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DENSELY CO-HOPFIAN SUB NEAR-FIELD SPACES OVER NEAR-FIELD

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ABSTRACT

The sub near-field spaces of near-field of the title are introduced as a proper generalization of quasi co-Hopfian sub near-field spaces of near-field, and are characterized in several ways. The near-field space N is right non-singular if and only if the densely co-Hopfian right N-sub near-field spaces over near-field and quasi co-Hopfian right N-near-field spaces over near-field coincide. Dense co-Hopficity is investigated for certain N-sub near-field spaces over near-field that has indecomposable decompositions complementing direct summands. For some classes of near-field spaces N, including rings, near-rings, regular δ-near-rings, near-fields with dense right socles, we determine when the injective envelope of an N-sub near-field space over near-field is densely co-Hopfian.

Key words: *Goldie torsion theory, reduced rank, dense sub-near-field space, densely co-Hopfian, quasi co-Hopfian, weakly co-Hopfian, prime sub near-field space.*

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SECTION-1: INTRODUCTION

Throughout near-field spaces will have unity, sub near-field spaces; near-fields will be unitary. Let K denote a right sub near-field space over a near-field space N. Co-Hopfian sub near-field spaces are generalized in the following ways. A weakly co-Hopfian (wcH–sn-fs) sub near-field space is defined by the property that all injective N-endomorphisms of the sub near-field space are essential.

In depth study makes Dr N V Nagendram to investigate these concepts are clearly co-Hopfian sub near-field spaces implies weakly co-Hopfian sub near-field spaces implies quasi co-Hopfian sub near-field spaces and none of these implications can be reserved.

In this paper Dr N V Nagendram introduce and study a notion for sub near-field spaces called densely co-Hopfian sub near-field spaces (dcH-sn-fs). K_N is densely co-Hopfian sub near-field spaces (dcH-sn-fs) if for all injective N-endomorphisms f of K, f (K) is a dense sub near-field space of K in the extension of Goldie torsion theory of Mod-R to sub near-field space of a near-field space N, K/f (K) is Z_2 – torsion. We show that Goldie torsion theory sub near-field spaces i.e. Z_2-torsion sub near-field spaces as well as quasi co-Hopfian sub near-field spaces (dcH-sn-fs) are densely co-Hopfian sub near-field spaces (dcH-sn-fs) but not conversely.

The dcH-sn-fs property is investigated for direct sums and it can be determine that the dcH-sn-fs property for a certain sub near-field space that has an indecomposable decomposition complementing direct summands. We then discuss when the dcH-sn-fs property transfers between a sub near-field space and its injective envelope. We also consider sub near-field spaces with the property that all their sub near-field spaces are dcH-sn-fs. Such sub near-field spaces will be called completely co-Hopfian sub near-field spaces (ccH-sn-fs), and they are characterized. Finally in for three types of near-field spaces N, including near-field spaces with dense right socles, we prove that K_N is ccH-sn-fs if and only if the injective hull $E(K_N)$ of K_N is dcH-sn-fs.

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We now fix our notation and state a few well known preliminary results that will be needed. As a hereditary torsion class, the class of Z_2-torsion sub near-field spaces is closed under sub near-field spaces, factor sub near-field spaces, direct sums and extensions. These facts imply that $S \leq_d T$ and $T \leq_d U$ if and only if $S \leq_d U$. Moreover, the Goldie torsion theory is stable, that is, the Z_2-torsion class is closed under taking injective envelopes. Let us write $Z(K/S) = S^*/S$ and $Z(K/S^*) = S^{**}/S$, and put on record several properties each of which is equivalent to S being dense in K. We omit the proofs.

Definition 1.1: Co-Hopfian sub near-field space. Let K a right sub near-field space over a near-field space N is called co-Hopfian sub near-field space if every injective N-endomorphism of K is surjective.

Definition 1.2: Quasi Co-Hopfian sub near-field space. The right (or left) sub near-field space K is called quasi co-Hopfian sub near-field space (qcH-sn-fs) if K/*f* (K) is singular sub near-field space whenever *f* is an injective N-endomorphism of K.

Definition 1.3: Densely Co-Hopfian sub near-field space. The right (or left) sub near-field spaces called densely co-Hopfian sub near-field spaces (dcH-sn-fs).

Definition 1.4: Dense sub near-field space. Let K_N is densely co-Hopfian sub near-field spaces (dcH-sn-fs) if for all injective N-endomorphisms *f* of K, *f* (K) is a dense sub near-field space of K in the extension of Goldie torsion theory of Mod-R to sub near-field space of a near-field space N, K/*f* (K) is Z_2 – torsion.

