International Journal of Mathematical Archive-7(10), 2016, 115-122
Available online through www.ijma.info ISSN 2229 – 5046

On Nano Regular Generalized Star Star b-Closed Sets in Nanotopological Spaces

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(Received On: 19-09-16; Revised & Accepted On: 24-10-16)

ABSTRACT

In this paper we introduce the concept of nano regular generalized star star b-closed sets and investigate some of its properties in nanotopological spaces. We also define nano $rg^{**}b$-continuity, nano $rg^{**}b$-open and some of its fundamental properties are given.

Keywords: *Nano $rg^{*}b$-closed set, Nano $rg^{**}b$-closed set, Nano $rg^{**}b$-open set, Nano $rg^{**}b$-continuity.*

INTRODUCTION

In 1970, Levine [8] introduced the concept of generalized closed sets in topological spaces. Ahmed and Mohd [1] (2009) studied the class of generalized b-closed sets. Sindhu and Indirani [6] (2013) introduced the concept of regular generalized star b-closed sets in topological spaces. Banupriya and indirani [2] (2014) introduced the concept of regular generalized star star b-closed sets in topological spaces. The notion of nanotopology was introduced by Lellis Thivagar which was defined in terms of approximations and boundary regions of a subset of an universe using an equivalence relation on it and also defined nano closed, nano interior and nano closure. Smitha and Indirani [7] (2015) introduced the concept of nano regular generalized star b-closed sets in nanotopological spaces. The aim of this paper, we introduce a new class of nano sets on nanotopological spaces called nano regular generalized star star b-closed sets, nano $rg^{**}b$-continuity, nano $rg^{**}b$-open and obtain their characteristics with counter examples.

2. PRELIMINARIES

Definition 2.1 [9]: Let Ube a non-emptyfinite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation, elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \cup_{x\in U}\{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.
ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \cup_{x\in U}\{R(x): R(x) \cap X \neq \emptyset\}$ where $R(x)$ denotes the equivalence class determined by x.
iii) The boundary region of X with respect to R is set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2[9]: Let U be a non-empty, finite universe of objects and R be an equivalence relation on U. Let $X \subseteq U$. $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$. Then $\tau_R(X)$ is a topology on$U$, called as the nanotopology with respect to X. Elements of nanotopology are known as the nanoopen sets in U and $(U, \tau_R(X))$ is called the nanotopological space. Elements of $(\tau_R(X))^c$ are called as nano closed sets.

Definition 2.3 [9]: If $\tau_R(X)$ is the nanotopology on U with respect to X, and then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

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Definition 2.4 [9]: If $(U, \tau_R(X))$ is a nanotopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of allnano open subsets of A and it is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest nano open subset of A.Thenano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$. That is $Ncl(A)$ is the smallest nano closed set containing A.

Definition 2.5 [9]: Let $(U, \tau_R(X))$ be a nanotopological space and$A \subseteq U$. Then A is said to be:

i) Nano semi-open if $A \subseteq Ncl(Nint(A))$
ii) Nano pre-open if $A \subseteq Nint(Ncl(A))$
iii) Nano α-open if $A \subseteq Nint(Ncl(Nint(A)))$
iv) Nano semi pre-open if $A \subseteq Ncl(Nint(Ncl(A)))$

$NSO(U,X), NPO(U,X), N\alpha O(U,X), NSPO(U,X)$ respectively denote the families of all nano semi-open, nano pre-open and nano semi pre-open subsets of U.

Let $(U, \tau_R(X))$ be a nanotopological space and$A \subseteq U$. A is said to be nano semi-closed, nano pre-closed, nano α-closed, nano semi pre-closed if its complement is respectively nano semi-open, nano pre-open, nano α-open, nano semi pre-open.

Definition 2.6: A subset A of $(U, \tau_R(X))$ is called

i) Nano generalized closed set [4] (briefly Ng-closed) if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open in $(U, \tau_R(X))$.
ii) Nano generalized star closed set [11] (briefly Ng^*-closed) if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano g-open in $(U, \tau_R(X))$.
iii) Nano generalized α-closed set [3] (briefly $Ng\alpha$-closed) if $N\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano α-open in $(U, \tau_R(X))$.
iv) Nano generalized pre-closed set [5] (briefly Ngp-closed) if $Npcl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano-open in $(U, \tau_R(X))$.
v) Nano generalized pre-regular closed set (briefly $Ngpr$-closed) if $Npcl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano r-open in $(U, \tau_R(X))$.
vi) Nano regular-generalized closed set (briefly Nrg-closed) if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano r-open in $(U, \tau_R(X))$.

