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ON μ_ψ - CLOSED SETS IN TOPOLOGICAL SPACES

K. ALLI[1], M. DHANALAKSHMI[2*]

[1]Assistant professor, Department of Mathematics,
The M. D. T. Hindu College, Tirunelveli, Tamil Nadu, India-627010.

[2]Research scholar, Department of Mathematics,
The M. D. T. Hindu College, Tirunelveli, Tamil Nadu, India-627010.

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ABSTRACT

In this paper, we introduce a new class of sets namely, μ_ψ-closed sets and their properties. Applying these sets, we introduce and study some seven new spaces namely, $T\mu_\psi$, $\alpha T\mu_\psi$, $sT\mu_\psi$, $pT\mu_\psi$, $spT\mu_\psi$, $\mu T\mu_\psi$ and $\psi T\mu_\psi$-spaces and some interrelationships between these spaces.

Keywords: *μ_ψ-closed set, μ_ψ-open set $T\mu_\psi$, $\alpha T\mu_{\psi, s}$ $T\mu_{\psi, p}$ $T\mu_\psi$, $spT\mu_\psi$, $\mu T\mu_\psi$ and $\psi T\mu_\psi$-spaces.*

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1. INTRODUCTION

N. Levine [4] introduced the class of g–closed sets in 1970. Andrijevic [1], N. Levine [4], Mashoor *et.al* [7], have respectively introduced semipre-closed sets, semi-closed sets, pre-closed sets which are some weak forms of closed sets.

M. K. R. S. Veerakumar has introduced several generalized closed sets namely, g*-closed sets, *g–closed sets, α*g–closed sets, *gs–closed sets, αg-closed sets, ψ-closed sets, μ-closed sets, μs–closed sets and μp–closed sets. In this paper we introduce μ_ψ -closed sets and applying these sets seven new spaces namely $T\mu_\psi$, $\alpha T\mu_\psi$, $sT\mu_\psi$, $pT\mu_\psi$, $spT\mu_\psi$, $\mu T\mu_\psi$, $\psi T\mu_\psi$ are introduced.

2. PRELIMINARIES

Throughout this paper, we consider spaces on which no separation axioms are assumed unless explicity stated. For $A \subset X$, the closure and interior of A is denoted by cl(A) and int(A) respectively. The complement of A is denoted by A^C, the power set of X is denoted by P(X).

Definition 2.1: A subset A of a topological space (X, τ) is called
1. a pre-open set [6] if $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$.
2. a semi-open set [3] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.
3. an α-open set [7] if $A \subseteq int(cl(int(A))$ and α-closed set if $cl(int(cl(A))) \subseteq A$.
4. a semipre-open set [1] if $A \subseteq cl(int(cl(A)))$ and a semipre-closed set if $(cl(int(A))) \subseteq A$.
5. a regular open set [9] if $A = int(cl(A))$ and a regular closed set [19] if $cl(int(A)) = A$.

The intersection of all semiclosed (resp. preclosed, semipreclosed, α-closed) sets containing a subset A of X is called semiclosure (resp. preclosure, semipreclosure, α-closure) of A is denoted by scl(A) (resp. pcl(A), spcl(A), αcl(A)).

The union of all semiopen sets containted in A is called semiinterior of A and is denoted by sint (A).

Corresponding Author: M. Dhanalakshmi[2]*
[2]Research scholar, Department of Mathematics,
The M. D. T. Hindu College, Tirunelveli, Tamil Nadu, India-627010.