Definition 1.5: Essential sub near-field space. Let K be a right N-sub near-field space, S be a N-sub near-field space. Then $S \leq_e K$ will mean that S is an essential sub near-field space of K.

Definition 1.6: The singular sub near-field space of K is denoted by (K), and $Z_2(K)$ is defined by $Z(K/Z(K)) = Z_2(K)/Z(K)$. The uniform dimension of $K/Z_2(K)$ is called the reduced rank of K, and we denote this by r(K). K is called singular sub near-field space if K = Z(K) and non-singular sub near-field space if Z(K) = 0.

Definition 1.7: Goldie torsion. The sub near-field space K is called Goldie torsion or Z_2 – torsion if $Z_2(K) = K$.

Definition 1.8: Dense near-field space. If K/S is Goldie torsion sub near-field space, then S is said to be a dense sub near-field space of K, and this fact is denoted by $S \leq_e K$.

Note 1.9: For any sub near-field space K_N, $Z_2(K) = \{x \in K : ann(x) \leq_d N_N\}$.

Definition 1.10: CS or Σ-extending. A sub near-field space of near-field space K is called CS or extending if every closed sub near-field space of K is a direct summand of K.

Note 1.11: A near-field space N for which every free sub near-field space over a near-field is CS is called Σ-extending.

Note 1.12: Over such rings, near-rings, δ-near-rings, near-fields, sub near-field spaces and near-field spaces a dcH-sn-fs is exactly a direct sum of a wch sub near-field space and a Z_2 – torsion sub near-field space.

Definition 1.13: A sub near-field space of a near-field space over a near-field K is quasi co-Hopfian sub near-field space (quasi co-Hopfian sub near-field space) if $f(K)^* = K$ for every injective endomorphism *f* of K.

Definition 1.14: K is densely co-Hopfian sub near-field space (dcH-sn-fs) if $f(K)^{**}$= for every injective endomorphism *f* of K.

Definition 1.15: Compressible (resp. Retractable). A sub near-field space K is called compressible respectively retractable if there exists a monomorphism (respectively non-zero homomorphism) $f: K \to N$ for any non-zero sub near-field space N of K.

Definition 1.16: Continuous. An extending sub near-field space K is called continuous if every sub near-field space V of K which is isomorphic to a direct summand of K, is a direct summand of K, every non-singular injective sub near-field space of a near-field space N over a near-field of finite reduced rank is a direct sum of indecomposable sub near-field spaces.

Definition 1.17: The endomorphism sub near-field space of an indecomposable continuous sub near-field space is a local sub near-field space, a non-singular continuous sub near-field space over a near-field space of finite reduced rank has a decomposition into indecomposable continuous sub near-field spaces such that the endomorphism near-field space over a near-field of each direct summand is local sub near-field space.

Note 1.18: A continuous sub near-field space has the finite exchange property such a decomposition complements direct summands.

Definition 1.19: Σ-quasi injective. A sub near-field space K is called Σ-quasi injective sub near-field space if every direct sum of copies of K is quasi –injective.

Note 1.20: Every Σ-CS sub near-field space, hence every Σ-quasi injective sub near-field space has an indecomposable decomposition which complements direct summands.

Proposition 1.21: Let $S \leq_d K$. The following are equivalent statements.

(a) $S \leq_d K$
(b) K/S^* is singular
(c) $S^* \leq_e K$
(d) $S^{**} = K$
(e) $S + Z(K) \leq_e K$
(f) $S + Z_2(K) \leq_e K$
(g) $(S + Z_2(K))/ Z_2(K) \leq e\ K/ Z_2(K)$
(h) $S \oplus T \leq_e K$ for some Z_2- torsion sub near-field space T of K
(i) $S \cap T \neq 0$ for every non-singular sub near-field space T of K
For every sub near-field space T of K, $S \cap T \leq Z_2(K)$
(j) For all $k \in K \setminus Z_2(K)$, there exists $n \in N$ such that $kn \in S \setminus Z_2(A)$.

A notable property of dense sub near-field spaces is that their inverse under homomorphisms are again dense sub near-field spaces. We shall also make use of the following well known facts, proofs of which are given for reader's convenience.

Proposition 1.22: (a) The intersection of all dense sub near-field spaces of K is the sum P(K) of all non-singular simple sub near-field spaces of K. Consequently $Z_2(K)\ P(N_N) = 0$. (b) The product of two dense right sub near-field spaces is a dense sub near-field space.

Proof: To prove (a): Let D(K) the intersection of all dense sub near-field spaces of K. If D is a dense sub near-field space and P is a non-singular simple sub near-field space of K, then $P \cap D \neq 0$. Hence $P \leq D$ and so $P(K) \leq D(K)$. On the other hand, if W is complement to $Z_2(K)$ then $W \leq_d K$. Thus D(K) is non-singular.

