International Journal of Mathematical Archive-7(10), 2016, 109-114
Available online through www.ijma.info ISSN 2229 – 5046

ON NANO (1, 2)* REGULAR-GENERALIZED CLOSED SETS IN NANO BITOPOLOGICAL SPACES

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(Received On: 19-09-16; Revised & Accepted On: 22-10-16)

ABSTRACT

The purpose of this paper is to define and study a new class of set called Nano (1,2) regular-generalized closed sets in nano bitopological spaces. Basic properties of nano (1,2)* regular-generalized closed sets are analyzed. Also we introduce the new notions of nano (1,2)* regular-generalized closure and their relation with already existing well known sets are also investigated.*

Keywords: *Nano (1,2)* Regular-Generalized Closed sets, Nano (1,2)* Regular-Closure, Nano (1,2)* Regular-Interior, Nano (1,2)* regular closed sets.*

1. INTRODUCTION

In 1970, Levine [5] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Later on N.Palaniappan [7] studied the concept of regular generalized closed set in a topological space. In 2011, Sharmistha Bhattacharya [8] have introduced the notion of generalized regular closed sets in topological space. The notion of nano topology was introduced by Lellis Thivagar[6]. In 1963, J.C.Kelly[4] initiated the study of bitopological spaces. In 2014 K.Bhuvaneswari *et al.*, [1, 2] have introduced the notion of nano regular generalized and generalized regular closed sets in nano topological spaces and Nano bitopological spaces. In this paper, we have introduced a new class of sets on nano bitopological spaces called nano (1,2)* regular generalized closed sets and the relation of these new sets with the existing sets.

2. PRELIMINARIES

Definition 2.1 [7]: A subset A of a topological space (X,τ) is called a regular open set if $A = Int[cl(A)]$. The complement of a regular open set of a space X is called regular closed set in X.

Definition 2.2 [7]: A regular-closure of a subset A of X is the intersection of all regular closed sets that contains A and it is denoted by rcl(A).

Definition 2.3 [7]: The union of all regular open subsets of X contained in A is called regular-interior of A and it is denoted by $rInt(A)$.

Definition 2.4 [7]: A subset A of (X,τ) is called a regular generalized closed set (briefly rg-closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open in X.

Definition 2.5 [7]: The regular-generalized closure of a subset A of a space X is the intersection of all regular-generalized closed sets containing A and is denoted by rgcl(A). The regular-generalized interior of a subset A of a space X is the union of all regular generalized open sets contained in A and is denoted by rgInt(A).

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Definition 2.6 [6]: Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then, $\tau_R(X)$ satisfies the following axioms:

- U and $\Phi \in \tau_R(X)$
- The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$

  Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. $(U, \tau_R(X))$ is called the nano topological space. Elements of the nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^C$ are called nano closed sets in $\tau_R(X)$.

Definition 2.7 [6]: If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by NInt(A). NInt(A) is the largest nano open subset of A.
- The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). Ncl(A) is the smallest nano closed set containing A.

Definition 2.8 [6]: Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be

- Nano regular open if $A \subseteq NInt[Ncl(A)]$
- Nano regular closed if $Ncl[NInt(A)] \subseteq A$

NRO(U,X), NRC(U,X) respectively denote the families of all nano regular open, nano regular closed subsets of U.

Definition 2.9 [6]: If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, Then

(i) The nano regular-closure of A is defined as the intersection of all nano regular closed sets containing A and it is denoted by Nrcl(A). Nrcl(A) is the smallest nano regular closed set containing A.
(ii) The nano regular-interior of A is defined as the union of all nano regular open subsets of A contained in A and it is denoted by NrInt(A). NrInt(A) is the largest nano regular open subset of A.

Definition 2.10 [1]: A subset A of $(U, \tau_R(X))$ is called nano regular-generalized closed set (briefly Nrg-closed) if $Nrcl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano regular open in $(U, \tau_R(X))$.

Definition 2.11 [3]: Let $(X, \tau_{1,2})$ be a bitopological space and $A \subseteq U$. Then A is said to be

- (1,2)* Regular open if $A \subseteq \tau_{1,2}Int[\tau_{1,2}cl(A)]$
- (1,2)* Regular closed if $\tau_{1,2}cl[\tau_{1,2}Int(A)] \subseteq A$

  (1,2)*RO(X), (1,2)*RC(X) respectively denote the families of all (1,2)* regular open, (1,2)* regular closed subsets of X.