Definition 2.2: A subset A of a topological space (X, τ) is called

1. a generalized closed set (briefly g–closed [4] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
2. an α-generalized closed set (briefly αg–closed) [6] if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
3. a semi generalized closed set (briefly sg–closed) [2] if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
4. a gˆ -closed set [11] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
5. a *g–closed set [12] if cl(A) ⊆ U whenever A ⊆ U and U is gˆ-open in (X, τ).
6. a g*-closed set [12] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
7. a g*-preclosed set (briefly g*p–closed) [13] if pcl(A) ⊆ U whenever A ⊆ U and U is g–open in (X, τ).
8. a *g- semiclosed set [17] (briefly *gs-closed) if scl(A) ⊆ U whenever A ⊆ U and U is g^ -open in (X, τ).
9. a α*g-closed set [17] if αcl(A) ⊆ U whenever A ⊆ U, and U is g^ -open in (X, τ).
10. a gα*-closed set [5] if αcl(A) ⊆ int (U) whenever A ⊆ U and U is α-open in (X, τ).
11. a ψ-closed set [15] if scl(A) ⊆ U whenever A ⊆ U and U is sg–open in (X, τ).
12. a g* ψ-closed set [15] if ψ cl(A) ⊆ U whenever A ⊆ U and U is g–open in (X, τ).
13. a μ-closed set [16] if cl(A) ⊆ U whenever A ⊆ U and U is gα*-open in (X, τ).
14. a μ-preclosed set (briefly μp–closed) [17] if pcl(A) ⊆ U whenever A ⊆ U and U is gα*- open in (X, τ).
15. a μ-semiclosed set (briefly μs–closed) [18] if scl(A) ⊆ U whenever A ⊆ U and U is gα*-open in (X, τ).

Notations 2.3

1. αC(X, τ) is the class of α-closed subsets of (X, τ).
2. sC(X, τ) is the class of semi-closed subsets of (X, τ).
3. pC(X, τ) is the class of pre-closed subsets of (X, τ).
4. spC(X, τ) is the class of semipre-closed subsets of (X, τ).
5. μC(X, τ) is the class of μ-closed subsets of (X, τ).
6. ψC(X, τ) is the class of ψ-closed subsets of (X, τ).

3. PROPERTIES OF μ_{ψ} –CLOSED SETS

We introduce the following definition.

Definition 3.1: A subset A of (X, τ) is called μ_{ψ} -closed set if μcl(A) ⊆ U whenever A⊆ U and U is ψ-open in (X, τ). The class of μ_{ψ}–closed subsets of X is denoted by μ_{ψ} C(X, τ).

Proposition 3.2: Every closed set is μ_{ψ} –closed. But the converse is not true which can be seen from the following examples.

Example3.3: Let X = {a, b, c} and τ = {x, φ, {c}, {a, b}}. Here, the set {b} is μ_{ψ} – closed but it is not closed.

Proportion 3.4: μ_{ψ} - closedness is independent of α-closedness and semi-closedness.

Proof: It follows from the following examples

Example 3.5: Let X = {a, b, c}, τ = {x, φ, {a}}. Here the set {b} is α-closed and semi-closed but it is not μ_{ψ} -closed.

Example 3.6: Let X = {a, b, c}, τ= {x, φ, {a, b}}. Here the set {b, c} is μ_{ψ} -closed but it is neither α- closed nor semi-closed.

Proportion 3.7: Every μ_{ψ} -closed set is g – closed (resp. αg- closed, gα- closed).But the converses are not true as can be seen from the following examples.

Example 3.8: Let X = {a, b, c}, τ = {φ, x, {b}}. Here the set {a} is g-closed (resp. αg- closed, gα- closed) but it is not μ_{ψ}- closed.

Proposition 3.9: μ_{ψ} – closedness is independent of *g- closedness, α*g – closedness, ψ-closedness, g*ψ-closedness *gs-closedness and μs-closedness.

Proof: It follows from the following examples.

Example 3.10: Let X = {a, b, c}, τ = {φ, x, {a}}. Here the set {b} is both *g -closed and α*g-closed but it is not μ_{ψ} -closed.

Example 3.11: Let X = {a, b, c}, τ = {φ, x, {a}, {b, c}}. Here the set {b} is μ_{ψ} -closed but it is not *g-closed and not α*g-closed.

Example 3.12: Let X = {a, b, c}, τ = {φ, x, {a}}. Here the set {b} is both ψ-closed and g*ψ-closed but it is not μ_{ψ} -closed.

Example 3.13: Let X = {a, b, c}, τ = {φ, x, {a}, {b, c}}. Here the set {b} is μ_{ψ} -closed but it is not ψ-closed and not g*ψ-closed.