Proposition 2.7 [9]: If(U, R) is an approximation space $X, Y \subseteq U$, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\emptyset) = \emptyset = U_R(\emptyset)$ and $L_R(U) = U = U_R(U)$
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$
8. $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$

3. Nano regular generalized star star b-closed sets ($Nrg^{**}b$-closed)

Definition 3.1: A subset A of a nanotopological space $(U, \tau_R(X))$ is called nano regular generalized star star b-closed set (briefly $Nrg^{**}b$-closed) if $Nrg^*bcl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open in $(U, \tau_R(X))$.

Theorem 3.2: Every nano closed set is nano $rg^{**}b$-closed.

Proof: If A is nano closed in $(U, \tau_R(X))$ and G is nano open in U such that $A \subseteq G$. Since A is nano closed, $Ncl(A) = A \subseteq G$. By theorem 3.7[7] " every nano closed set is nano rg^*b-closed", $Nrg^*bcl(A) \subseteq Ncl(A) = A$. Therefore $Nrg^*bcl(A) \subseteq G$. Hence A is nano$rg^{**}b$-closed.

Converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$. Then the nanotopology $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Then the sets $\{a\}$ and $\{c\}$ are nano $rg^{**}b$-closed but not nano closed.

Theorem 3.4: Every nano semi closed set is nano $rg^{**}b$-closed.

Proof: Let$A$ benano semi closed in $(U, \tau_R(X))$ and G is nano open in U such that $A \subseteq G$. Since A is nano semi closed, $Nscl(A) = A \subseteq G$. By theorem 3.7[7] " every nano semi closed set is nano rg^*b-closed", $Nrg^*bcl(A) \subseteq Nscl(A) = A$. Therefore $Nrg^*bcl(A) \subseteq G$. Hence A is nano $rg^{**}b$-closed.

Converse of the above theorem need not be true as seen from the following example.

Example 3.5: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$. Then the nanotopology $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Then the sets $\{a, b, c\}$ and $\{a, b, d\}$ are nano $rg^{**}b$-closed but notnanosemi closed.

Theorem 3.6: Let $(U, \tau_R(X))$ be nanotopological space and $A \subseteq U$. Then
- i) Every nano pre closed set is nano $rg^{**}b$-closed.
- ii) Every nano α-closed set is nano $rg^{**}b$-closed.
- iii) Every nano regular closed set is nano $rg^{**}b$-closed.

Reverse implications of the theorem need not be true as seen from the following examples.

Example 3.7: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$. Then the nanotopology $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Then the set $\{a\}$is nano $rg^{**}b$-closed but not nano pre closed.

Example 3.8: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$. Then the nanotopology $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Then the set $\{a\}$ and $\{a\}$ are nano $rg^{**}b$-closed but notnanoα- closed. Also the sets $\{a\}, \{b\}$ and $\{a, d\}$ are nano $rg^{**}b$-closed but not nano regular closed.

Theorem 3.9: Every nano$g$-closed set is nano $rg^{**}b$-closed.

Proof: Let A be nano$g$-closed in $(U, \tau_R(X))$ and G is nano open in U such that $A \subseteq G$. Since A is nano g-closed, $Ncl(A) \subseteq G$. By theorem 3.7[7] " every nano closed set is nano rg^*b-closed", $Nrg^*bcl(A) \subseteq Ncl(A) = A$. Therefore $Nrg^*bcl(A) \subseteq G$. Hence A is nano $rg^{**}b$-closed.

Converse of the above theorem need not be true as seen from the following example.

Example 3.10: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$. Then the nanotopology $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Then the sets $\{c\}$ and $\{c, d\}$ are nano $rg^{**}b$-closed but notnano$g$-closed.