Definition 2.12 [3]: If $(X, \tau_{1,2})$ is a bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The (1,2)* regular-closure of A is defined as the intersection of all (1,2)* regular closed sets containing A and it is denoted by $\tau_{1,2}$ rcl(A). $\tau_{1,2}$ rcl(A) is the smallest (1,2)* regular closed set containing A.
(ii) The (1,2)* regular-interior of A is defined as the union of all (1,2)* regular open subsets of A contained in A and it is denoted by $\tau_{1,2}$ rInt(A). $\tau_{1,2}$ rInt(A) is the largest (1,2)*regular open subset of A.

Definition 2.13 [3]: A subset A of $(X, \tau_{1,2})$ is called (1, 2)* regular-generalized closed set (briefly (1,2)* rg-closed) if $\tau_{1,2}rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is (1,2)* regular open in $(X, \tau_{1,2})$.

Definition 2.14 [2]: Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = \cup\{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and $X \subseteq U$ Then $\tau_R(X)$ satisfies the following axioms:

- U and $\Phi \in \tau_R(X)$
- The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $(U, \tau_{R_{1,2}}(X))$ is called the nano bitopological space. Elements of the nano bitopology are known as nano (1, 2)* open sets in U. Elements of $[\tau_{R_{1,2}}(X)]^c$ are called nano (1, 2)* closed sets in $\tau_{R_{1,2}}(X)$.

Definition 2.15 [2]: If $(U, \tau_{R_{1,2}}(X))$ is a nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- The nano (1, 2)* closure of A is defined as the intersection of all nano (1, 2)* closed sets containing A and it is denoted by $N\tau_{1,2}cl(A)$. $N\tau_{1,2}cl(A)$ is the smallest nano (1, 2)* closed set containing A.
- The nano (1, 2)* interior of A is defined as the union of all nano (1, 2)* open subsets of A contained in A and it is denoted by $N\tau_{1,2}Int(A)$. $N\tau_{1,2}Int(A)$ is the largest nano (1, 2)* open subset of A.

3. NANO (1,2)* REGULAR GENERALIZED CLOSED SETS

In this section, the definition of nano (1,2)* regular closed sets and nano (1,2)* regular-generalized closed sets are introduced and studied some of its properties.

Definition 3.1: A subset A of $(U, \tau_{R_{1,2}}(X))$ is called **nano (1,2)* regular open** set if $A \subseteq N\tau_{1,2}Int[N\tau_{1,2}cl(A)]$. The complement of a nano (1, 2)* regular open set of a space U is called **nano (1,2)* regular closed** set in $(U, \tau_{R_{1,2}}(X))$.

Definition 3.2: If $(U, \tau_{R_{1,2}}(X))$ is a nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The **nano (1,2)* regular-closure** of A is defined as the intersection of all nano (1,2)* regular closed sets containing A and it is denoted by $N\tau_{1,2}rcl(A)$. $N\tau_{1,2}rcl(A)$ is the smallest nano (1,2)* regular closed set containing A.

(ii) The **nano (1,2)* regular-interior** of A is defined as the union of all nano (1,2)* regular open subsets of A contained in A and it is denoted by $N\tau_{1,2}rInt(A)$. $N\tau_{1,2}rInt(A)$ is the largest nano (1,2)* regular open subsets of A.

Definition 3.3: A subset A of $(U, \tau_{R_{1,2}}(X))$ is called **nano (1,2)* regular-generalized closed** set (briefly N(1,2)*rg-closed) if $N\tau_{1,2}rcl(A)) \subseteq V$ whenever $A \subseteq V$ and V is nano (1,2)* regular open in $(U, \tau_{R_{1,2}}(X))$.

Example 3.4: Let $U = \{a, b, c, d\}$ with $U / R = \{\{c\}, \{d\}, \{a, b\}\}$

$X_1 = \{a, c\}$ and $\tau_{R_1}(X) = \{U, \phi, \{c\}, \{a, b, c\}, \{a, b\}\}$

$X_2 = \{a, d\}$ and $\tau_{R_2}(X) = \{U, \phi, \{d\}, \{a, b, d\}, \{a, b\}\}$

Then $\tau_{R_{1,2}}(X) = \{U, \phi, \{c\}, \{d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ which are (1,2)* open sets.

The nano (1,2)* closed sets = $\{U, \phi, \{c\}, \{d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$.