Example 3.14: Let X = {a, b, c}, τ = {φ, x, {a}, {a, b}}. Here the set {b} is *gs-closed and μs-closed but it is not

Example 3.15: Let X = {a, b, c}, τ = {φ, x, {a}, {b, c}}. Here the set {b} is μψ -closed but is not *gs-closed and not

Proposition 3.16: Every μ_{ψ} - closed set is g*p-closed. But the converse is not true as can be seen from the following example.

Example 3.17: Let X = {a, b, c}, τ = {φ, x, {b, c}}. Here the set {b} is g*p-closed but it is not μ_{ψ} -closed.

Proposition 3.18: Every μ-closed set is μ_{ψ} -closed. But the converse is not true as can be seen from the following example.

Example 3.19: Let X = {a, b, c}, τ = {φ, x, {c}, {a, b}}.Here the set {b} is μ_{ψ} -closed but it is not μ-closed.

Proposition 3.20: Every μ_{ψ} -closed set is μp-closed. But the converse is not true as can be seen from the following example.

Example 3.21: Let X = {a, b, c}, τ = {φ, x, {a}, {a, b}}.Here the set {b} is μp-closed but it is not μ_{ψ} -closed.

Theorem 3.22: The union (intersection) of any two μ_{ψ}-closed sets is also an μ_{A}–closed set.

Proposition 3.23: Let A and B be any two subsets of the topology (X, τ).Then

1. A is μ_{ψ} -closed, then μcl(A)\A does not contain any non empty ψ -closed set.
2. A is μ_{ψ} -closed and A ⊂ B ⊂ μcl(A), then B is μ_{ψ} -closed.

Proof: Let A be μ_{ψ} –closed and suppose μcl(A)\A contain a non empty ψ-closed set F. Therefore, F⊂ μcl(A)\A implies A⊂ F^c, which is ψ-open. Since A is μ_{ψ}-closed, μcl(A)⊂ F^c implies F ⊂ $(\mu cl(A))^c$, also F⊂μcl(A) therefore F ⊂ μcl(A)∩ $(\mu cl(A))^c$ = φ.

Let U be a ψ-open set such that B⊂U. Since A⊂B⊂U and U is ψ-open μcl(A)⊂U. Since B⊂μcl(A), cl(B)⊂μcl(μcl(A)) implies μcl(B) ⊂ μcl(A) ⊂ U therefore B is μψ-closed.

Theorem 3.24: Let A be a μ_{ψ}-closed set of a topological space (X, τ). Then

1. Sint(A) is μ_{ψ}-closed.
2. Pcl(A) is μ_{ψ}-closed.
3. If A is regular open, then pint(A) and scl(A) are also μ_{ψ}-closed sets.

Proof: First we note that for a subset A of (X, τ), scl(A) = A ∪ int(cl(A)) and pcl(A) = A ∪ cl(int(A)). Moreover sint(A) = A ∩ cl(int(A)) and pint(A) = A ∩ int(cl(A)).

Since cl(int(A)) is a closed set, then A and cl(int(A)) are μ_{ψ}-closed sets. By the theorem 3.22, A ∩ cl(int(A)) is also a μ_{ψ}-closed set.

1. Pcl(A) is the union of two μ_{ψ}-closed sets A and cl(int(A)). Again by the theorem 3.22, pcl(A) is μ_{ψ}-closed.
2. Since A is regular open, then A = int(cl(A)). Then scl(A) = A ∪ int(cl(A)) = A. Thus, scl(A) is μ_{ψ}-closed. Similarly pint(A) is also a μ_{ψ}-closed set.