Theorem 3.11: Every nanog^*-closed set is nano $rg^{**}b$-closed.

Proof: Let A be nanog^*-closed in $(U, \tau_R(X))$. Therefore $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano g-open. As every nano open set is nano g-open, we get$Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open. But it is true that "Every nano closed set is nano rg^*b-closed set. Therefore $Nrg^*bcl(A) \subseteq Ncl(A) \subseteq G$. Hence A is nano $rg^{**}b$-closed.

Converse of the above theorem need not be true as seen from the following example.

Example 3.12: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$. Then the nanotopology $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Then the sets $\{a\}$ and $\{d\}$ are nano $rg^{**}b$-closed but notnanog^*-closed.

Theorem 3.13: Every nano$g\alpha$-closed set is nano $rg^{**}b$-closed.

Proof: Let$A$ benano$g\alpha$-closed in $(U, \tau_R(X))$ and G is nano open in U such that $A \subseteq G$. Since A is nano $g\alpha$-closed, $N\alpha cl(A) \subseteq G$. By theorem 3.7[7] " every nano α-closed set is nano rg^*b-closed", $Nrg^*bcl(A) \subseteq N\alpha cl(A) = A$.

Therefore $Nrg^*bcl(A) \subseteq G$. Hence A is nano $rg^{**}b$-closed.

Converse of the above theorem need not be true as seen from the following example.

Example 3.14: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$. Then the nanotopology $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Then the sets $\{a\}$ and $\{c, d\}$ are nano $rg^{**}b$-closed but notnano$g\alpha$-closed.

Theorem 3.15: Every nanogp-closed set is nano $rg^{**}b$-closed.

Proof: Let$A$ benanogp-closed in $(U, \tau_R(X))$ and G is nano open in U such that $A \subseteq G$. Since A is nano gp-closed, $Npcl(A) \subseteq G$. By theorem 3.7[7] " every nano pre-closed set is nano $rg^{*}b$-closed", $Nrg^{*}bcl(A) \subseteq Npcl(A) = A$. Therefore $Nrg^{*}bcl(A) \subseteq G$. Hence A is nano $rg^{**}b$-closed.

Converse of the above theorem need not be true as seen from the following example.

Example 3.16: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$. Then the nanotopology $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Then the sets $\{a\}$ is nano $rg^{**}b$-closed but notnanogp-closed.

Remark 3.17: The nano$rg^{**}b$-closed set is an independent of the nano gpr-closed and nano rg-closed as shown by the following example.

The set $\{a\}$ is nano $rg^{**}b$-closed but not nanogpr-closed. The set $\{a, c, d\}$ is nano gpr-closed but notnano$rg^{**}b$-closed. Also the set $\{d\}$ is nano $rg^{**}b$-closed but notnanorg-closed. The set $\{a, c, d\}$ is nano rg-closed but notnano$rg^{**}b$-closed.

Remark 3.18: The union of two nano $rg^{**}b$-closed sets need not be nano $rg^{**}b$-closed set can be seen from the following example.

Example 3.19: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$. Then the nanotopology $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Then the sets $\{a\}$ and $\{c\}$ are nano $rg^{**}b$-closed sets but $\{a\} \cup \{c\} = \{a, c\}$ is notnano$rg^{**}b$-closed.

Remark 3.20: The intersection of two nano $rg^{**}b$-closed sets need not be nano $rg^{**}b$-closed set can be seen from the following example.

Example 3.21: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$. Then the nanotopology $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Then the sets $\{a, b, c\}$ and $\{a, b, d\}$ are nano $rg^{**}b$-closed sets but $\{a, b, c\} \cap \{a, b, d\} = \{a, b\}$ is notnano$rg^{**}b$-closed.

Theorem 3.22: A set A is nano $rg^{**}b$-closed if and only if $Nrg^{*}bcl(A) - A$ contains no non-empty nano closed set.

