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ON μ_ψ - CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of sets namely, μ_ψ-closed sets and their properties. Applying these sets, we introduce and study some seven new spaces namely, $T\,\mu_\psi$, $\alpha\,T\,\mu_\psi$, $s\,T\,\mu_\psi$, $p\,T\,\mu_\psi$, $spT\,\mu_\psi$,$\mu\,T\,\mu_\psi$ and $\psi\,T\,\mu_\psi$-spaces and some interrelationships between these spaces.

Keywords: *μ_ψ-closed set, μ_ψ-open set $T\,\mu_\psi$, $\alpha\,T\,\mu_{\psi, s}\,T\,\mu_{\psi, p}\,T\,\mu_\psi$, $spT\,\mu_\psi$, $\mu\,T\,\mu_\psi$ and $\psi\,T\,\mu_\psi$-spaces.*

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1. INTRODUCTION

N. Levine [4] introduced the class of g–closed sets in 1970. Andrijevic [1], N. Levine [4], Mashoor *et.al* [7], have respectively introduced semipre-closed sets, semi-closed sets, pre-closed sets which are some weak forms of closed sets.

M. K. R. S. Veerakumar has introduced several generalized closed sets namely, g*-closed sets, *g–closed sets, α*g–closed sets, *gs–closed sets, αg-closed sets, ψ-closed sets, μ-closed sets, μs–closed sets and μp–closed sets. In this paper we introduce μ_ψ -closed sets and applying these sets seven new spaces namely $T\,\mu_\psi$, $\alpha\,T\,\mu_\psi$, $s\,T\,\mu_\psi$, $p\,T\,\mu_\psi$, $spT\,\mu_\psi$, $\mu\,T\,\mu_\psi$, $\psi\,T\,\mu_\psi$ are introduced.

2. PRELIMINARIES

Throughout this paper, we consider spaces on which no separation axioms are assumed unless explicity stated. For $A \subset X$, the closure and interior of A is denoted by cl(A) and int(A) respectively. The complement of A is denoted by A^C, the power set of X is denoted by P(X).

Definition 2.1: A subset A of a topological space (X, τ) is called
1. a pre-open set [6] if $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$.
2. a semi-open set [3] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.
3. an α-open set [7] if $A \subseteq int(cl(int(A))$ and α-closed set if $cl(int(cl(A))) \subseteq A$.
4. a semipre-open set [1] if $A \subseteq cl(int(cl(A)))$ and a semipre-closed set if $(cl(int(A))) \subseteq A$.
5. a regular open set [9] if $A = int(cl(A))$ and a regular closed set [19] if $cl(int(A)) = A$.

The intersection of all semiclosed (resp. preclosed, semipreclosed, α-closed) sets containing a subset A of X is called semiclosure (resp. preclosure, semipreclosure, α-closure) of A is denoted by scl(A) (resp. pcl(A), spcl(A), αcl(A)).

The union of all semiopen sets containted in A is called semiinterior of A and is denoted by sint (A).

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Definition 2.2: A subset A of a topological space (X, τ) is called

1. a generalized closed set (briefly g–closed [4] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
2. an α-generalized closed set (briefly αg–closed) [6] if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
3. a semi generalized closed set (briefly sg–closed) [2] if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
4. a gˆ -closed set [11] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
5. a *g–closed set [12] if cl(A) ⊆ U whenever A ⊆ U and U is gˆ-open in (X, τ).
6. a g*-closed set [12] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
7. a g*-preclosed set (briefly g*p–closed) [13] if pcl(A) ⊆ U whenever A ⊆ U and U is g–open in (X, τ).
8. a *g- semiclosed set [17] (briefly *gs-closed) if scl(A) ⊆ U whenever A ⊆ U and U is g^ -open in (X, τ).
9. a α*g-closed set [17] if αcl(A) ⊆ U whenever A ⊆ U, and U is g^ -open in (X, τ).
10. a gα*-closed set [5] if αcl(A) ⊆ int (U) whenever A ⊆ U and U is α-open in (X, τ).
11. a ψ-closed set [15] if scl(A) ⊆ U whenever A ⊆ U and U is sg–open in (X, τ).
12. a g* ψ-closed set [15] if ψ cl(A) ⊆ U whenever A ⊆ U and U is g–open in (X, τ).
13. a μ-closed set [16] if cl(A) ⊆ U whenever A ⊆ U and U is gα*-open in (X, τ).
14. a μ-preclosed set (briefly μp–closed) [17] if pcl(A) ⊆ U whenever A ⊆ U and U is gα*- open in (X, τ).
15. a μ-semiclosed set (briefly μs–closed) [18] if scl(A) ⊆ U whenever A ⊆ U and U is gα*-open in (X, τ).