The nano (1,2)* regular closed sets = $\{U, \phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$

The nano (1,2)* regular open sets = $\{U, \phi, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{a, b\}, \{d\}, \{c\}\}$

The nano (1,2)* regular-generalized open sets are

$$\{U, \phi, \{a\}\{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}.$$

The nano (1,2)* regular-generalized closed sets are

$$\{U, \phi, \{a\}\{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}.$$

Theorem 3.5: Let $(U, \tau_{R_{1,2}}(X))$ be a nano bitopological space. If a subset A of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is nano (1,2)* closed set in $(U, \tau_{R_{1,2}}(X))$, then A is a nano (1,2)*regular closed set in $(U, \tau_{R_{1,2}}(X))$.

Proof: Let A be a nano (1,2)* closed set in X such that $A \subseteq U$; whenever U is (1,2)* regular open. That is $N\tau_{1,2}cl(A) = A$. To prove that $N\tau_{1,2}cl(N\tau_{1,2}Int(A)) \subseteq A$. Since A is nano (1,2)* regular open in U.

Therefore $N\tau_{1,2}cl(A) = A$ which implies $N\tau_{1,2}cl(N\tau_{1,2}Int(A)) = N\tau_{1,2}cl(A) = A$.

Hence A is a nano (1,2)* regular closed set. Also every nano (1,2)* open set is nano (1,2)* regular open set.

The converse of the above Theorem 3.5 is not true from the following example.

Example 3.6: Let $U = \{a, b, c, d\}$ with $U / R = \{\{c\}, \{d\}, \{a, b\}\}$

$X_1 = \{a, c\}$ and $\tau_{R_1}(X) = \{U, \phi, \{c\}, \{a, b, c\}, \{a, b\}\}$

$X_2 = \{a, d\}$ and $\tau_{R_2}(X) = \{U, \phi, \{d\}, \{a, b, d\}, \{a, b\}\}$

Then $\tau_{R_{1,2}}(X) = \{U, \phi, \{c\}, \{d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ which are (1,2)* open sets.

The nano (1,2)* closed sets = $\{U, \phi, \{c\}, \{d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$.

The nano (1,2)* regular closed sets = $\{U, \phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$

Let A={c, d} be a nano (1,2)* closed set.

$$N\tau_{1,2}cl(A) = \{c, d\}.$$

$N\tau_{1,2}Int(N\tau_{1,2}cl(A)) = \{c, d\}$ which implies $N\tau_{1,2}Int(N\tau_{1,2}cl(A)) \subseteq A$.

Hence every nano (1,2)* closed set is a nano (1,2)* regular closed set.

Here {a, b} is nano (1,2)* regular closed sets but it is not nano (1,2)* closed set.

Theorem 3.7: Let $(U, \tau_{R_{1,2}}(X))$ be a nano bitopological space. If a subset A of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is nano (1,2)* regular closed set in $(U, \tau_{R_{1,2}}(X))$, then A is a nano (1,2)* regular-generalized closed set in $(U, \tau_{R_{1,2}}(X))$.

Proof: Let A be a nano (1, 2)* regular closed set in X and $A \subseteq V$, V is nano (1, 2)* regular open in U. That is $N\tau_{1,2}cl[N\tau_{1,2}Int(A)] = A$. Since A is nano (1,2)* regular open, $N\tau_{1,2}Int(A) = A$. Every nano (1,2)* open set is nano (1,2)* regular open. Therefore $N\tau_{1,2}cl(A) = A \subseteq V$ implies $N\tau_{1,2}cl(A) \subseteq V$. Since $A \subseteq V$ then $N\tau_{1,2}cl(A) \subseteq V$ whenever V is nano (1,2)* regular open in U. Hence A is a nano (1,2)* regular generalized closed set.

The converse of the above Theorem 3.7 is not true from the following example.

Example 3.8: Let $U = \{a,b,c,d\}$ with $U / R = \{\{c\},\{d\},\{a,b\}\}$

$X_1 = \{a,c\}$ and $\tau_{R_1}(X) = \{U,\phi,\{c\},\{a,b,c\},\{a,b\}\}$

$X_2 = \{a,d\}$ and $\tau_{R_2}(X) = \{U,\phi,\{d\},\{a,b,d\},\{a,b\}\}$

Then $\tau_{R_{1,2}}(X) = \{U,\phi,\{c\},\{d\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$ which are (1,2)* open sets.

Here {{a},{b},{a, c},{a, d},{b, c},{b, d},{a, c, d},{b, c, d}}are nano (1,2)* regular generalized closed sets but it is not nano (1,2)* regular closed.