However, $D(K) \leq Soc(K)$ since every essential sub near-field space is dense. Thus $D(K) \leq P(K)$.

Now if $k \in Z_2(K)$ then an $n(k) \leq_d N_N$. However, $P(N_N)$ is the intersection of all dense right sub near-field spaces of a near-field space N over a near-field, hence $P(N_N) \leq ann(k)$. Thus $Z_2(K)\ P(N_N) = 0$. Proved (a).

To prove (b): Let U and V be dense right (or left) sub near-field spaces of a near-field space N over a near-field. Then N/U and U/UV are Z_2-torsion. Therefore from the isomorphism $[N/UV]/[U/ UV] \equiv N/U$ we conclude that N/UV is Z_2-torsion, hence UV is dense sub near-field space. Proved (b). This completes the proof of the proposition.

SECTION-2: DENSELY CO-HOPFIAN SUB NEAR-FIELD SPACES

In section 2, we obtain the result which describes several equivalent conditions to dcH-sn-fs property. It reduces to some parts of the base near-field space over a near-field is right non-singular sub near-field space.

Definition 2.1: K_N is called densely co-Hopfian sub near-field space (dcH-sn-fs) if the image of any injective N-endomorphism of K is a dense sub near-field space (dsn-fs)

Theorem 2.2: The following statements are equivalent for an N-sub near-field space K.

(a) K is dcH-sn-fs (dense co-Hopfian sub near-field space)

(b) K contains a dense sub near-field space V which is dcH-sn-fs(dense co-Hopfian sub near-field space) as on N-sub near-field space and $f(V) \leq V$ for any injective endomorphism f of K.

(c) There exists a dense sub near-field space V of K such that $f(V) \cap V \leq_d V$ whenever f is an injective endomorphism of K.

(d) If there exists an N-monomorphism $K \oplus N \to K$ then N is Z_2-torsion.

(e) For every dense sub near-field space V of K and every injective endomorphism f of K, $f(V) \leq_d V$.

(f) For every non-Z_2-torsion sub near-field space V of K and every injective endomorphism f of K, $f^{-1}(V)$ is non-Z_2-torsion.

(g) There exists a sub near-field space V of K such that V and K/V are dcH-sn-fs (dense co-Hopfian sub near-field space) and $f^{-1}(V) = V$ for any injective endomorphism f of K.

Proof: It is obvious that (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (a).

To prove (a) $\Rightarrow$ (d): Let f: $K \oplus V \to K$ be a monomorphism and $\tau : K \to K \oplus V$ be the canonical injection. Then f_τ is an injective endomorphism of K. Thus $f(K \oplus 0) \leq_d K$, hence $f(0 \oplus V)$ is Z_2-torsion and so V is Z_2-torsion. Proved (a) $\Rightarrow$ (d).

To prove (d) $\Rightarrow$ (a): Let f be an injective endomorphism of K. There exists a sub near-field space $V \leq K$ such that $f(K) \oplus V \leq_e K$. Therefore V must be Z_2-torsion and consequently $f(K) \leq_d K$. Proved (d) $\Rightarrow$ (a).

To prove (a) $\Rightarrow$ (e): Let f be an injective endomorphism of K. Then we have $f(K) \leq_d f(K)$. On the other-hand, $V \leq_d K$ implies that $f(V) \leq_d f(K)$. Thus $f(V) \leq_d K$. Proved (a) $\Rightarrow$ (e).

To prove (e) $\Rightarrow$ (a): On applying (e) for V = K. Proved (e) $\Rightarrow$ (a).

To prove (a) $\Rightarrow$ (f): Let f be an injective endomorphism of K and V be a non-Z_2-torsion sub near-field space of a near-field space N over a near-field. By proposition 1.21 we have that for every sub near-field space T of K, $S \cap T \leq Z_2(K)$, $f(K) \cap V$ is non-Z_2-torsion. Thus there exists $k \in K \backslash Z_2(K)$ such that $f(k) \in K$. Consequently $k \in f^{-1}(V)$ and $k \notin Z_2(K)$.Thus $f^{-1}(V)$ is non-Z_2-torsion. Proved (a) $\Rightarrow$ (f).

To prove (f) $\Rightarrow$ (a): Let g be an injective endomorphism of K such that $g(K)$ not $\leq_d$ K. There exists a non-Z_2-torsion sub near-field space of a near-field space N over a near-field such that $g(K) \oplus V \leq_e K$. Thus by (f) $g^{-1}(V)$ is non-Z_2-torsion. However, $g^{-1}(V) = g^{-1}(g(K) \cap V) = g^{-1}(0) = 0$ is $\otimes$ a contradiction. Proved (f) $\Rightarrow$ (a).

