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ON δ(δg)^-CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, a new class of generalized closed sets called δ(δg)^ -closed sets is introduced and its properties are studied in topological spaces. Moreover the relation between δ(δg)^ -closed sets and various other classes of closed sets already defined are investigated.

Keywords: *g-closed sets, $\hat{g}$ -closed sets, δg -closed sets, δg* -closed sets, δ(δg)* -closed sets, $\delta\hat{g}$ - closed sets and δ(δg)^ -closed sets.*

AMS subject classification: *54A05.*

1. INTRODUCTION

The concept of generalized closed (briefly g-closed) sets was introduced and investigated by Norman Levine [6] in 1970. Velicko [10] introduced δ –open sets in 1968 which are stronger than open sets. By combining the concepts of δ- closedness and g –closedness, Julian Dontchev [2] proposed a class of generalised closed sets called δg –closed sets in 1996. Lellis Thivagar [5] defined a new class of closed set called $\delta\hat{g}$ - closed set in 2010. Veerakumar [8] and [9] introduced $\hat{g}$ -closed sets in 2003 and $\delta g^{\#}$ -closed sets in 2006. Meena and Sivakamasundari [7] defined a new class of generalised closed sets called δ(δg)* -closed sets and various properties were analysed.

Motivated by the development of various classes of δ-closed sets, we extend the concept of δ-generalized closed sets to a new class of closed sets called δ(δg)^ -closed sets and investigate their relationship with other existing closed sets in topological spaces. This new class contains the class of δ(δg)*-closed sets. The following inclusion relation holds.

δ(δg)* -closed sets ⊂ δ(δg)^ -closed ⊂ $\delta g^{\#}$ -closed sets

2. PRELIMINARIES

Definition 2.1 [4]: A Topology on a set X is a collection τ of subsets of X having the following properties:

a. φ and X are in τ.
b. The union of elements of any sub collection of τ is in τ.
c. The intersection of the elements of any finite sub collection of τ is in τ.

A set X for which a topology τ has been specified is called a Topological space.

Definition 2.2[7]: A subset A of a Topologica l space (X, τ) is called

1. Regular open if A= int(cl(A))
2. Semi-open if A $\subseteq$ cl(int(A))
3. Pre-open if A $\subseteq$ int (cl(A))
4. α-open if A $\subseteq$ int(cl(int(A)))
5. semi preopen if A $\subseteq$ cl(int(cl(A)))
6. π- open if it is the finite union of regular open sets.
7. δ- open if it is the union of regular open sets.

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The complement of a regular open (resp.Semi open, pre-open, α-open, semi preopen, π-open, δ-open) set is called regular closed (resp.semi closed, pre-closed, α- closed, semi preclosed, π-closed and δ-closed).

Definition 2.3[7]: The intersection of all regular closed (resp.semi-closed, pre-closed, α-closed, semi preclosed, π-closed, δ-closed) subsets of (X, τ) containing A is called the regular closure (resp.semi-closure, pre-closure, α-closure, semi preclosure, π-closure, δ-closure) of A and is denoted by rcl(A) ((resp. scl(A), pcl(A), αcl(A), spcl(A), πcl(A) and δcl(A)).