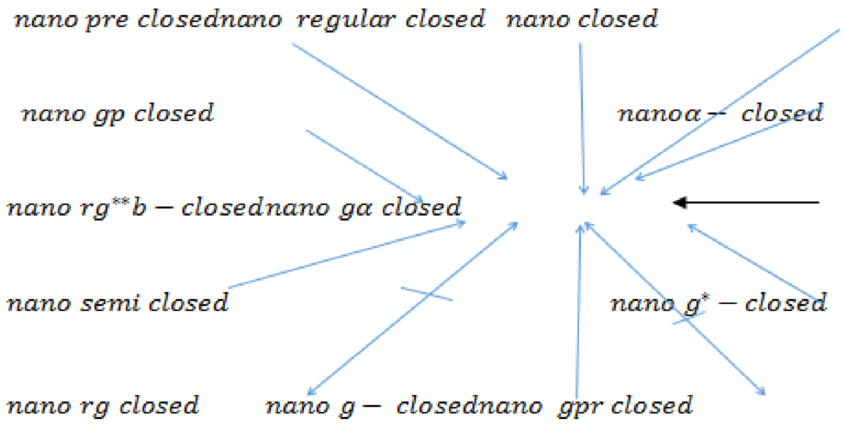
Proof: Necessity: Let F be a nano closed set in $(U, \tau_R(X))$ such that $F \subseteq Nrg^{*}bcl(A) - A$. Then $A \subseteq X - F$. Since A is nano regular generalized star star b-closed set and $X - F$ is nano open then $Nrg^{*}bcl(A) \subseteq X - F$.That is $F \subseteq X - Nrg^{*}bcl(A)$. So $F \subseteq X - Nrg^{*}bcl(A) \cap Nrg^{*}bcl(A) - A$. Therefore $F = \emptyset$.

Sufficiency: Let us assume that $Nrg^{*}bcl(A) - A$ contains no non-empty nano closed set. Let $A \subseteq G$ and G is nano open. Suppose that $Nrg^{*}bcl(A)$ is not contained in G, $Nrg^{*}bcl(A) \cap G^C$ is non-empty nano closed set of $Nrg^{*}bcl(A) - A$ which is a contradiction. Therefore $Nrg^{*}bcl(A) \subseteq G$. Hence A is nano $rg^{**}b$-closed.

Theorem 3.23: If A is nano $rg^{**}b$-closed and $A \subseteq B \subseteq Nrg^{*}bcl(A)$ then B is alsonano$rg^{**}b$-closed.

Proof: Let B ⊆ G where G is nano open in $(U, \tau_R(X))$. Then $A \subseteq B$ implies $A \subseteq G$. Since A is nano $rg^{**}b$-closed, $Nrg^{*}bcl(A) \subseteq G$. Also B ⊆ $Nrg^{*}bcl(A)$ implies $Nrg^{*}bcl(B) \subseteq Nrg^{*}bcl(A)$. Thus $Nrg^{*}bcl(B) \subseteq G$ and B is nano$rg^{**}b$-closed.

Remark 3.24: The above discussions are summarized in the following diagram



4. Nano regular generalized star star b-open sets($Nrg^{**}b-open\ set$)

Definition 4.1: A subset A of a nanotopological space $(U,\tau_R(X))$ is called nano regular generalized star star b-open set (briefly $Nrg^{**}b$-open) if A^C is nano $rg^{**}b$-closed.

Theorem 4.2: Every nano open set is nano $rg^{**}b$-open.

Proof follows from the Theorem 3.2.

Remark 4.3: For subsets A,B of a nanotopological space$(U,\tau_R(X))$

i) $U-Nrg^*bint(A)=Nrg^*bcl(U-A)$

ii) $U-Nrg^*bcl(A)=Nrg^*bint(U-A)$

Theorem 4.4: A subset $A\subseteq U$ is nano $rg^{**}b$-open if and only if $F\subseteq Nrg^*bint(A)$ whenever F is nano closed set and $F\subseteq A$.

Proof: Let A benano$rg^{**}b$-open and suppose $F\subseteq A$ where F is nano closed. Then $U-A$ is nano $rg^{**}b$-closed set containing in the nano open set $U-F$. Hence $Nrg^*bcl(U-A)\subseteq U-F$ and $U-Nrg^*bint(A)\subseteq U-F$. Thus $F\subseteq Nrg^*bint(A)$.Conversely, if F is a nano closed set with $F\subseteq Nrg^*bint(A)$ and $F\subseteq A$, then $U-Nrg^*bint(A)\subseteq U-F$. Thus $Nrg^*bcl(U-A)\subseteq U-F$. Hence $U-A$ is nano$rg^{**}b$-closed set and A benano$rg^{**}b$-open.