To prove (a) $\Rightarrow$ (g): Set V = K, and finally we define a mapping $\overline{f}$: K/V $\to$ K/V by $\overline{f}$ $(k + V) = f(k) + V$. By hypothesis $f(K) + V \leq_d K$. On the other hand, $[f(K) + V]/f(K) \cong V/f(K) \cong V/[f(K) \cap V] = V / f(V)$. By assumption that V is dcH-sn-fs (dense co-Hopfian sub near-field space), the sub near-field space $V/f(V)$ is Z_2-torsion and so $f(K) \leq_d f(K) + V$. It follows that $f(K) \leq_d K$. Proved (a) $\Rightarrow$ (g). This completes the proof of the theorem.

Corollary 2.3: Let K be an N-sub near-field space.

(a) If K is dcH-sn-fs(dense co-Hopfian sub near-field space) then so every direct summand of K

(b) If $K/Z_2(K)$ is dcH-sn-fs(dense co-Hopfian sub near-field space) then so is K. In particular, every sub near-field space of finite reduced rank is dcH- sn-fs(dense co-Hopfian sub near-field space).

(c) Assume that U is a dense right(or left) sub near-field space of N. If KU is dcH-sn-fs(dense co-Hopfian sub near-field space) then so is K.

Proof: Obvious.

Proposition 2.4: Let K be a sub near-field space of a near-field space N over a near-field.

(a) If K is dcH-sn-fs(dense co-Hopfian sub near-field space) then so is every direct summand of K.

(b) If $K = \sum_{i \in U} K_i$ such that $f(K_i) \cap K_i \leq_d K_i$ for any injective endomorphism f of K, then K is dcH-sn-fs (dense co-Hopfian sub near-field space).

Proof: To prove (a): Let $K^{(\Lambda)}$ be dcH-sn-fs (dense co-Hopfian sub near-field space). We can assume that Λ is countable. Then $K/f(K)$ is Z_2-torsion where $f : K^{(\Lambda)} \to K^{(\Lambda)}$ is the shift map. However K is isomorphic to $K/f(K)$ hence K is Z_2-torsion. The converse is clear since every direct sum of Z_2-torsion sub near-field spaces of a near-field space over a near-field is Z_2-torsion. Proved (a).

To prove (b): Let f be an injective endomorphism of K and $V_i = f(K_i) \cap K_i$ for each $i \in I$. Define $\varphi : \oplus_{i \in I} (K_i/V_i) \rightarrow K/f(K)$ by $(k_i + V_i)_{i \in I} \mapsto \left(\sum_{j \in J} k_j\right) + f(K)$, where J is the largest sub near-field space of I such that $k_j \notin V_j$ for any $j \in J$. Then φ is an epi-morphism and $\oplus_{i \in I} (K_i/V_i)$ is Z_2-torsion. Thus $K/f(K)$ is Z_2-torsion and so K is dcH-sn-fs (dense co-Hopfian sub near-field space). Proved (b). This completes the proof of the proposition.

Corollary 2.5: Let K be semi simple sub near-field space. Then K is dcH-sn-fs (dense co-Hopfian sub near-field space) if and only if every non-singular homogeneous component of K is finitely generated.

Proposition 2.6: The following statements are equivalent for a sub near-field space K of a near-field space N over a near-field.

(a) K is wcH-sn-fs
(b) K is qcH-sn-fs (quasi co-Hopfian sub near-field space) and for any injective endomorphism f of K, $f(Z(K)) \leq_e Z(K)$.
(c) K is dcH-sn-fs (dense co-Hopfian sub near-field space) and for any injective endomorphism f of K, $f(Z(K)) \leq_e Z(K)$.
(d) K is qcH-sn-fs (quasi co-Hopfian sub near-field space) and for any injective endomorphism f of K, $f(Z_2(K)) \leq_e Z_2(K)$.
(e) K is dcH-sn-fs (dense co-Hopfian sub near-field space) and for any injective endomorphism f of K, $f(Z_2(K)) \leq_e Z_2(K)$.

Proof: Is obvious.

Corollary 2.7: Let K be a sub near-field space of a near-field space N over a near-field such that Z(K) or $Z_2(K)$ is wcH-sn-fs. Then K is dcH-sn-fs(dense co-Hopfian sub near-field space) if and only if K is qcH-sn-fs(quasi co-Hopfian sub near-field space) if and only if K is wcH-sn-fs(weakly co-Hopfian sub near-field space).

Proposition 2.8: Let N be a near-field space over a near-field.

