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AN OPTIMUM INVENTORY MODEL FOR TIME DEPENDENT DEMAND WITH SHORTAGES

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ABSTRACT

In this study we develop an inventory model for deteriorating items with time dependent demand. Shortages are allowed in the inventory model and are completely backlogged. This model is to determine the expected total cost which is minimized. The model is illustrated by a numerical example.

INTRODUCTION

Inventory is required at different locations within many locations of a facility or within many locations of a supply network to precede the regular and planned course of production and stock of materials. Possessing a high amount of inventory for a long time is usually not advantageous for a business because of large amount of storage cost, the possibility of obsolescence and spoilage costs. Demand is a major factor in inventory which are classified into four types such as constant demand, time dependent demand, probabilistic demand and stock demand. Nowadays time dependent inventory model plays a vital role in present situation.

Babukrishnaraj and K.Ramsamy [2] studied an inventory model for weibull time dependent demand rate with completely backlogged. Goyal and Giri [11] proposed recent trends in modeling of deteriorating inventory. Khanra and Chaudhuri [13] studied the production model for time dependent demand for a deteriorating item with shortage. Muniappan *et al.* developed an economic lot sizing production model with two level trade credits for deteriorating items. Muniappan and Uthayakumar [15] studied mathematical analyse technique for computing optimal replenishment policies. Padmanabhan and Vrat [17] studied EOQ models for perishable items under stock dependent selling rate. Swaminathan and Muniappan [25] introduced an mathematical model for optimum production inventory with deteriorating items. Vikas sharma and Rekha rani chaudhary [27] proposes an inventory model for deteriorating items with weibull deterioration with time dependent demand and shortages.

ASSUMPTIONS AND NOTATIONS

1. Lead time zero.
2. The demand rate at any time t is D= a+ bt where $a \geq 0$, $b > 0$, $t \geq 0$.
3. Production rate is constant P > D
4. Replenishment rate and time horizon T is zero.
5. t_1 denotes the production time.
6. Deterioration rate θ is constant.
7. $I_1(t)$ is the inventory level at time $0 \leq t \leq t_1$.
8. $I_2(t)$ is the inventory level at time $t_1 \leq t \leq T$.
9. C_1 is the deterioration cost per unit per unit time.
10. C_2 is the shortage cost per unit per unit time.
11. A is the order cost per unit order is known and constant.
12. h is the holding cost per unit per unit time.

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MATHEMATICAL FORMULATION

The length of the cycle is T. At the time t_1 the inventory level becomes zero and shortages occur in the inventory at the period (t_1, T) which is completely backlogged. Let I(t) be the inventory level at time t ($0 \le t \le t_1$). The differential equations are given by

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = P - (a + bt); \quad 0 \le t \le t_1 \quad (1)$$

$$\frac{dI_2(t)}{dt} = -(a + bt); \quad t_1 \le t \le T \quad (2)$$

With boundary conditions $I_1(t) = 0$ and $I_2(T) = 0$. The solutions of the equation (1) and (2) are given by

$$I_1(t) = \left(\frac{P-a}{\theta} - \frac{bt}{\theta} + \frac{b}{\theta^2}\right) - \left(\frac{P-a}{\theta} - \frac{bt}{\theta} + \frac{b}{\theta^2}\right)e^{\theta(t_1 - t)}; \quad 0 \le t \le t_1 \quad (3)$$

$$I_2(t) = a(T - t) + \frac{b}{2}(T^2 - t^2); \quad t_1 \le t \le T \quad (4)$$

The components of the system are as follows

i) Average setup cost = $\frac{A}{T}$ (5)

ii) Average holding cost = $\frac{h}{T}\left[\int_0^{t_1} I_1(t)\,dt + \int_{t_1}^{T} I_2(t)\,dt\right]$

$$= \frac{h}{T}\left[\left(\frac{P-a}{\theta}t_1 - \frac{bt_1^2}{2\theta} + \frac{b}{\theta^2}t_1\right) + \left(\frac{P-a}{\theta^2} - \frac{bt_1}{\theta^2} + \frac{b}{\theta^3}\right)(1 - e^{\theta t_1}) + \frac{a}{2}(T - t_1)^2 + \frac{b}{6}(t_1^3 + 2T^3 - 3Tt_1^2)\right] \quad (6)$$