Remark 4.5: The union of two nano $rg^{**}b$-open sets need not be nano $rg^{**}b$-open can be seen from the following example.

Example 4.6: Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{b\},\{c,d\}\}$ and $X=\{a,b\}$. Then the nanotopology $\tau_R(X)=\{U,\phi,\{a,b\},\{a,b\},\emptyset\}$. Then the sets $\{c\}$ and $\{d\}$ are nano $rg^{**}b$-open sets but $\{c\}\cup\{d\}=\{c,d\}$ is notnano$rg^{**}b$-open.

Remark 4.7: The intersection of two nano $rg^{**}b$-open sets need not be nano $rg^{**}b$-open set can be seen from the following example.

Example 4.8: Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{b\},\{c,d\}\}$ and $X=\{a,c\}$. Then the nanotopology $\tau_R(X)=\{U,\phi,\{a\},\{a,c,d\},\{c,d\}\}$. Then the sets $\{b,c,d\}$ and $\{a,b,d\}$ are nano $rg^{**}b$-open sets but $\{b,c,d\}\cap\{a,b,d\}=\{b,d\}$ is notnano$rg^{**}b$-open

Theorem 4.9: $Nrg^*bint(A)\subseteq B\subseteq A$ and if A is nano $rg^{**}b$-open then B is also nano$rg^{**}b$-open.

Proof: Let $Nrg^*bint(A)\subseteq B\subseteq A$. Now$A^C\subseteq B^C\subseteq Nrg^*bcl(A)$ where A^C is nano $rg^{**}b$-closed set and hence B^C is also nano $rg^{**}b$-closed by theorem (3.23) B is nano $rg^{**}b$-open.

Theorem 4.10: If A is nano $rg^{**}b$-closed then $Nrg^{*}bcl(A) - A$ is nano $rg^{**}b$-open.

Proof: Let A be nano $rg^{**}b$-closed.Let F be a nano closed set in $(U, \tau_R(X))$ such that $F \subseteq Nrg^{*}bcl(A) - A$. Then $F = \emptyset$. Since $Nrg^{*}bcl(A) - A$ cannot have any non-empty nano closed set. Therefore $F \subseteq Nrg^{*}bint(Nrg^{*}bcl(A) - A$. Hence $Nrg*bclA-A$ is nano $rg**b$-open.

Theorem 4.11: If A is nano $rg^{**}b$-open, G is nano open and $Nrg^{*}bint(A) \cup A^{C} \subseteq G$, then $G = U$.

Proof: Let A be nano $rg^{**}b$-open and G is nano open such that $Nrg^{*}bint(A) \cup A^{C} \subseteq G$. Then $G^{C} \subseteq A \cap Nrg^{*}bcl(A^{C}) \subseteq Nrg^{*}bcl(A^{C}) - A^{C}$. Since A^{C} is nano $rg^{**}b$-closed $Nrg^{*}bcl(A^{C}) - A^{C}$ cannot contain any non-empty nano closed set but G^{C} is nano closed subset of $Nrg^{*}bcl(A^{C}) - A^{C}$. Therefore $G^{C} = \emptyset$. That is $G = U$.

Theorem 4.12: A subset $A \subseteq U$ is nano $rg^{**}b$-open if and only if $F \subseteq Nrg^{*}bint(A)$ whenever F is nano closed set and $F \subseteq A$.

Proof: Let A be nano $rg^{**}b$-open and suppose $F \subseteq A$ where F is nano closed. Then $U - A$ is nano $rg^{**}b$-closed contained in the nano open set $U - F$. Hence $Nrg^{*}bcl(U - A) \subseteq U - F$ and $U - Nrg^{*}bint(A) \subseteq U - F$. Thus $F \subseteq Nrg^{*}bint(A)$.Conversely, if F is nano closed set with $F \subseteq Nrg^{*}bint(A)$ and $F \subseteq A$. Then $U - Nrg^{*}bint(A) \subseteq U - F$. Thus $Nrg^{*}bint(U - A) \subseteq U - F$. Hence $U - A$ is nano $rg^{**}b$-closed set and A is nano $rg^{**}b$-open set.