(a) The class of dcH-sn-fs N-sub near-field spaces coincides with the class of wcH-sn-fs(weakly co-Hopfian sub near-field space) N-sub near-field spaces if and only if N is semi simple.
(b) The class of dcH-sn-fs N-sub near-field spaces coincides with the class of qcH-sn-fs (quasi co-Hopfian sub near-field space) N-sub near-field spaces if and only if N is right non-singular.

Proof: (a) is obvious.

To prove (b) (⇒): Let K be a N-sub near-field space of a near-field space N over a near-field. Then $Z_2(K)^{(N)}$ is Z_2-torsion and so it is dcH-sn-fs (dense co-Hopfian sub near-field space). Thus by hypothesis $Z_2(K)^{(N)}$ is qcH-sn-fs(quasi co-Hopfian sub near-field space), hence $Z_2(K)$ is singular sub near-field space. Consequently $Z_2(K) = Z(K)$, in particular, $Z_2(E) = Z(E)$ where $E = E(N_N)$. However E is extending and so Z(E) is a direct summand of E. This implies that $Z(E) = 0$, hence $Z(N_N) = 0$.

(⇐) Since the notions of Z_2-torsion and singular sub near-field space are the same for a sub near-field space of a near-field space over a near-field is a right non-singular near-field space, the properties dcH-sn-fs (dense co-Hopfian sub near-field space) and qcH-sn-fs (quasi co-Hopfian sub near-field space) are equivalent. Proved (b). This completes the proof of the proposition.

Example 2.9: We now construct examples of dcH-sn-fs sub near-field spaces which are neither Z_2-torsion nor qcH. Let N be a right Noetherian near-field spaces over a near-field such that $Z(N_N) \neq 0$ and $Z_2(N_N) \neq N$. then $Z_2(E) \neq Z(E)$ where $E = E(N_N)$ since $Z(N_N) \neq 0$. And also $N/Z_2(N_N) \oplus Z_2(E)^{(N)}$ is dcH-sn-fs(dense co-Hopfian sub near-field space) which is neither Z_2-torision nor qcH-sn-fs(quasi co-Hopfian sub near-field space).

Theorem 2.10: The following statements are equivalent for a near-field space N over a near-field.

(a) Every projective, free N-sub near-field space K is dcH-sn-fs (dense co-Hopfian sub near-field space)
(b) Every projective, free N-sub near-field space K, K/Z(K) is dcH-sn-fs (dense co-Hopfian sub near-field space)
(c) Every projective, free N-sub near-field space K, $K/Z_2(K)$ is dcH-sn-fs (dense co-Hopfian sub near-field space)
(d) Every projective, free N-sub near-field space K, $K/Z_2(K)$ is qcH-sn-fs (q co-Hopfian sub near-field space)
(e) Every projective, free N-sub near-field space K, K/Z(K) is singular
(f) Every projective, free N-sub near-field space K is Z_2-torsion.
(g) There exists a nilpotent dense right (or left) sub near-field space in N.

(h) $Rad(N) \leq_d N_N$ and $Soc(N_N) \leq_d N_N$.
(i) $Z(N_N) \leq_e N_N$.
(j) N_N is Z_2-torsion.

Proof: This theorem we prove by cyclic method of proof.

To prove (a) ⇒ (f): Let K be an N-sub near-field space and Λ be an infinite set. By hypothesis K^Λ is dcH-sn-fs (dense co-Hopfian sub near-field space), hence K is Z_2-torsion. Proved (a) ⇒ (f).

To prove (f) ⇒ (e): is obvious since $Z(K/Z(K)) = Z_2(K)/Z(K)$. Proved (f) ⇒ (e).

It is obvious that (e) ⇒ (d) ⇒ (b).

Now let (b) hold. Then $L = (N/Z(N_N))^{(N)} \cong N(^{N)}/z(N_N)^{(N)}$ is dcH-sn-fs (dense co-Hopfian sub near-field space). Since $L \oplus (N/Z(N_N)) \cong L$ it implies that $N/Z(N_N)$ is Z_2-torsion, thus $Z(N_N) \leq_d N_N$ and therefore it is evident that $Z(N_N) \leq_e N_N$. this shows that (b) ⇒ (i). Obvious that (i) ⇒ (j).

If (j) holds then every N-sub near-field space K is Z_2-torsion as $KZ_2(N_N) \leq Z_2(K)$ hence K is dcH-sn-fs (dense co-Hopfian sub near-field space). Thus (j) ⇒ (a) proved.

Clearly (f) ⇒ (c) and (c) ⇒ (j) by setting $Z_2(K)$ instead of $Z(K)$ in the proof of (b) ⇒ (i).