Theorem 3.9: The union of two nano (1,2)* regular-generalized closed sets in $(U,\tau_{R_{1,2}}(X))$ is also a nano (1,2)* regular-generalized closed set in $(U,\tau_{R_{1,2}}(X))$.

Proof: Let A and B be two nano (1,2)* regular-generalized closed sets in $(U,\tau_{R_{1,2}}(X))$. Let V be any nano (1,2)* regular open set in U such that $A \subseteq V$ and $B \subseteq V$. Then we have $A \bigcup B \subseteq V$. As A and B are nano (1,2)* regular-generalized closed sets in $(U,\tau_{R_{1,2}}(X))$. Therefore $N\tau_{1,2}rcl(A) \subseteq V$ and $N\tau_{1,2}rcl(B) \subseteq V$.

Now $N\tau_{1,2}rcl(A \bigcup B) = N\tau_{1,2}rcl(A) \bigcup N\tau_{1,2}rcl(B) \subseteq V$. Thus we have $N\tau_{1,2}rcl(A \bigcup B) \subseteq V$ whenever $A \bigcup B \subseteq V$, V is nano (1,2)* regular open set in $(U,\tau_{R_{1,2}}(X))$ which implies $A \bigcup B$ is a nano (1,2)* regular-generalized closed set in $(U,\tau_{R_{1,2}}(X))$.

Theorem 3.10: The intersection of any two subsets of nano (1,2)* regular-generalized closed sets in $(U,\tau_{R_{1,2}}(X))$ is nano (1,2)* regular-generalized closed set in $(U,\tau_{R_{1,2}}(X))$.

Proof: Let A and B are any two nano (1,2)* regular-generalized closed sets. $A \subseteq V$; V is an nano (1,2)* regular open and $B \subseteq V$; V is nano (1,2)* regular open. Then $N\tau_{1,2}rcl(A) \subseteq V$ and $N\tau_{1,2}rcl(B) \subseteq V$. Therefore $N\tau_{1,2}rcl(A \bigcap B) \subseteq V$. V is nano (1,2)* regular open in X. Since A and B nano (1,2)* regular generalized closed sets. Hence $A \bigcap B$ is a nano (1,2)* regular generalized closed set.

Theorem 3.11: If a set A is nano (1,2)* regular generalized closed set iff $N\tau_{1,2}rcl(A) - A$ contains no non-empty, nano (1,2)* regular closed set.

Proof:

***Necessity*:** Let F be a nano (1,2)* regular closed set in $(U,\tau_{R_{1,2}}(X))$ such that $F \subseteq N\tau_{1,2}rcl(A) - A$. Then $A \subseteq X - F$. Since A is nano (1,2)* regular generalized closed set and X-F is nano (1,2)* regular open then $N\tau_{1,2}rcl(A) \subseteq X - F$. That is $F \subseteq X - N\tau_{1,2}rcl(A)$. So $F \subseteq (X - N\tau_{1,2}rcl(A)) \bigcap (N\tau_{1,2}rcl(A) - A)$. Therefore $F = \varphi$.

***Sufficiency*:** Let us assume that $N\tau_{1,2}rcl(A)-A$ contains no non empty nano (1,2)* regular closed set. Let $A \subseteq V$; V is nano (1,2)* regular open.

Suppose that $N\tau_{1,2}rcl(A)$ is not contained in V, $N\tau_{1,2}rcl(A)\cap V^c$ is non empty, nano (1,2)* regular closed set of $N\tau_{1,2}rcl(A)-A$ which is contradiction therefore $N\tau_{1,2}rcl(A)\subseteq V$. Hence A is nano (1,2)* regular generalized closed.

Theorem 3.12: If A is both nano (1,2)* regular open and nano (1,2)* regular generalized closed set in X, then A is nano (1,2)* regular closed set.

Proof: Since A is nano (1,2)* regular open and nano (1,2)* regular generalized closed in X, $N\tau_{1,2}rcl(A)\subseteq V$ But $A\subseteq N\tau_{1,2}rcl(A)$. Therefore A= $N\tau_{1,2}rcl(A)$. Since A is nano (1,2)* closed $N\tau_{1,2}Int(A)=A$. Implies $N\tau_{1,2}rcl(A)=A$. Hence A is nano (1,2)* regular closed.

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Source of support: Nil, Conflict of interest: None Declared