Definition 2.4: A subset A of a topological space (X, τ) is called

1. generalized closed(briefly g-closed) [6] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, U is open in (X, τ).
2. regular generalized closed(briefly rg-closed) [7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, U regular is open in (X, τ).
3. δ-generalized closed (briefly δg - closed) [2] if δcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is open in (X, τ).
4. δ-generalized semi closed(briefly δgs- closed) [3] if δscl(A) $\subseteq$ U whenever A $\subseteq$ U, U is δ-open in (X, τ).
5. δg*- closed [7] if δcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is g-open in (X, τ).
6. $\hat{g}$ -closed [10] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, U is semi open in (X, τ).
7. $\delta\hat{g}$ -closed [5] if δcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is $\hat{g}$ -open in (X, τ).
8. δ(δg)*- closed [7] if δcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is δg-open in (X, τ).
9. α-generalized closed (briefly αg-closed) [7] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is open in (X, τ).
10. $\alpha\hat{g}$ - closed [7] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is $\hat{g}$ -open in (X, τ).
11. generalised pre-closed(briefly gp- closed) [7] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is open in (X, τ).
12. generalised pre regular closed(briefly gpr- closed) [7] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is regular open in (X, τ).
13. g*p- closed [7] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is g open in (X, τ).
14. *g- closed [7] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is $\hat{g}$ -open in (X, τ).
15. g*s- closed [7] if scl(A) $\subseteq$ U whenever A $\subseteq$ U, U is gs open in (X, τ).
16. generalised semi pre regular closed(briefly gspr- closed) [7] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is regular open in (X, τ).
17. (gs)*- closed [7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, U is gs- open in (X, τ).
18. regular weakly generalised closed(briefly rwg- closed) [7] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U, U is regular open in (X, τ).
19. generalised δ-closed(briefly gδ- closed) [7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, U is δ-open in (X, τ).
20. #gs- closed [5] if scl(A) $\subseteq$ U whenever A $\subseteq$ U, U is *g-open in (X, τ).
21. $\delta g^{\#}$ - closed [9] if δcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is δ-open in (X, τ).
22. π-generalised closed(briefly πg- closed) [3] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, U is π- open in (X, τ).
23. π-generalised pre closed(briefly πgp- closed) [3] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is π-open in (X, τ).
24. π-generalised semi pre closed(briefly πgsp- closed) [3] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is π- open in (X, τ).
25. π-generalised b-closed(briefly πgb- closed) [3] if bcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is π- open in (X, τ).
26. π-generalised semi closed(briefly πgs- closed) [3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U, U is π-open in (X, τ).
27. π-generalised α-closed(briefly πgα- closed) [3] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U, U is π-open in (X, τ).
28. ψ-closed set [1] if scl(A) ⊆ U whenever A ⊆ U and U is sg-open in X.
29. ψg-closed set [1] if ψcl(A) ⊆ U whenever A ⊆ U and U is open in X.
30. ψg*-closed set [1] if ψcl(A) ⊆ U whenever A ⊆ U and U is g*-open in X.
31. g*-closed set [1] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in X.

Remark 2.5: r-closed(open) → π-closed(open) →δ-closed(open) → δg*-closed(open) → δ(δg)* - closed(open) → $\delta g^{\#}$ -closed(open) → gδ-closed(open) [7].

Remark 2.6: For every subset A of X,

i. spcl(A) $\subseteq$ pcl(A) $\subseteq$ δcl(A) [7].
ii. spcl(A) $\subseteq$ scl(A) $\subseteq$ δscl(A) $\subseteq$ δcl(A) (Lemma 3.4 of [3]).
iii. bcl(A) $\subseteq$ δscl(A) (Corollary 3.28 of [3]).

Remark 2.7:

i. Every $\delta\hat{g}$ -closed set is g-closed and δg –closed (Proposition 3.5 and 3.14 of [5]).
ii. Every δ-closed set is $\delta\hat{g}$ -closed (Proposition 3.2 of [5]).

3. δ(δg)^-CLOSED SETS

In this section we introduce a new class of closed sets called δ(δg)^-closed sets which lie between the class of δ(δg)*-closed sets and the class of $\delta g^{\#}$ -closed sets.

Definition 3.1: A subset A of a topological space (X, τ) is said to be δ(δg)^-closed sets if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$, U is $\delta\hat{g}$ -open in (X, τ). The class of all δ(δg)^-closed sets of (X, τ) is denoted by δ(δg)^C(X, τ).

Theorem 3.2: Let A and B are δ(δg)^-closed sets in a topological space (X, τ), then

- i. $A \cup B$ is δ(δg)^-closed in (X, τ).
- ii. $A \cap B$ need not be δ(δg)^-closed in (X, τ).

Proof:

- i. Suppose that $A \cup B \subseteq U$ where U is any $\delta\hat{g}$ -open in (X, τ). Then $A \subseteq U$ and $B \subseteq U$. Since A and B are δ(δg)^-closed sets of (X, τ), $\delta cl(A) \subseteq U$ and $\delta cl(B) \subseteq U$. Also, $\delta cl(A \cup B) = \delta cl(A) \cup \delta cl(B)$. It follows that, $\delta cl(A \cup B) \subseteq U$. Therefore $A \cup B$ is a δ(δg)^-closed set in (X, τ).
- ii. Let X = {a, b, c}, τ = {X, φ, {a}, {b}, {a, b}}. In this topology, the set {c} and {a, c} are δ(δg)^-closed but their intersection {c} is not δ(δg)^-closed.

Theorem 3.3: In a topological space (X, τ), every δ-closed set is δ(δg)^-closed but the converse need not be true.

Proof: Let A be a δ-closed set and let U be any $\delta\hat{g}$ -open set containing A in (X, τ). Since A is δ-closed, δcl(A)=A. Therefore $\delta cl(A) = A \subseteq U$ and hence A is δ(δg)^-closed.

