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# SOME MATHEMATICAL MISTAKES <br> ON RATIO AND PROPORTION (ALGEBRAIC TREATMENT) 

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#### Abstract

In this paper, four problems concerning conceptual and operational mistakes are generally done by the learners in dealing with 'Ratio and Proportion (Algebraic Treatment)' and their proofs/solutions \& explanation of the proofs/solutions of mistakes have been presented. This study has been done through 'text book scanning', 'class room interaction', 'answer scripts of achievement test' and 'experts' opinions'. This study will help the learners to understand important mathematical concept of 'Ratio and Proportion (Algebraic Treatment) as well as entire teaching learning process of mathematics.


Key words: mathematical problem, solution, proof, mistake, mathematical concept, ratio, proportion, algebraic treatment..

MSC AMS Subject Classification 2010: 97A 20, 97C10, 97C70, 97D70, 97H99.

## 1. INTRODUCTION

Ratio and proportion in algebra are important major concepts which are introduced in the syllabus and text books of mathematics at the upper primary and secondary level in different Boards in our country. Many real life problems are solved easily by the knowledge of ratio and proportion. To make the happy and prosperous life in future of each learner and to study the higher mathematics, the concepts of ratio and proportion should be acquired by the learners. In this regard, it may be said that the author has identified and sequenced the elementary concepts of ratio and proportion (algebraic treatment) [10, 12]. But it is a matter of pity that a large number of learners do mistakes while they solve the problems of ratio and proportion in algebra on account of conceptual gaping and lack of knowledge for application the basic concepts properly in any appropriate place. Hence, their achievements in the subject and interest towards the subject deteriorate which is major hindrance to learn mathematics as well as science. So, the researcher has tried to emphasize to frame the problems involving conceptual mistakes of mathematics are generally done by the learners. This study will help the learners for better understanding. Some works have been done but that is sporadic, not systematic [1, 2, 3, 8, 9, 15]. But the author has already done some works on mathematical mistake at school level [11, 13, 14].

In this paper, three problems with proper solutions and their explanations \& four problems concerning conceptual and operational mistakes with proof and their proper explanations on 'Ratio and Proportion (Algebraic Treatment) have been presented.

## 2. OBJECTIVE OF THE STUDY

To frame the problems involving conceptual mistakes on 'Ratio and Proportion (Algebraic Treatment) including their solutions/proofs \& explain the solutions/proofs of the mistakes.

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## 3. MATERIALS AND METHODS

The following materials and methods have been adopted:

1. Text books of mathematics of different Boards were collected at first [4, 5, 6, 7]. Selection of basic concepts 'Ratio and Proportion (Algebraic Treatment) and then identification of important elementary concepts of these basic concepts from class room interaction, answer scripts of the achievement test where the learners do mistake, have been done.
2. Three problems of 'Ratio and Proportion (Algebraic Treatment) have been set and their solutions \& proofs and the explanation of the solutions \& proofs have been done.
3. Framing the problems of mathematical mistake on 'Ratio and Proportion (Algebraic Treatment) and their solutions and explanation of the proofs/solutions of the mistakes have been done by the researcher keeping in view the logical order of the subject and the psychological order of learners.
4. Experts' opinions were taken on these. Their opinions were incorporated for the improvement of the study.
5. Then the final forms of mathematical mistake on 'Ratio and Proportion (Algebraic Treatment) have been done.

## 4. SALIENT POINTS OF THE STUDY

Adopting the above materials and methods, four mistakes and their solutions /proofs and explanations of the proofs of mistakes on 'Ratio and Proportion (Algebraic Treatment) have been presented below. For the better understanding of these mistakes, three problems and their proper solutions and explanations of the solutions on 'Ratio and Proportion (Algebraic Treatment) have also been presented below.

Problem-1: Verify 2, 3, 4 and 6 are in proportion.
Solution: Here $2 \times 6=12$ and $3 \times 4=12$
$\therefore 2,3,4$ and 6 are in proportion.