Notations 2.3

1. αC(X, τ) is the class of α-closed subsets of (X, τ).
2. sC(X, τ) is the class of semi-closed subsets of (X, τ).
3. pC(X, τ) is the class of pre-closed subsets of (X, τ).
4. spC(X, τ) is the class of semipre-closed subsets of (X, τ).
5. μC(X, τ) is the class of μ-closed subsets of (X, τ).
6. ψC(X, τ) is the class of ψ-closed subsets of (X, τ).

3. PROPERTIES OF μ_ψ –CLOSED SETS

We introduce the following definition.

Definition 3.1: A subset A of (X, τ) is called μ_ψ -closed set if μcl(A) ⊆ U whenever A⊆ U and U is ψ-open in (X, τ). The class of μ_ψ–closed subsets of X is denoted by μ_ψ C(X, τ).

Proposition 3.2: Every closed set is μ_ψ –closed. But the converse is not true which can be seen from the following examples.

Example3.3: Let X = {a, b, c} and τ = {x, φ, {c}, {a, b}}. Here, the set {b} is μ_ψ – closed but it is not closed.

Proportion 3.4: μ_ψ - closedness is independent of α-closedness and semi-closedness.

Proof: It follows from the following examples

Example 3.5: Let X = {a, b, c}, τ = {x, φ, {a}}. Here the set {b} is α-closed and semi-closed but it is not μ_ψ -closed.

Example 3.6: Let X = {a, b, c}, τ= {x, φ, {a, b}}. Here the set {b, c} is μ_ψ -closed but it is neither α- closed nor semi-closed.

Proportion 3.7: Every μ_ψ -closed set is g – closed (resp. αg- closed, gα- closed).But the converses are not true as can be seen from the following examples.

Example 3.8: Let X = {a, b, c}, τ = {φ, x, {b}}. Here the set {a} is g-closed (resp. αg- closed, gα- closed) but it is not μ_{ψ}- closed.

Proposition 3.9: μ_ψ – closedness is independent of *g- closedness, α*g – closedness, ψ-closedness, g*ψ-closedness *gs-closedness and μs-closedness.

Proof: It follows from the following examples.

Example 3.10: Let X = {a, b, c}, τ = {φ, x, {a}}. Here the set {b} is both *g -closed and α*g-closed but it is not μ_ψ -closed.

Example 3.11: Let X = {a, b, c}, τ = {φ, x, {a}, {b, c}}. Here the set {b} is μ_ψ -closed but it is not *g-closed and not α*g-closed.

Example 3.12: Let X = {a, b, c}, τ = {φ, x, {a}}. Here the set {b} is both ψ-closed and g*ψ-closed but it is not μ_ψ -closed.

Example 3.13: Let X = {a, b, c}, τ = {φ, x, {a}, {b, c}}. Here the set {b} is μ_ψ -closed but it is not ψ-closed and not g*ψ-closed.

Example 3.14: Let X = {a, b, c}, τ = {φ, x, {a}, {a, b}}. Here the set {b} is *gs-closed and μs-closed but it is not

Example 3.15: Let X = {a, b, c}, τ = {φ, x, {a}, {b, c}}. Here the set {b} is μψ -closed but is not *gs-closed and not

Proposition 3.16: Every μ_ψ - closed set is g*p-closed. But the converse is not true as can be seen from the following example.

Example 3.17: Let X = {a, b, c}, τ = {φ, x, {b, c}}. Here the set {b} is g*p-closed but it is not μ_ψ -closed.

Proposition 3.18: Every μ-closed set is μ_ψ -closed. But the converse is not true as can be seen from the following example.

