpha$ -open set $\{a, b\}$ in Y, $f^{-1}(\{a, b\}) = \{a, b\}$ is not regular open in X.*

Theorem 4.31: *If a function $f : X \to Y$ is completely continuous and Y is $T_{g\omega\alpha}$ -space, then f is completely $g\omega\alpha$ -continuous.*

Proof: Let G be a completely $g\omega\alpha$ -open set in Y. Since Y is $T_{g\omega\alpha}$ -space, G is an open in Y. Since f is completely continuous, $f^{-1}(G)$ is regular open in X. Therefore, f is completely $g\omega\alpha$ -continuous function.

Theorem 4.32: *If a function $f : X \to Y$ is completely $g\omega\alpha$ -continuous if the graph function $g : X \times X \to Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, is completely $g\omega\alpha$ -continuous.*

Proof: Let V be any $g\omega\alpha$ -open set in Y. Then $X \times V$ is a $g\omega\alpha$ -open set of $X \times Y$. Since g is completely $g\omega\alpha$ -continuous, $f^{-1}(V) = g^{-1}(X \times V)$ is regular open in X. Thus f is completely $g\omega\alpha$ -continuous.

Lemma 4.33 [18]: *Let Y be preopen subset of X. Then $Y \cap U$ is regular open in Y for each regular open set U of X.*

Theorem 4.34: *Let A be preopen of X. If $f : X \to Y$ is completely $g\omega\alpha$ -continuous, then the restricted function $f|_A : A \to Y$ is perfectly $g\omega\alpha$ -continuous.*

Proof: Let A be a $g\omega\alpha$ -open set of Y. Then, $(f|_A)^{-1}(V) = A \cap f^{-1}(V)$. Since $f^{-1}(V)$ is regular open and A is preopen, by lemma 4.33, $(f|_A)^{-1}(V)$ is regular open in the relative topology of A. Hence $f|_A$ is completely $g\omega\alpha$ -continuous.

Theorem 4.14: *Let $f : X \to Y$ and $g : Y \to Z$ be two functions. Then*

(i) If f is completely continuous and g is completely $g\omega\alpha$ -continuous, then $(g \circ f)$ is completely $g\omega\alpha$ -continuous.
(ii) If f is completely $g\omega\alpha$ -continuous and g is $g\omega\alpha$ -irresolute, then $(g \circ f)$ is completely $g\omega\alpha$ -continuous.
(iii) If f is completely $g\omega\alpha$ -continuous and g is strongly $g\omega\alpha$ -continuous, then $(g \circ f)$ is completely $g\omega\alpha$ -continuous.

Proof.

(i) Let G be a $g\omega\alpha$ -open set in Z. Then $g^{-1}(G)$ is regular open in Y as g is completely $g\omega\alpha$ -continuous. So, $g^{-1}(G)$ is open in Y. Since f is completely continuous, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is regular open in X. Hence $(g \circ f)$ is completely $g\omega\alpha$ -continuous.
(ii) Let G be a $g\omega\alpha$ -open set in Z. Since g is $g\omega\alpha$ -irresolute, $g^{-1}(G)$ is $g\omega\alpha$ -open in Y. Since f is completely $g\omega\alpha$ -continuous, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is regular open in X. Hence $(g \circ f)$ is completely $g\omega\alpha$ -continuous.
(iii) Let G be a $g\omega\alpha$ -open set in Z. As g is strongly $g\omega\alpha$ -continuous, $g^{-1}(G)$ is open and hence $g\omega\alpha$ -open in Y. Again Since f is completely $g\omega\alpha$ -continuous, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is regular open in X. Hence $(g \circ f)$ is completely $g\omega\alpha$ -continuous.

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