5. On $Nrg^{**}b$- Continuity

Definition 5.1: Let $(U,\ \tau_R(X))$ and $(V,\ \tau_R(Y))$ be nanotopological spaces $f: (U,\ \tau_R(X)) \to (V,\ \tau_R(Y))$ is said to be

i) nano continuous[10] if $f^{-1}(B)$ is nano closed in U for every nano closed set B in V.
ii) nanoα- continuous if $f^{-1}(B)$ is nano α- closed in U for every nano closed set B in V.
iii) nano semi continuous if $f^{-1}(B)$ is nano semi closed in U for every nano closed set B in V.
iv) nano pre continuous if $f^{-1}(B)$ is nano pre closed in U for every nano closed set B in V.
v) nanoregular continuous if $f^{-1}(B)$ is nano regular closed in U for every nano closed set B in V.
vi) nano$g$- continuous if $f^{-1}(B)$ is nano$g$- closed in U for every nano closed set B in V.
vii) nanog^{*}- continuous if $f^{-1}(B)$ is nano g^{*}-closed in U for every nano closed set B in V.
viii) nano$g\alpha$- continuous if $f^{-1}(B)$ is nano$g\alpha$- closed in U for every nano closed set B in V.
ix) nanogp- continuous if $f^{-1}(B)$ is nanogp- closed in U for every nano closed set B in V.

Definition 5.2: Let $(U,\ \tau_R(X))$ and $(V,\ \tau_R(Y))$ be nanotopological spaces. Then a mapping $f: (U,\ \tau_R(X)) \to (V,\ \tau_R(Y))$ is said to be nano regular generalized star star$b$-closed continuous ($Nrg^{**}b$- continuous) if the inverse images of every nano closed set in V is nano $rg^{**}b$-closed in U.

Theorem 5.3: Let $(U,\ \tau_R(X))$ and $(V,\ \tau_R(Y))$ be nanotopological spaces and a mapping $f: (U,\ \tau_R(X)) \to (V,\ \tau_R(Y))$. Then every nano continuous, nano α- continuous, nano semi continuous, nano pre continuous, nano regular continuous, nano$g$- continuous, nano g^{*}- continuous, nano$g\alpha$- continuous, nanogp- continuous functions are$Nrg^{**}b$- continuous.

Remark 5.4: The converse of the above theorem need not be true whichcan be seen from the following example.

Example 5.5: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{a, c, d\}\}$ and $X = \{a, c\}$.Then the nano topology is defined as $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R' = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$.Then the nanotopology is defined as $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = d, f(b) = c, f(c) = b, f(d) = a$. Then f is $Nrg^{**}b$- continuous, but f is not nano continuous and nano semi continuous since $f^{-1}\{b, c, d\} = \{a, b, c\}$ and $f^{-1}\{b\} = \{c\}$ which not nano closed are and nano semi closed in U where as $\{b, c, d\}, \{b\}$ are nano closed in V. Thus a$Nrg^{**}b$- continuous function is not nano continuous and nano semi continuous.

Example 5.6: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{a, c, d\}\}$ and $X = \{a, c\}$.Then the nano topology is defined as $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R' = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$.Then the nanotopology is defined as $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = c, f(b) = a, f(c) = d, f(d) = b$. Then f is $Nrg^{**}b$- continuous but f is not nano α- continuous since $f^{-1}\{a, b\} = \{b, d\}$which is not nano α- closed in U where as $\{a, b\}$ are nano closed in V. Thus a $Nrg^{**}b$- continuous is not nanoα- continuous.

Example 5.7: Let $U = \{a, b, c, d\}$ with $U/R = \left\{\{a\}, \{b\}, \{a, c, d\}\right\}$ and $X = \{a, c\}$.Then the nano topology is defined as $\tau_R(X) = \left\{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\right\}$. Let $V = \{a, b, c, d\}$ with $V/R' = \left\{\{a\}, \{b, c\}, \{d\}\right\}$ and $Y = \{a, b\}$.Then the nanotopology is defined as $\tau_R(Y) = \left\{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\right\}$. Define $f: U \to V$ as $f(a) = c, f(b) = d, f(c) = a, f(d) = b$. Then f is $Nrg^{**}b$- continuous but f is not nano pre- continuous since $f^{-1}\{a, b\} = \{c, d\}$ which is not nano pre-closed in U where as$\{a, b\}$ are nano closed in V. Thus a $Nrg^{**}b - continuous$ function is not nano pre-continuous.