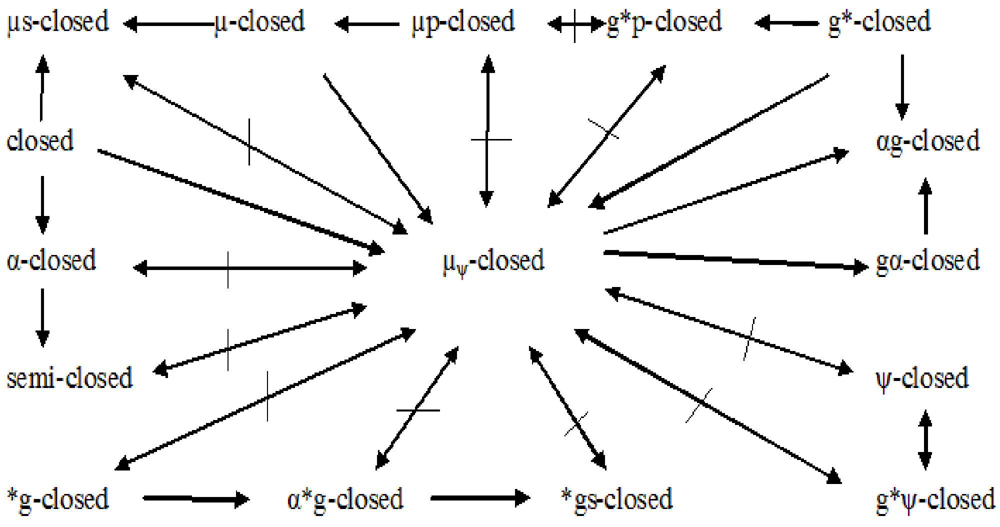
The converses of the statements in the above theorem are not true as we see from the following examples.

Example 3.25: Let (X, τ) be the space as in the example 3.14. B = {b} is not μ_{ψ}-closed set. However sint(B) = φ is a μ_{ψ}-closed set.

Example 3.26: Let X = {a, b, c} and τ = {φ, x, {a}, {b}, {a, b}, {a, c}}. Consider A = {c}. Clearly A is not regular open. However A is μ_{ψ}-closed and scl(A) = pint(A) = φ is μ_{ψ}-closed.

Remark 3.27: The following diagram shows the relationship established between μ_{ψ}-closed set and some other sets A → B (resp. A ↮ B) represents A implies B but not conversely (resp. A and B are independent of each other).

From the above Propositions and Examples, we have the following diagram.



Definition 3.28: A subset A of a space X is said to be μ_ψ–open if A^c is μ_ψ-closed. The class of all μ_ψ-open subsets of X is denoted by $\mu_\psi O(X, \tau)$.

Proposition 3.29: A subset A of a topological space X is said to be μ_ψ-open if and only if F ⊂ μint(A) whenever A ⊃ F and F is ψ-closed in X.

Proof: Suppose that A is μ_ψ-open in X and A ⊃ F, where F is ψ-closed in X. Then $A^c \subset F^c$, where F^c is ψ-open in X. Hence we get $\mu cl(A^c) \subset F^c$ implies μint(A) ⊃ F.

Conversely, suppose that $A^c \subset U$ and U is ψ-open in X then $A \supset U^c$ and U^c is ψ-closed then by hypothesis $\mu int(A) \supset U^c$ implies $(\mu int(A))^c \subset U$. Hence $\mu cl(A^c) \subset U$ gives A^c is μ_ψ-closed.

Proposition 3.30: In a topological space X, for each x ∈ X, either {x} is ψ–closed or μ_ψ –open in X.

Proof: Suppose that {x} is not ψ-closed in X. then X – {x} is not ψ-open and the only ψ-open set containing X – {x} is the space X itself. Therefore, μcl(X – {x}) ⊂ X and so X – {x} is μ_ψ –closed gives {x} is μ_ψ – open.

4. APPLICATION OF μ_ψ-CLOSED SETS

As an applications of μ_ψ-closed sets, new spaces namely, T μ_ψ, α T μ_ψ, s T μ_ψ, p T μ_ψ, sp T μ_ψ, μ T μ_ψ, ψ T μ_ψ spaces are introduced. First we introduce the following definitions.

Definition 4.1: A topological space (X, τ) is called a

1. T μ_ψ-space if every μ_ψ –closed set is closed.
2. αT μ_ψ-space if every μ_ψ –closed set is α-closed.
3. sT μ_ψ-space if every μ_ψ –closed set is semi-closed.
4. pT μ_ψ-space if every μ_ψ –closed set is pre-closed.
5. spT μ_ψ-space if every μ_ψ –closed set is semipre-closed.
6. μT μ_ψ-space if every μ_ψ –closed set is μ-closed.
7. ψT μ_ψ-space if every μ_ψ –closed set is ψ-closed.