Example 3.19: Let X = {a, b, c}, τ = {φ, x, {c}, {a, b}}.Here the set {b} is μ_ψ -closed but it is not μ-closed.

Proposition 3.20: Every μ_ψ -closed set is μp-closed. But the converse is not true as can be seen from the following example.

Example 3.21: Let X = {a, b, c}, τ = {φ, x, {a}, {a, b}}.Here the set {b} is μp-closed but it is not μ_ψ -closed.

Theorem 3.22: The union (intersection) of any two μ_ψ-closed sets is also an μ_A–closed set.

Proposition 3.23: Let A and B be any two subsets of the topology (X, τ).Then

1. A is μ_ψ -closed, then μcl(A)\A does not contain any non empty ψ -closed set.
2. A is μ_ψ -closed and A ⊂ B ⊂ μcl(A), then B is μ_ψ -closed.

Proof: Let A be μ_ψ –closed and suppose μcl(A)\A contain a non empty ψ-closed set F. Therefore, F⊂ μcl(A)\A implies A⊂ F^c, which is ψ-open. Since A is μ_ψ-closed, μcl(A)⊂ F^c implies F ⊂ $(\mu cl(A))^c$, also F⊂μcl(A) therefore F ⊂ μcl(A)∩ $(\mu cl(A))^c$ = φ.

Let U be a ψ-open set such that B⊂U. Since A⊂B⊂U and U is ψ-open μcl(A)⊂U. Since B⊂μcl(A), cl(B)⊂μcl(μcl(A)) implies μcl(B) ⊂ μcl(A) ⊂ U therefore B is μψ-closed.

Theorem 3.24: Let A be a μ_ψ-closed set of a topological space (X, τ). Then

1. Sint(A) is μ_ψ-closed.
2. Pcl(A) is μ_ψ-closed.
3. If A is regular open, then pint(A) and scl(A) are also μ_ψ-closed sets.

Proof: First we note that for a subset A of (X, τ), scl(A) = A ∪ int(cl(A)) and pcl(A) = A ∪ cl(int(A)). Moreover sint(A) = A ∩ cl(int(A)) and pint(A) = A ∩ int(cl(A)).

Since cl(int(A)) is a closed set, then A and cl(int(A)) are μ_ψ-closed sets. By the theorem 3.22, A ∩ cl(int(A)) is also a μ_ψ-closed set.

1. Pcl(A) is the union of two μ_ψ-closed sets A and cl(int(A)). Again by the theorem 3.22, pcl(A) is μ_ψ-closed.
2. Since A is regular open, then A = int(cl(A)). Then scl(A) = A ∪ int(cl(A)) = A. Thus, scl(A) is μ_ψ-closed. Similarly pint(A) is also a μ_ψ-closed set.

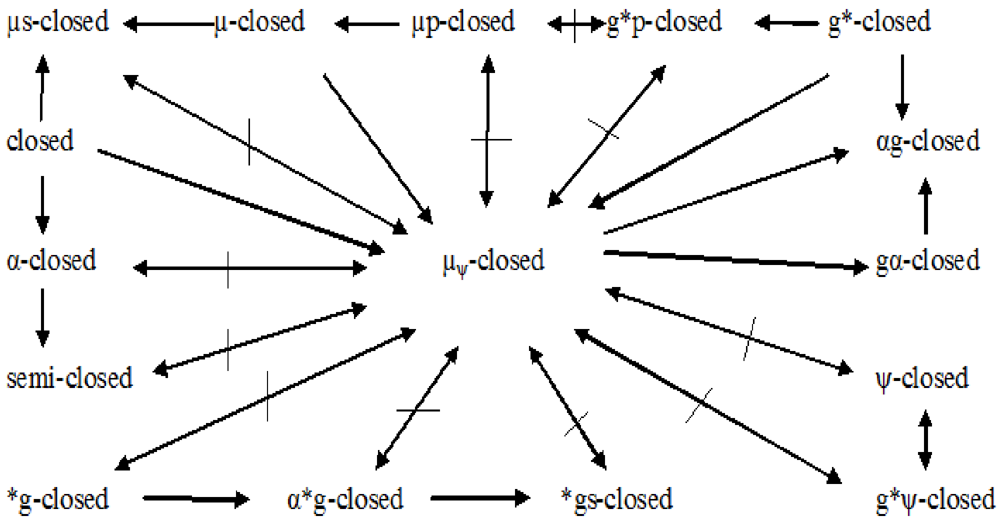
The converses of the statements in the above theorem are not true as we see from the following examples.