Example 5.8: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{a, c, d\}\}$ and $X = \{a, c\}$.Then the nano topology is defined as $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R' = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$.Then the nanotopology is defined as $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = d, f(b) = c, f(c) = a, f(d) = b.$ Then f *is* $Nrg^{**}b$- continuous is but f is not nano g- continuous and nano g^*-continuous since $f^{-1}\{a, b\} = \{c, d\}$and $f^{-1}\{b\} = \{d\}$ which are not nano g- closed and nanog^*-closed in U where as $\{a, b\}$ and $\{b\}$nano closed in V. Thus a $Nrg^{**}b -$ *continuous* function is not nano g- continuous and nano g^*-continuous.

Example 5.9: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{a, c, d\}\}$ and $X = \{a, c\}$.Then the nano topology is defined as $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R' = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$.Then the nanotopology is defined as $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = d, f(b) = c, f(c) = a, f(d) = b.$ Then f is $Nrg^{**}b -$ *continuous* but f is not nano $g\alpha$- continuous and nano gp-continuous since $f^{-1}\{a, b\} = \{c, d\}$ is not nano $g\alpha$- closed and nano gp-closed in U where as $\{a, b\}$ is nano closed in V. Thus a nano $Nrg^{**}b -$ continuous function is not nano $g\alpha$- continuous and nano gp-continuous.

Theorem 5.10: If $(U, \tau_R(X))$ and $(V, \tau_R(Y))$ be nanotopological spaces with respect to $X \subseteq U$ and $Y \subseteq V$ respectively, then any function $f: U \to V$, the following conditions are equivalent:

i) f is $Nrg^{**}b -$ continuous.
ii) The inverse of each nano open set in Y is $Nrg^{**}b$ −open in X.
iii) For each subset A of X, $f(Nrg^{**}b\ cl(A)) \subseteq Ncl(f(A))$.

Proof:
$(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$: Let B be an nano open subset of Y. Then $Y - B$ is nano closed in Y. Since f is $Nrg^{**}b -$ continuous, $f^{-1}(Y - B)$ is $Nrg^{**}b -$ closed in X. That is, $X - f^{-1}(B)$ is $Nrg^{**}b$ −closed in X. Hence $f^{-1}(B)$ is $Nrg^{**}b -$ open in X.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{iii})$: Let A be a subset of X. Since $A \subset f^{-1}(f(A))$, $A \subset f^{-1}\left(Ncl(f(A))\right)$. Now $Ncl(f(A))$ is a nano closed set in Y. Then by $(ii), f^{-1}\left(Ncl(f(A))\right)$ is $Nrg^{**}b -$ closed in X containing A. But $Nrg^{**}b\ cl(A)$ is smallest $Nrg^{**}b -$ closed set in X containing A. Therefore $Nrg^{**}b\ cl(A) \subseteq f^{-1}\left(Ncl(f(A))\right)$. Hence $f(Nrg^{**}b\ cl(A)) \subseteq Ncl(f(A))$.

$(\boldsymbol{iii}) \Rightarrow (\boldsymbol{ii})$: Let B be a nano closed subset of Y. Then $f^{-1}(B)$ is a subset of X. By $(iii) f\left(Nrg^{**}b\ cl\left(f^{-1}(B)\right)\right) \subseteq Ncl\left(f\left(f^{-1}(B)\right)\right) \subseteq Ncl(B) = B$. This implies $Nrg^{**}b\ cl\left(f^{-1}(B)\right) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq Nrg^{**}b\ cl\left(f^{-1}(B)\right)$. Hence $f^{-1}(B) = Nrg^{**}b\ cl\left(f^{-1}(B)\right)$ and $f^{-1}(B)$ is $Nrg^{**}b -$ closed in X. This implies that f is $Nrg^{**}b$ −continuous.