Example: Let X = {a, b, c}, τ = {x, φ, {a, c}}. In this topology the set {b} is δ(δg)^-closed but not δ-closed.

Theorem 3.4: Let (X, τ) be a topological space and $A \subseteq X$. Then the class of δg*-closed sets and the class of δ(δg)*-closed sets are proper subsets of the class of δ(δg)^-closed sets.

Proof:

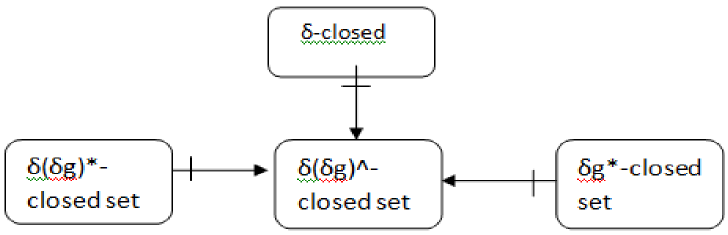
- i. Let A be δg*-closed set and U be any $\delta\hat{g}$ -open set containing A in (X, τ). By Remark 2.7(i), every $\delta\hat{g}$ -open set is g-open. Since A is δg*-closed, $\delta cl(A) \subseteq U$. Hence A is δ(δg)^-closed.
- ii. Let A be δ(δg)*-closed set and U be any $\delta\hat{g}$ -open set containing A in (X, τ). By Remark 2.7(i), every $\delta\hat{g}$ -open set is δg-open. Since A is δ(δg)*-closed, $\delta cl(A) \subseteq U$. Hence A is δ(δg)^-closed.

Remark 3.5: A δ(δg)^-closed set need not be a δg*-closed and need not be δ(δg)*-closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, b}, {b, c}, {b}}. In this topology, the set {b} is δ(δg)^-closed but not δg* closed.

Example: Let X = {a, b, c}, τ = {X, φ, {c}, {a, b}}. In this topology the set {a} is δ(δg)^-closed but not δ(δg)* closed.

Remark 3.6: The following diagram gives a diagrammatic representation of the above Theorems.



In the above diagram, A ⟶ B means, A implies B but, B does not imply A.

Remark 3.7: r-closed(open) → π -closed(open) → δ -closed(open) → δg* -closed(open) → δ(δg)* -closed(open) → δ(δg)^ -closed(open) → $\delta g^{\#}$ -closed(open) → gδ-closed(open).

Theorem 3.8: Let (X, τ) be a topological space and A ⊆ X be a δ(δg)^-closed set. Then A is

i. gδ-closed ii. gpr-closed iii. gspr-closed.

The converse part of this Theorem need not be true.

Proof:

i. Let A be δ(δg)^-closed set and U be any δ-open set containing A in (X, τ). By Remark 2.7(ii), every δ-open is $\delta\hat{g}$ -open. Since A is δ(δg)^-closed, δcl(A) ⊆ U. For every subset A of X, cl(A) ⊆ δcl(A). Therefore cl(A) ⊆ U. Hence A is gδ-closed.
ii. Let A be δ(δg)^-closed set and U be any regular open set containing A in (X, τ). By Remark 2.5&7(ii), every regular open is $\delta\hat{g}$ -open and A is δ(δg)^-closed, δcl(A) ⊆ U. By Remark 2.6(i), For every subset A of X, pcl(A) ⊆ δcl(A) and so we have pcl(A) ⊆ U. Hence A is gpr-closed.
iii. Let A be δ(δg)^-closed set and U be any regular open set containing A in (X, τ). By Remark 2.5 and 2.7(ii), every regular open is $\delta\hat{g}$ -open and A is δ(δg)^-closed, δcl(A) ⊆ U. By Remark 2.6(i), spcl(A) ⊆ δcl(A). And so we have, spcl(A) ⊆ U. Hence A is gspr –closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, c}}. In this topology, the set {a} is gδ-closed but not δ(δg)^-closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, b}, {b, c}, {b}}. In this topology, the set {b} is gpr-closed but not δ(δg)^-closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, b}, {b, c}, {b}}. In this topology, the set {b} is gspr-closed but not δ(δg)^ closed.

Theorem 3.9: Let (X, τ) be a topological space and A ⊆ X. Then,

i. A is δ(δg)^-closed set implies, A is $\delta g^{\#}$ -closed.
ii. A is δ(δg)^-closed set implies, A is δgs-closed..

The converse part of (i) and (ii) need not be true.