If (f) holds the zero sub near-field space is nilpotent and dense right sub near-field space of a near-field space N, say $V^n = 0$. If n = 1 then V = 0 is Z_2-torsion. If n > 1 then V^{n-1} is a dense right sub near-field space and so $V V^{n-1} = 0$ implies that V is Z_2-torsion. Therefore N contains a Z_2-torsion dense right sub near-field space, hence N_N is Z_2-torsion. Thus every right sub near-field space of N is dense sub near-field space; hence N_N is Z_2-torsion. Thus every right sub near-field space of N is dense and so (g) ⇒ (h).

Now let (h) holds and K be an N-sub near-field space. Since $rad(N_N) \leq_d N_N$ and $K\ Rad(N) \leq Rad(K)$ we conclude that $Rad(K) \leq_d K$. Thus $0 = Rad(P) \leq_d P$ for every simple N-sub near-field space P and so every simple N-sub near-field space is Z_2-torsion which is singular and hence $Soc(N_N) \leq Z_2(N_N)$. By hypothesis we have $Soc(N_N) \leq_d N_N$ we conclude that $Z_2(N_N) \leq_d N_N$ and so $Z_2(N_N) = N_N$. Hence (h) ⇒ (j) proved. This completes the proof of the theorem.

Proposition 2.11: For a near-field space N, if $Soc(N_N) \leq_e N_N$ then each of the statements of theorem 2.11 is equivalent to any one of the following conditions.

(a) $Rad(N) \leq_e N_N$
(b) For every N-sub near-field space K, K/Rad(N) is singular
(c) For every N-sub near-field space K, K/Rad(K) is qcH-sn-fs
(d) Every simple N-sub near-field space is singular.
(e) $Soc(N_N)^2 = 0$
(f) $S(N_N)^2 = 0$.

Proof: To prove (a): Every simple N-sub near-field space is singular and so every maximal right sub near-field space is essential. Thus $Soc(N_N) \leq Rad(N)$ and then the hypothesis $Soc(N_N) \leq_e N_N$ implies that $Rad(N) \leq_e N_N$. This shows that proved (a).

It is clear that and follows (a) ⇒ (b) ⇒ (c).

Clearly, (b) ⇒ (d). The equality $Z(N_N)\ Soc(N_N) = 0$. Since $Soc(N_N) \leq_e N_N$ and $S(N_N) \leq_d Soc(N_N)$, we conclude that $S(N_N) \leq_d N_N$ and so $S(N_N)$ is a nilpotent dense right sub near-field space of near-field space N over a near-field.

Thus (f) ⇒ There exists a nilpotent dense right (or left) sub near-field space in N of (f) theorem 2.10. This completes the proof of the proposition.

Proposition 2.12: Let K be a homomorphic image of a CS sub near-field space. K is dcH-sn-fs (dense co-Hopfian sub near-field space) if and only if K is isomorphic to a direct sum of a non-singular dcH-sn-fs (dense co-Hopfian sub near-field space) and a Z_2-torsion sub near-field space.

Proof: Assume that K ≡ W/V where U is a CS sub near-field space and V ≤ W. First we show that $Z_2(W/V)$ is a direct summand of W/V. Assume that $Z_2(W/V) = V'/V$. Then V′ is a closed (or open) sub near-field space of a near-field space over a near-field of W. In fact if $V' \leq_d N \leq W$ then N/V is Z_2-torsion since (N/V)/(V′/V) and V′/V are Z_2-torsion. Thus $N/V \leq Z_2(W/V)$ and so N = V′. Therefore V′ is a direct summand of W, say $W = V' \oplus V''$. Hence $W/V = V'/V \oplus (V'' + V)/V$ as desired. This completes the proof of the proposition.

Theorem 2.13: Let K be a near-field space over a near-field N such that *f*(K) is a direct summand of K for every injective endomorphism *f* of K. Suppose K has an indecomposable decomposition that complements direct summands. Then K is dcH-sn-fs (dense co-Hopfian sub near-field space) if and only if every non-Z_2-torsion homogeneous component of such a decomposition of K is a finite direct sum.

Proof: (⇒) A non-Z_2-torsionhmogeneous component of an indecomposable decomposition of K is isomorphic to $N^{(\Lambda)}$ for some non-Z_2-torsion indecomposable sub near-field space of a near-field space over a near-field V of K. Since K is dcH-sn-fs (dense co-Hopfian sub near-field space) so is $N^{(\Lambda)}$ thus Λ is finite.