Remark 5.11: If $f: (U, \tau_R(X)) \to (V, \tau_R(Y))$ is nano$rg^{**}b$- continuous then $f(Nrg^{**}b\ cl(A))$ is not necessarily equal to$Ncl(f(A))$.

Example 5.12: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{a, c, d\}\}$ and $X = \{a, c\}$.Then the nano topology is defined as $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R' = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$.Then the nanotopology is defined as $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = d, f(b) = c, f(c) = b, f(d) = a.$ Then f is $Nrg^{**}b$ −continuous.

i) Let $A = \{a\} \subseteq V$. Then $f(Nrg^{**}b\ cl(A)) = f(\{a\}) = d$. But $Ncl(f(A)) = Ncl(\{d\}) = \{b, c, d\}$. Thus $f(Nrg^{**}b\ cl(A)) \neq Ncl(f(A))$, even though f is $Nrg^{**}b$ −continuous.

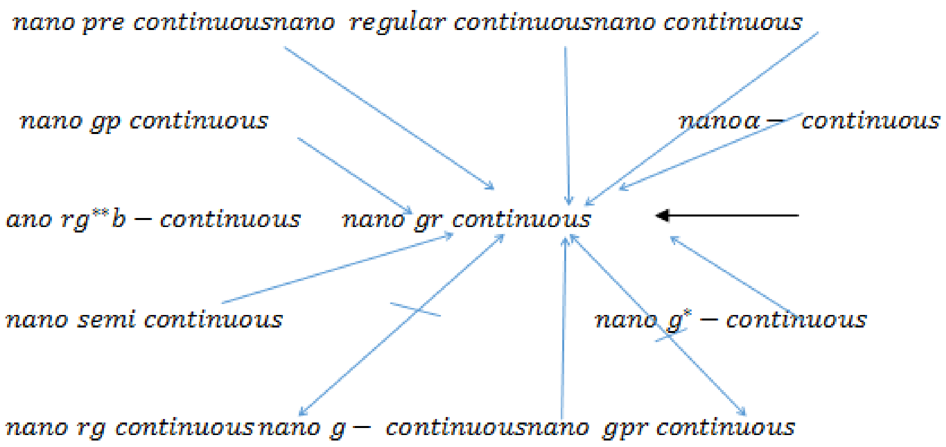
Remark 5.13: If $f: (U, \tau_R(X)) \to (V, \tau_R(Y))$ is nano$rg^{**}b$- continuous then is not $Nrg^{**}b\ cl\left(f^{-1}(B)\right)$ is not necessarily equal to$f^{-1}(Ncl(B)$.

Example 5.14: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{a, c, d\}\}$ and $X = \{a, c\}$.Then the nano topology is defined as $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R' = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$.Then the nanotopology is defined as $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = d, f(b) = c, f(c) = a, f(d) = b.$ Then f is $Nrg^{**}b$ −continuous.
i) Let $B = \{c\} \subseteq V$. Then $f^{-1}(Ncl(B)) = f^{-1}(Ncl\{c\}) = f^{-1}(b, c, d) = \{a, b, c\}$. But $Nrg^{**}b\ cl\left(f^{-1}(B)\right) = \{d\}$. Thus $Nrg^{**}b\ cl\left(f^{-1}(B)\right) \neq f^{-1}(Ncl(B)$, even though f is $Nrg^{**}b$ −continuous.

Theorem 5.15: If $f : X \to Y$ is $Nrg^{**}b-$ continuous and $g : Y \to Z$ is nano continuous then their composition $f \circ g : X \to Z$ is $Nrg^{**}b-$ continuous.

Proof: Let $f : X \to Y$ is $Nrg^{**}b-$ continuous and $g : Y \to Z$ is nano continuous. Let U be a nano closed set in Z.Therefore $g^{-1}(U)$ is nano closed in Y and $f^{-1}(g^{-1}(U))$ is $Nrg^{**}b-$ closed in X. Therefore $f \circ g$ is $Nrg^{**}b-$ continuous.

Remark 5.16: The above discussions are summarized in the following diagram



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Source of support: Nil, Conflict of interest: None Declared