Example 3.25: Let (X, τ) be the space as in the example 3.14. B = {b} is not μ_ψ-closed set. However sint(B) = φ is a μ_ψ-closed set.

Example 3.26: Let X = {a, b, c} and τ = {φ, x, {a}, {b}, {a, b}, {a, c}}. Consider A = {c}. Clearly A is not regular open. However A is μ_ψ-closed and scl(A) = pint(A) = φ is μ_ψ-closed.

Remark 3.27: The following diagram shows the relationship established between μ_ψ-closed set and some other sets A → B (resp. A ↮ B) represents A implies B but not conversely (resp. A and B are independent of each other).

From the above Propositions and Examples, we have the following diagram.



Definition 3.28: A subset A of a space X is said to be μ_{ψ}–open if A^c is μ_{ψ}-closed. The class of all μ_{ψ}-open subsets of X is denoted by $\mu_{\psi}O(X, \tau)$.

Proposition 3.29: A subset A of a topological space X is said to be μ_{ψ}-open if and only if $F \subset \mu int(A)$ whenever $A \supset F$ and F is ψ-closed in X.

Proof: Suppose that A is μ_{ψ}-open in X and $A \supset F$, where F is ψ-closed in X. Then $A^c \subset F^c$, where F^c is ψ-open in X. Hence we get $\mu cl(A^c) \subset F^c$ implies $\mu int(A) \supset F$.

Conversely, suppose that $A^c \subset U$ and U is ψ-open in X then $A \supset U^c$ and U^c is ψ-closed then by hypothesis $\mu int(A) \supset U^c$ implies $(\mu int(A))^c \subset U$. Hence $\mu cl(A^c) \subset U$ gives A^c is μ_{ψ}-closed.

Proposition 3.30: In a topological space X, for each $x \in X$, either {x} is ψ–closed or μ_{ψ} –open in X.

Proof: Suppose that {x} is not ψ-closed in X. then X – {x} is not ψ-open and the only ψ-open set containing X – {x} is the space X itself. Therefore, $\mu cl(X - \{x\}) \subset X$ and so X – {x} is μ_{ψ} –closed gives {x} is μ_{ψ} – open.

4. APPLICATION OF μ_{ψ}-CLOSED SETS

As an applications of μ_{ψ}-closed sets, new spaces namely, T μ_{ψ}, α T μ_{ψ}, s T μ_{ψ}, p T μ_{ψ}, sp T μ_{ψ} ,μ T μ_{ψ} , ψ T μ_{ψ} spaces are introduced. First we introduce the following definitions.

Definition 4.1: A topological space (X, τ) is called a
1. T μ_{ψ}-space if every μ_{ψ} –closed set is closed.
2. αT μ_{ψ}-space if every μ_{ψ} –closed set is α-closed.
3. sT μ_{ψ}-space if every μ_{ψ} –closed set is semi-closed.
4. pT μ_{ψ}-space if every μ_{ψ} –closed set is pre-closed.
5. spT μ_{ψ}-space if every μ_{ψ} –closed set is semipre-closed.
6. μT μ_{ψ}-space if every μ_{ψ} –closed set is μ-closed.
7. ψT μ_{ψ}-space if every μ_{ψ} –closed set is ψ-closed.

Example 4.2: Let X = {a, b, c} and τ = {φ, x, {a}}. Here $\mu_{\psi}C(X, \tau)$ = {x, φ, {b, c}}. Then (X, τ) is T μ_{ψ} -space. The space in the following example is not a T μ_{ψ}-space. Let X = {a, b, c} and τ = {φ, x,{a, b}}. Here $\mu_{\psi}C(X, \tau)$ = {x, φ, {c}, {b, c}, {a, c}}.