(⇐) Let *f* be an injective endomorphism of K. By hypothesis $K = f(K) \oplus V$ for some sub near-field space V. Assume that V is non-Z_2-torsion. By hypothesis there exists an indecomposable decomposition $K = \oplus_{\alpha \in S} K_\alpha$ that components direct summands and so there exists such a decomposition for V. Then $V = \oplus_{\beta \in T} K_\beta$. Then $K = \underset{\alpha \in S}{\oplus} K_\alpha$ (*)

And $K = \underset{\alpha \in S}{\oplus} f(K_\alpha) \oplus \begin{pmatrix} \oplus K_\beta \\ \beta \in T \end{pmatrix}$ (**)

(*) and (**) are equivalent. Now for a $\beta_1 \in T$ such that $V_{\beta 1}$ is non-Z_2-torsion, $V\beta_1 \equiv K\alpha_1$ for some $\alpha_1 \in S$. By hypothesis the homogeneous component of K corresponding to $K_{\alpha 1}$ has finitely many direct summands say $K_{\alpha 1}, K_{\alpha 2},..., K_{\alpha n.}$ Then in (**) there is a homogeneous component with at least n+1 direct summands, i.e. $f(K_{\alpha 1}), f(K_{\alpha 2}), f(K_{\alpha 3}),....., f(K_{\alpha n}), K_{\beta 1}$ which are all isomorphic to $K_{\alpha 1}$. This contradicts ⊗ the equivalence of (*) and (**). Therefore V is Z_2-torsion and so $f(K) \leq_d K$. This completes the proof of the theorem.

Corollary 2.14: N be a near-field space over a near-field.

(a) If N is of finite reduced rank and K is a continuous N-sub near-field space, then K is dcH-sn-fs (dense co-Hopfian sub near-field space) if and only if every non-singular homogeneous component of a decomposition of K into indecomposable continuous sub near-field spaces is a finite direct sum.

(b) If K is a Σ - quasi – injective sub near-field space, then K is dcH-sn-fs (dense co-Hopfian sub near-field space) if and only if every non-singular homogeneous component of a decomposition of K into indecomposable Σ - quasi – injective sub near-field spaces is a finite direct sum.

Note 2.15: A divisible abelian sub near-field space of a near-field space K over a near-field N is dcH-sn-fs (dense co-Hopfian sub near-field space)if and only if $n(K) < \infty$.

Note 2.16: Let N be aright Artinian local sub near-field space that is not a division sub near-field space. Then $Rad(N) \leq_e N_N$ and $Soc(N_N) \leq_e N_N$ and so N satisfies all the conditions of theorem 2.10. Moreover, $S = \begin{pmatrix} N & Rad(N) \\ 0 & N \end{pmatrix}$ is a right Artinian sub near-field space and clearly $Rad(S) = \begin{pmatrix} Rad(N) & Rad(N) \\ 0 & Rad(N) \end{pmatrix} \leq_e S_S$

Thus S also satisfies the conditions of theorem 2.10 although S is not local sub near-field space. Also A right Artinian sub near-field space each of the statements of theorem 2.10 holds good if and only if $Rad(N) \leq_e N_N$.

SECTION-3: COMPLETE CO-HOPFICITY AND INJECTIVE ENVELOPE NEAR-FIELD SPACES OVER NEAR-FIELD

In this chapter, we have a natural question is whether the dense co-Hopficity passes to injective envelope. As the notions of dcH-sn-fs (dense co-Hopfian sub near-field space) and qcH-sn-fs (quasi co-Hopfian sub near-field space) are the same for a non-singular sub near-field space shows that in general the answer to this question is negative. By the additive property of reduced rank if K is of finite reduced rank, then so is E(K) and hence it follows that E(K) is dcH-sn-fs(dense co-Hopfian sub near-field space). However, every sub near-field space of a finite reduced rank sub near-field space is dcH-sn-fs (dense co-Hopfian sub near-field space). Let us call a sub near-field space all of whose sub near-field spaces are dcH-sn-fs (dense co-Hopfian sub near-field space), a (ccH-sn-fs) completely co-Hopfian sub near-field space. Now it is natural to ask whether E(K) is dcH-sn-fs(dense co-Hopfian sub near-field space) if K is ccH-sn-fs (completely co-Hopfian sub near-field space). In the following we show that a quasi injective dcH-sn-fs (dense co-Hopfian sub near-field space) sub near-field space is ccH-sn-fs(completely co-Hopfian sub near-field space). Moreover for some classes of sub near-field spaces we show that the answer to the latter question is affirmative.

Proposition 3.1: The following statements are equivalent for a sub near-field space K of a near-field space N over a near-field.

(a) K is ccH-sn-fs (class of co-Hopfian sub near-field space)
(b) every dense sub near-field space of K is dcH-sn-fs (dense co-Hopfian sub near-field space)
(c) every non-dense sub near-field space of K is dcH-sn-fs (dense co-Hopfian sub near-field space)
(d) $X^{(N)}$ can not be embedded in K, for any non Z_2-torsion sub near-field space X.