Average shortage cost = $\frac{c_2}{T}\int_{t_1}^{T} I_2(t)\,dt$

$$= \frac{c_2}{T}\left[\frac{a}{2}(T - t_1)^2 + \frac{b}{6}(t_1^3 + 2T^3 - 3Tt_1^2)\right] \quad (7)$$

iii) Average production cost = $\frac{c_1}{T}Q_0$

$$= \frac{c_1}{T}\left[\left(\frac{P-a}{\theta} + \frac{b}{\theta^2}\right) - \left(\frac{P-a}{\theta} - \frac{bt_1}{\theta} + \frac{b}{\theta^2}\right)e^{\theta t_1} + aT + \frac{bT^2}{2}\right] \quad (8)$$

Therefore the average total inventory cost is given by

TC = setup cost + holding cost + production cost + shortage cost

$$TC = \frac{1}{T}\Big[A + h\Big[\left(\frac{P-a}{\theta}t_1 - \frac{bt_1^2}{2\theta} + \frac{b}{\theta^2}t_1\right) + \left(\frac{P-a}{\theta^2} - \frac{bt_1}{\theta^2} + \frac{b}{\theta^3}\right)(1 - e^{\theta t_1})\Big]$$
$$+ c_1\left[\left(\frac{P-a}{\theta} + \frac{b}{\theta^2}\right) - \left(\frac{P-a}{\theta} - \frac{bt_1}{\theta} + \frac{b}{\theta^2}\right)e^{\theta t_1} - at_1 - \frac{bt_1^2}{2}\right] + (h + c_2)\left[\frac{a}{2}(t_1 - T)^2 + \frac{b}{6}(2t_1^3 + T^3 - 3Tt_1^2)\right]\Big] \quad (9)$$

The necessary condition for TC in (9) to be minimized is $\frac{\partial TC}{\partial T} = 0$

$$-\frac{1}{T^2}\Big[A + h\Big[\left(\frac{P-a}{\theta}t_1 - \frac{bt_1^2}{2\theta} + \frac{b}{\theta^2}t_1\right) + \left(\frac{P-a}{\theta^2} - \frac{bt_1}{\theta^2} + \frac{b}{\theta^3}\right)(1 - e^{\theta t_1})\Big] + c_1\left[\frac{P-a}{\theta} + \frac{b}{\theta^2} - \left(\frac{P-a}{\theta} - \frac{bt_1}{\theta} + \frac{b}{\theta^2}\right)e^{\theta t_1} - aT - \frac{bT^2}{2}\right]$$
$$+ (h + c_2)\left[\frac{a}{2}(T - t_1)^2 + \frac{b}{6}(t_1^3 + 2T^3 - 3Tt_1^2)\right]\Big] - \frac{1}{T}\left[(a + bT)\left[c_1 + (h + c_2)(T - t_1)\right]\right] = 0 \quad (10)$$

provided $\left(\frac{\partial^2 P}{\partial T^2}\right) > 0$

Numerical Example:

Consider an inventory system with parameters in proper unit A= 100, a = 25, p = 50, b=1.1, h=0.5, θ=0.1, C_1 = 0.2, C_2=0.3, t_1= 150/365 we get T=31.3318 and TC = 599.4689.

Sensitivity Analysis:

t_1	T	TC
150/365	31.3318	599.4689
125/365	31.4058	603.7879
100/365	31.4798	608.1201
75/365	31.5538	612.4655
50/365	31.6279	616.8267
25/365	31.7020	621.2010
10/365	31.7464	623.8300

CONCLUDING REMARKS

In this study we developed an inventory model for deteriorating items with time dependent demand and shortages. The rate of deterioration is constant. The demand rate is assumed to be time dependent. The shortages are allowed and are completely backlogged. The shortage cost, holding cost and, deterioration cost are considered in this model and the total cost is minimized.

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