Example 4.3: Let X = {a, b, c} and τ = {φ, x, {b}}. Here $\mu_{\psi}C(X, \tau)$ = {x, φ, {a, c}}. Then (X, τ) is αTμ_{ψ}-space. The space in the following example is not aαT μ_{ψ}-space. Let X = {a, b, c} and τ = {φ, x, {a, b}}. Here $\mu_{\psi}C(X, \tau)$ = {x, φ, {c}, {b, c}, {a, c}} and αC(X, τ) = {φ, x, {a, b}}.

Proposition 4.4: If (X, τ) is a αTμ_{ψ}-space then every singleton of X is either ψ–closed or μ-open.

Proof: Let $x \in$ X. Suppose {x} is not ψ–closed, then X-{x} is not ψ–open. This implies that X is the only ψ–open set containing X-{x}. So X-{x} is μ_{ψ}–closed of (X, τ). Since (X, τ) is αTμ_{ψ} –space, X-{x} is α–closed and every α–closed is μ-closed implies X-{x} is μ–closed or equivalently {x} is μ–open. The converse of the above proposition is not true as it can be seen from the following example.

Example 4.5: Let X = {a, b, c} and τ = {φ, x, {a, c}}. Here every singleton of X is either ψ–closed or μ–open but is not $\alpha T\mu_{\psi}$-space.

Proposition 4.6: Every $\alpha T\mu_{\psi}$ (resp. $sT\mu_{\psi}$)-space is $pT\mu_{\psi}$-space.

Proof: It follows from the fact that every α-closed (resp. semi-closed) is pre-closed. The converse of the above proposition is not true as it can be seen by the following example.

Example 4.7: Let X = {a, b, c} and τ = {φ, x, {a}, {b, c}}. Here (X, τ) is $pT\mu_{\psi}$ –space but it is not a $\alpha T\mu_{\psi}$ (resp. not a $sT\mu_{\psi}$)-space.

Proposition 4.8: Every $T\mu_{\psi}$-space is $pT\mu_{\psi}$-space, $spT\mu_{\psi}$-space, $\mu T\mu_{\psi}$ –space and $\psi T\mu_{\psi}$-space but not conversely.

Example 4.9: The space (X, τ) in Example 4.5 is $pT\mu_{\psi}$-space, $spT\mu_{\psi}$-space, $\mu T\mu_{\psi}$ –space and $\psi T\mu_{\psi}$-space but not $T\mu_{\psi}$-space.

Proposition 4.10: Every $T\mu_{\psi}$ (resp. $\alpha\, T\, \mu_{\psi}$) space is $\mu\, T\, \mu_{\psi}$ -space, but not conversely.

Proof: Let A be μ_{ψ} -closed set in a topological space X, which is $T\, \mu_{\psi}$ -space. Hence A is closed implies A is μ-closed. Therefore $T\, \mu_{\psi}$ -space is $\mu\, T\, \mu_{\psi}$ -space. Similarly A is μ_{ψ} -closed set in topological space X which is $\alpha\, T\, \mu_{\psi}$ -space. Hence A is α-closed implies A is μ-closed. Therefore $\alpha T\mu_{\psi}$ -space is $\mu\, T\, \mu_{\psi}$ -space.

Converse is not true as it can be seen by the following example. The space (X, τ) in the example 4.9 is $\mu T\, \mu_{\psi}$ –space but it is neither $T\, \mu_{\psi}$ –space nor $\alpha T\, \mu_{\psi}$-space.

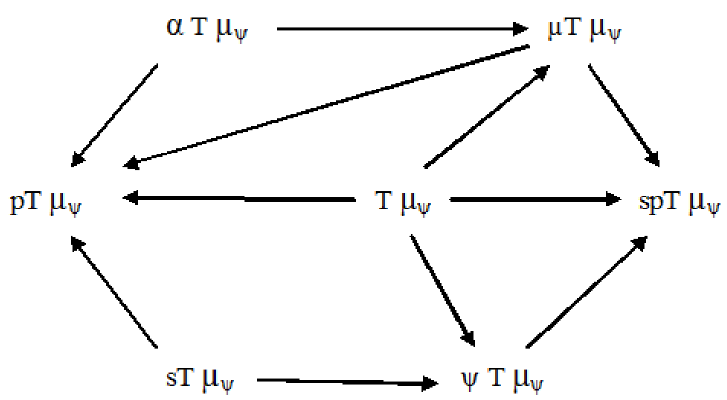
Theorem 4.11: The following statements are true but the respective converses are not true in general.