Example 4.2: Let X = {a, b, c} and τ = {φ, x, {a}}. Here $\mu_\psi C(X, \tau)$ = {x, φ, {b, c}}. Then (X, τ) is T μ_ψ -space. The space in the following example is not a T μ_ψ-space. Let X = {a, b, c} and τ = {φ, x,{a, b}}. Here $\mu_\psi C(X, \tau)$ = {x, φ, {c}, {b, c}, {a, c}}.

Example 4.3: Let X = {a, b, c} and τ = {φ, x, {b}}. Here $\mu_\psi C(X, \tau)$ = {x, φ, {a, c}}. Then (X, τ) is αTμ_ψ-space. The space in the following example is not aαT μ_ψ-space. Let X = {a, b, c} and τ = {φ, x, {a, b}}. Here $\mu_\psi C(X, \tau)$ = {x, φ, {c}, {b, c}, {a, c}} and αC(X, τ) = {φ, x, {a, b}}.

Proposition 4.4: If (X, τ) is a αTμ_ψ-space then every singleton of X is either ψ–closed or μ-open.

Proof: Let x ∈ X. Suppose {x} is not ψ–closed, then X-{x} is not ψ–open. This implies that X is the only ψ–open set containing X-{x}. So X-{x} is μ_ψ–closed of (X, τ). Since (X, τ) is αTμ_ψ –space, X-{x} is α–closed and every α–closed is μ-closed implies X-{x} is μ–closed or equivalently {x} is μ–open. The converse of the above proposition is not true as it can be seen from the following example.

Example 4.5: Let X = {a, b, c} and τ = {φ, x, {a, c}}. Here every singleton of X is either ψ–closed or μ–open but is not $\alpha T\mu_\psi$-space.

Proposition 4.6: Every $\alpha T\mu_\psi$ (resp. $sT\mu_\psi$)-space is $pT\mu_\psi$-space.

Proof: It follows from the fact that every α-closed (resp. semi-closed) is pre-closed. The converse of the above proposition is not true as it can be seen by the following example.

Example 4.7: Let X = {a, b, c} and τ = {φ, x, {a}, {b, c}}. Here (X, τ) is $pT\mu_\psi$ –space but it is not a $\alpha T\mu_\psi$ (resp. not a $sT\mu_\psi$)-space.

Proposition 4.8: Every $T\mu_\psi$-space is $pT\mu_\psi$-space, $spT\mu_\psi$-space, $\mu T\mu_\psi$ –space and $\psi T\mu_\psi$-space but not conversely.

Example 4.9: The space (X, τ) in Example 4.5 is $pT\mu_\psi$-space, $spT\mu_\psi$-space, $\mu T\mu_\psi$ –space and $\psi T\mu_\psi$-space but not $T\mu_\psi$-space.

Proposition 4.10: Every $T\mu_\psi$ (resp. α T μ_ψ) space is μ T μ_ψ -space, but not conversely.

Proof: Let A be μ_ψ -closed set in a topological space X, which is T μ_ψ -space. Hence A is closed implies A is μ-closed. Therefore T μ_ψ -space is μ T μ_ψ -space. Similarly A is μ_ψ -closed set in topological space X which is α T μ_ψ -space. Hence A is α-closed implies A is μ-closed. Therefore $\alpha T\mu_\psi$ -space is μ T μ_ψ -space.

Converse is not true as it can be seen by the following example. The space (X, τ) in the example 4.9 is μT μ_ψ –space but it is neither T μ_ψ –space nor αT μ_ψ-space.

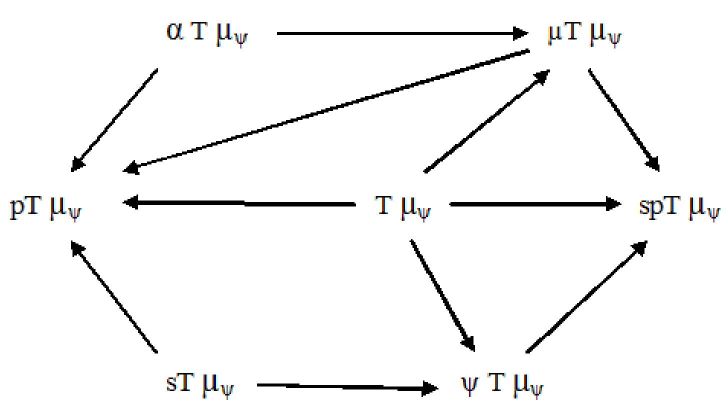
Theorem 4.11: The following statements are true but the respective converses are not true in general.