Proof:

i. Let A be δ(δg)^-closed and U be any δ-open set containing A in (X, τ). By Remark 2.7(ii), every δ-open set is $\delta\hat{g}$ -open and A is δ(δg)^-closed, δcl(A) ⊆ U. Hence A is $\delta g^{\#}$ -closed.
ii. Let A be δ(δg)^-closed set and U be any δ-open set containing A in (X, τ). By Remark 2.7(ii), every δ-open is $\delta\hat{g}$ -open and A is δ(δg)^-closed, δcl(A) ⊆ U. By Remark 2.6 (iii), δscl(A) ⊆ δcl(A). And hence, δscl(A) ⊆ U and A is δgs -closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, c}}. In this topology, the set {c} is $\delta g^{\#}$ -closed but not δ(δg)^-closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, c}}. In this topology, the set {c} is δgs-closed but not δ(δg)^-closed

Theorem 3.10: Let (X, τ) be a topological space and A ⊆ X. Then the class of δ(δg)^-closed sets is a proper subset of each of the classes of rg-closed, rwg-closed, πg-closed, πgp-closed and πgb-closed sets.

Proof:

i. Let A be δ(δg)^-closed set and U be any regular open set containing A in (X, τ). By Remark 2.5 and 2.7(ii), every regular open is $\delta\hat{g}$ -open and A is δ(δg)^-closed, δcl(A) ⊆ U. For every subset A of X, cl(A) ⊆ δcl(A) and so we have cl(A) ⊆ U. Hence A is rg-closed.
ii. Let A be δ(δg)^-closed set and U be any regular open set containing A in (X, τ). By Remark 2.5 and 2.7(ii), every regular open is $\delta\hat{g}$ -open and A is δ(δg)^-closed, δcl(A) ⊆ U. As int(A) ⊆ U, we have cl(int(A)) ⊆ cl(A) ⊆ δcl(A). Then cl(int(A)) ⊆ U. Hence A is rwg-closed.
iii. Let A be δ(δg)^-closed set and U be any π-open set containing A in (X, τ). By Remark 2.5 and 2.7(ii), every π-open is $\delta\hat{g}$ -open and A is δ(δg)^-closed, δcl(A) ⊆ U. By Remark 2.6, cl(A) ⊆ δcl(A). And so we have, cl(A) ⊆ U. Hence A is πg -closed.
iv. Let A be δ(δg)^-closed set and U be any π-open set containing A in (X, τ). By Remark 2.5 and 2.7(ii), every π-open is $\delta\hat{g}$ -open and A is δ(δg)^-closed, δcl(A) ⊆ U. By Remark 2.6, pcl(A) ⊆ δcl(A). And so we have, pcl(A) ⊆ U. Hence A is πgp -closed.

v. Let A be δ(δg)^-closed set and U be any π-open set containing A in (X,τ). By Remark 2.5 and 2.7(ii), every π-open is $\hat{\delta g}$ -open and A is δ(δg)^-closed, δcl(A)$\subseteq$U. By Remark 2.6(iii), bcl(A)$\subseteq$δcl(A). And so we have, bcl(A)$\subseteq$U. Hence A is πgb-closed.

Remark 3.11: The converse of the above Theorem need not be true.

Example: Let X = {a, b, c}, τ = {X, φ, {a, c}}. In this topology, the set {a} is rg-closed but not δ(δg)^-closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, c}}. In this topology, the set {a} is rwg-closed but not δ(δg)^-closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, c}}. In this topology, the set {a} is πg-closed but not δ(δg)^-closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, b}, {b, c}, {b}}. In this topology, the set {b} is πgp-closed but not δ(δg)^closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, c}}. In this topology, the set {a} is πgb-closed but not δ(δg)^-closed.

Theorem 3.12: Let (X, τ) be a topological space and A$\subseteq$X be a δ(δg)^-closed set. Then A is
(i) πgα-closed. (ii) πgs-closed (iii) πgsp-closed.
The converse need not be true.

Proof:

i. Let A be δ(δg)^-closed set and U be any π-open set containing A in (X, τ). By Remark 2.5 and 2.7(ii), every π-open is $\hat{\delta g}$ -open and A is δ(δg)^-closed, δcl(A)$\subseteq$U. For every subset A of X, αcl(A)$\subseteq$δcl(A). And so we have, αcl(A)$\subseteq$U. Hence A is πgα -closed.

ii. Let A be δ(δg)^-closed set and U be any π-open set containing A in (X, τ). By Remark 2.5 and 2.7(ii), every π-open is $\hat{\delta g}$ -open and A is δ(δg)^-closed, δcl(A)$\subseteq$U. By Remark 2.6(ii), scl(A)$\subseteq$δcl(A). And so we have, scl(A)$\subseteq$U. Hence A is πgs -closed.