1. If (X, τ) is a $T\, \mu_{\psi}$-space, then every singleton of X is either ψ-closed or open.
2. If (X, τ) is a $\alpha T\, \mu_{\psi}$-space, then every singleton of X is either ψ-closed or pre-open.
3. If (X, τ) is a $sT\, \mu_{\psi}$-space, then every singleton of X is either ψ-closed or μ-open.
4. If (X, τ) is a $\mu T\, \mu_{\psi}$-space, then every singleton of X is either gα*-closed or μ_{ψ} -open.
5. If (X, τ) is a $\psi T\, \mu_{\psi}$-space, then every singleton of X is either sg-closed or μ_{ψ}-open.

Proof:

1. Let x ∈ X and suppose that {x} is not a ψ-closed of (X, τ). This implies that X-{x} is not ψ–open set. So X is the only ψ –open set such that X-{x} ⊆ X. Then X-{x} is a μ_{ψ} –closed set of(X, τ). Since is a $T\, \mu_{\psi}$ –space, then X-{x} is closed or equivalently {x} is open.
2. The proofs for the first assertions of 2 to 5 are similar to as that of the first assertions of (1). The space (X, τ) as in the example 4.7 shows that the converses of 1 to 5 need not be true.

Remark 4.12: The following diagram shows relationship among the spaces considered in this paper.



REFERENCES

1. Andrijevic.D, Semipreopen sets, mat. vesnik, 38 (1986), 24 – 32.
2. Bhattacharya.P, and Lahiri .B. K., Semi–generalized closed set in topology, IndianJ. Math.29 (1987), no.3, 375–382.
3. Levine .N, Semiopen sets and semi continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36 – 41.
4. Levine N, Generalized closed sets in topology, Rend. Circ. Mat. Ser. III, 10 (1975), 347 – 350.
5. Maki. H, Devi. R and Balachandran. K, Generalized α-closed sets in topology, Bull. Fukuoka Univ. Ed PartIII 42(1993), 13 – 21.
6. Maki. H, Devi. R and Balachandran. K Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math. 15 (1994), 51- 63.
7. Mashour. A. S, Abd EI-Monsef. M. E and EI-Deeb. S. N, On precontinuous and weak pre continuous mappings, Proc. Math. Phys. Soc. Egypt. 53 (1982), 47 – 53.
8. Pious Missier.S, and Sucila.E, On $\hat{\mu}$ - closed sets in topological spaces.
9. Pious Missier.S, and Alli.K, g#p-closed sets in topological spaces.
10. Veerakumar. M. K. R. S, Between closed sets and g–closed sets, Mem, Fac. Sci Kochi Univ. Ser A. Math 21 (2000), 1 – 19.
11. Veerakumar. M. K. R. S, on g^-closed sets in topological spaces, Bull. Allahabad Math. Soc, (18) (2003)
12. Veerakumar. M. K. R. S, Between g*-closed sets and g–closed sets Antartica J. Math, Reprint.
13. Veerakumar. M. K. R. S, g*-preclosed sets Acta Ciencia Indica (Mathematics) Meerut, XXV111 (M) (1) (2002), 51 – 60.
14. Veerakumar. M. K. R. S , Presemiclosed Indian J. Math. 44(2) (2002), 165 – 181
15. Veerakumar M. K. R. S., Between ψ-closed sets and gsp-closed sets, Antartica J. Math., 2(1) (2005), 123 – 141.
16. Veerakumar. M. K. R. S, μ-closed sets in topological spaces, Antartica J. Math, 2(1)(2005), 1 – 18.
17. Veerakumar. M. K. R. S, μp–closed sets in topological spaces, Antartica J. Math. 2(1) (2005), 31 – 52.
18. Veerakumar. M. K. R. S, μs–closed sets in topological spaces, Antartica J. Math. 2(1) (2005), 91 – 109.

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