1. If (X, τ) is a T μ_ψ-space, then every singleton of X is either ψ-closed or open.
2. If (X, τ) is a αT μ_ψ-space, then every singleton of X is either ψ-closed or pre-open.
3. If (X, τ) is a sT μ_ψ-space, then every singleton of X is either ψ-closed or μ-open.
4. If (X, τ) is a μT μ_ψ-space, then every singleton of X is either gα*-closed or μ_ψ -open.
5. If (X, τ) is a ψT μ_ψ-space, then every singleton of X is either sg-closed or μ_ψ-open.

Proof:

1. Let x $\in$ X and suppose that {x} is not a ψ-closed of (X, τ). This implies that X-{x} is not ψ–open set. So X is the only ψ –open set such that X-{x} $\subseteq$ X. Then X-{x} is a μ_ψ –closed set of(X, τ). Since is a T μ_ψ –space, then X-{x} is closed or equivalently {x} is open.
2. The proofs for the first assertions of 2 to 5 are similar to as that of the first assertions of (1). The space (X, τ) as in the example 4.7 shows that the converses of 1 to 5 need not be true.

Remark 4.12: The following diagram shows relationship among the spaces considered in this paper.



REFERENCES

1. Andrijevic.D, Semipreopen sets, mat. vesnik, 38 (1986), 24 – 32.
2. Bhattacharya.P, and Lahiri .B. K., Semi–generalized closed set in topology, IndianJ. Math.29 (1987), no.3, 375–382.
3. Levine .N, Semiopen sets and semi continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36 – 41.
4. Levine N, Generalized closed sets in topology, Rend. Circ. Mat. Ser. III, 10 (1975), 347 – 350.
5. Maki. H, Devi. R and Balachandran. K, Generalized α-closed sets in topology, Bull. Fukuoka Univ. Ed PartIII 42(1993), 13 – 21.
6. Maki. H, Devi. R and Balachandran. K Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math. 15 (1994), 51- 63.
7. Mashour. A. S, Abd EI-Monsef. M. E and EI-Deeb. S. N, On precontinuous and weak pre continuous mappings, Proc. Math. Phys. Soc. Egypt. 53 (1982), 47 – 53.
8. Pious Missier.S, and Sucila.E, On $\hat{\mu}$ **-** closed sets in topological spaces.
9. Pious Missier.S, and Alli.K, g#p-closed sets in topological spaces.
10. Veerakumar. M. K. R. S, Between closed sets and g–closed sets, Mem, Fac. Sci Kochi Univ. Ser A. Math 21 (2000), 1 – 19.
11. Veerakumar. M. K. R. S, on g^-closed sets in topological spaces, Bull. Allahabad Math. Soc, (18) (2003)
12. Veerakumar. M. K. R. S, Between g*-closed sets and g–closed sets Antartica J. Math, Reprint.
13. Veerakumar. M. K. R. S, g*-preclosed sets Acta Ciencia Indica (Mathematics) Meerut, XXV111 (M) (1) (2002), 51 – 60.
14. Veerakumar. M. K. R. S , Presemiclosed Indian J. Math. 44(2) (2002), 165 – 181
15. Veerakumar M. K. R. S., Between ψ-closed sets and gsp-closed sets, Antartica J. Math., 2(1) (2005), 123 – 141.
16. Veerakumar. M. K. R. S, μ-closed sets in topological spaces, Antartica J. Math, 2(1)(2005), 1 – 18.
17. Veerakumar. M. K. R. S, μp–closed sets in topological spaces, Antartica J. Math. 2(1) (2005), 31 – 52.
18. Veerakumar. M. K. R. S, μs–closed sets in topological spaces, Antartica J. Math. 2(1) (2005), 91 – 109.

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