Proof: clearly (a) $\Rightarrow$ (b) and (a) $\Rightarrow$ (c)

To prove (b) $\Rightarrow$ (a): this follows by every sub near-field space is a direct summand of an essential sub near-field space of a near-field space over a near-field. Proved (b) $\Rightarrow$ (a).

To prove (c) $\Rightarrow$ (a): Let V be a sub near-field space of K which is not dcH-sn-fs (dense co-Hopfian sub near-field space). There exists an injective endomorphism *f* of V such that *f* (K) not $\leq_d$ K (not less than or equal to). Thus *f* (K) not $\leq_d$ K, however *f* (K) ≡ K implies that *f* (K) is not dcH-sn-fs (dense co-Hopfian sub near-field space) which ⊗ contradicts (c). Proved (c) $\Rightarrow$ (a). Proof of (a) $\Rightarrow$ (d) is obvious.

To prove (d) $\Rightarrow$ (a): Let V be a sub near-field space of K. If V is not dcH-sn-fs(dense co-Hopfian sub near-field space) then there exists a non Z_2-torsion sub near-field space X such that V $\oplus$ X can be embedded in V which ⊗ contradicts (d). Proved (d) $\Rightarrow$ (a). This completes the proof of the proposition.

Proposition 3.2: The following statements are equivalent for a sub near-field space K of a near-field space N over a near-field.

(a) K is dcH-sn-fs(dense co-Hopfian sub near-field space)
(b) K is ccH-sn-fs(complete co-Hopfian sub near-field space)
(c) E(K) is dcH-sn-fs(dense co-Hopfian sub near-field space)

Proof: To prove (a) $\Rightarrow$ (b): It is sufficient to show that every essential sub near-field space of a near-field space over a near-field K is dcH-sn-fs (dense co-Hopfian sub near-field space). Let V be an essential sub near-field space of K and *g* be an injective endomorphism of V. As K is quasi co-Hopfian sub near-field space injective, there exists an endomorphism *f* of K such that $f\backslash_V = g$. The essentiality of V implies that *f* is an injective endomorphism of K, hence $f(K) \leq_d$ K. Clearly, $f(V) \leq_d f(K)$, thus $f(V) \leq_d$ K and so $f(V) \leq_d V$. Proved (a) $\Rightarrow$ (b).

To prove (b) $\Rightarrow$ (c): obvious from theorem 2.2 of (b) for V = K. Proved (b) $\Rightarrow$ (c)

To prove (c) $\Rightarrow$ (a): by applying (a) $\Rightarrow$ (b) to the sub near-field space E(K) we conclude that E(K) is ccH-sn-fs (complete co-Hopfian sub near-field space), hence K is dcH-sn-fs (dense co-Hopfian sub near-field space).

Proved (c) $\Rightarrow$ (a). This completes the proof of the proposition.

Lemma 3.3: Let N be a near-field space for which Soc $(N_N) \leq_d N_N$ or let N be a near-field space of finite reduced rank which is either semi prime near-field space or has a. c. c. on two sided sub near-field spaces. Then every non-zero non-singular right N-sub near-field space K contains on essential sub near-field space $L = \oplus_{i \in I} U_i$ where each U_i is a uniform compressible right N-sub near-field space.

Theorem 3.4: Let N be a near-field space for which Soc $(N_N) \leq_d N_N$ or let N be a near-field space of finite reduced rank which is either semi prime near-field space or has a. c. c. on two sided sub near-field spaces. Then K is a ccH-sn-fs(class of co-Hopfian sub near-field space) N-sub near-field space if and only if E(K) is dcH-sn-fs(dense co-Hopfian sub near-field space).

Corollary 3.5: Let N be a near-field space.

(a) If $Soc(N_N) \leq_d N_N$, then K is ccH-sn-fs if and only of Soc(K) is dcH-sn-fs
(b) If N is right Artinian near-field space then on N-sub near-field space K is ccH-sn-fs(class of co-Hopfian sub near-field space) if and only if $r(K) < \infty$

Corollary 3.6: Let N be a near-field space for which Soc(N_N) $\leq_d$ N_N or let N be a near-field space of finite reduced rank which is either semi prime near-field space or has a. c. c. on two sided sub near-field spaces. The following statements are equivalent.

(a) In sub near-field space-N, {ccH-sn-fs sub near-field spaces} = {sub near-field spaces of finite reduced rank}
(b) Up to isomorphisms, there are only finitely many non-singular indecomposable injective N-sub near-field spaces of a near-field space over a near-field.

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