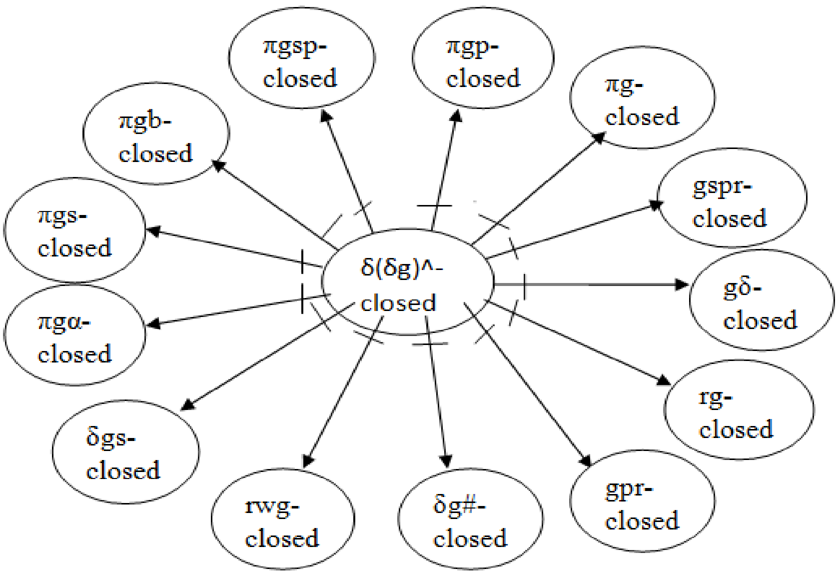
iii. Let A be δ(δg)^-closed set and U be any π-open set containing A in (X, τ). By Remark 2.5 and 2.7(ii), every π-open is $\hat{\delta g}$ -open and A is δ(δg)^-closed, δcl(A)$\subseteq$U. By Remark 2.6(i), spcl(A) $\subseteq$δcl(A). And so we have, spcl(A)$\subseteq$U. Hence A is πgsp -closed.

Example: X = {a, b, c}, τ = {X, φ, {a, b}, {b, c},{b}}. In this topology, the set {b} is πgα-closed but not δ(δg)^-closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, b}, {b, c}, {b}}. In this topology, the set {b} is πgs-closed but not δ(δg)-losed

Example: Let X = {a, b, c}, τ = {X, φ,{a, b}, {a},{b}}. In (X, τ) the set {a, b} is πgsp-closed but not δ(δg)^-closed.

Remark 3.13: The results of Theorem 3.7 to Theorem 3.12 are illustrated in the following diagram.



In the above diagram, A ⟶ B means, A implies B, but B does not imply A.

Remark 3.14: The following examples show that, the class δ(δg)^-closed sets are independent from the classes of g-closed sets, δg-closed sets, sg-closed sets, gs-closed sets, αg-closed sets, *g-closed sets, $\alpha\hat{g}$ -closed sets, #gs-closed sets, g*p –closed sets, $\delta\hat{g}$ -closed sets, gp-closed sets, gsp-closed sets, ψ-closed sets, ψg-closed sets, gb-closed sets, ψg^* -closedness and g*s-closed sets.

Example: Let X = {a, b, c}, τ = {X, φ, {a}}. In this topology the set {c} is g-closed, δg-closed, sg-closed, gs-closed, αg-closed, *g-closed, $\alpha\hat{g}$ -closed, #gs-closed, g*p -closed, $\delta\hat{g}$ -closed, gp-closed, gsp-closed, ψ-closed, ψg-closed, gb-closed and ψg^* -closed but not δ(δg)^-closed.

Example: Let X = {a, b, c}, τ = {X, φ, {a, b}, {b, c}, {b}}. In this topology the set {a, b} is δ(δg)^-closed but not g-closed, δg-closed, sg-closed, gs-closed, αg-closed, *g-closed, $\alpha\hat{g}$ -closed, g*p -closed, $\delta\hat{g}$ -closed, gp-closed, gsp-closed, ψ-closed, ψg-closed, gb-closed and but not δ(δg)^-closed.

Example: Let X = {X, φ, {a}}, τ = {X, φ, {a}}. In this topology the set {b} is g*s-closed but not δ(δg)^-closed.

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