## Explanation of the solution of the problem-1:

Explanation: We know that if a, b, c and d are in proportion, then a : b : : c : d i.e. $\frac{\mathrm{a}}{\mathrm{b}}=\frac{\mathrm{c}}{\mathrm{d}}$ i.e. $\mathrm{ad}=\mathrm{bc}$.
Now, here $a=2, d=6, b=3, c=4$
$\therefore$ ad $=2 \times 6=12$ and $b c=3 \times 4=12$
$\therefore 2,3,4$ and 6 are in proportion.
Problem-2: Find the fourth proportional from the following members. $x^{2} y, y^{2} z, z^{2} x$
Solution: Let us consider the fourth proportion is m .

$$
\begin{aligned}
& \therefore x^{2} y: y^{2} z:: z^{2} x: m \\
& m= \\
& \begin{aligned}
z^{2} x \times y^{2} z \\
x^{2} y
\end{aligned} \\
& \\
& =\frac{y z^{3}}{x}
\end{aligned}
$$

## Explanation of the solution of the problem-2:

Explanation: We know that if $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and d are in proportion, then $\mathrm{a}: \mathrm{b}:: \mathrm{c}: \mathrm{d}$ i.e. $\frac{\mathrm{a}}{\mathrm{b}}=\frac{\mathrm{c}}{\mathrm{d}}$ i.e. $\mathrm{d}=\frac{\mathrm{b} \times \mathrm{c}}{\mathrm{a}}$.
Now, here $\mathrm{a}=\mathrm{x}^{2} \mathrm{y}, \mathrm{b}=\mathrm{y}^{2} \mathrm{z}, \mathrm{c}=\mathrm{z}^{2} \mathrm{x}, \mathrm{d}=\mathrm{m}$ (considered)

$$
\begin{aligned}
& \therefore \mathrm{d}=\frac{\mathrm{b} \times \mathrm{c}}{\mathrm{a}} \\
& \begin{aligned}
\therefore \mathrm{m} & =\frac{\mathrm{z}^{2} \mathrm{x} \times \mathrm{y}^{2} \mathrm{z}}{\mathrm{x}^{2} \mathrm{y}} \\
& =\frac{\mathrm{yz}^{3}}{\mathrm{x}}
\end{aligned}
\end{aligned}
$$

Problem-3: Find the mean proportion of 4 and 9.
Solution: Let us consider the mean proportion of 4 and 9 be $m$.
Therefore, 4, m, 9 are in continued proportion.

|  | $\therefore 4: \mathrm{m}:: \mathrm{m}: 9$ |
| :--- | :--- |
| or | $4: \mathrm{m}=\mathrm{m}: 9$ |
| or, | $\frac{4}{\mathrm{~m}}=\frac{\mathrm{m}}{9}$ |
|  |  |
| or, | $\mathrm{m}^{2}=36$ |
| or | $m=\sqrt{36}$ |
| or, | $m=6$ |

$\therefore$ The mean proportion of 4 and 9 is 6 .

## Explanation of the solution of the problem-3:

Explanation: It has been seen that $\mathrm{m}^{2}=36$ implies that $\mathrm{m}=\sqrt{36}$ i.e $\mathrm{m}=6$. Actually, $\mathrm{m}^{2}=36$ implies that $\mathrm{m}= \pm \sqrt{36}$ i.e $\mathrm{m}= \pm 6$.

Since, 4 and 9 are positive numbers then value of $m$ will be positive i.e. 6 .
Mistake-1: Show that a proportion having different ratios.
Or
Show that inequality of ratio=inequality of ratio where the ratios are different.
Or
Show that ratio is changed if the terms of the ratio are multiplied by the same number.
Proof: Let us consider a ratio=a:1
Now, $\mathrm{a}: 1=\frac{\mathrm{a}}{1}=\frac{\mathrm{a} \times 0}{1 \times 0}=0: 0$

$$
\therefore \mathrm{a}: 1=0: 0
$$

Therefore, a proportion having different ratios.

## Explanation of the proof of the mistake-1:

## Explanation:

Here, both the antecedent and consequent of the ratio multiplied by the same number 0 . But, the ratio is only changed after the antecedent and consequent of a ratio is multiplied by zero only. Actually, ratio is not changed only when the antecedent and consequent of the ratio multiplied by the same number other than zero. Therefore, a proportion does not provide different ratio.

Mistake-2: Show that $\frac{\mathrm{a}}{0}=$ defined.

Proof: Here, $\frac{\mathrm{a}}{0}=\mathrm{a}: 0=$ The ratio of a and 0 .
Now, we consider Ram and Rahim have a rupees and 0 rupees in their pockets respectively.
Therefore, the ratio between the rupees of Ram and Rahim is a: 0 .

## Explanation of the proof of the mistake-2:

## Explanation:

The ratio between two numbers is undefined when the consequent is zero.
So, here the ratio between the rupees of Ram and Rahim is undefined.
Hence, $\frac{\mathrm{a}}{0}=$ undefined.
Mistake-3: Solution when a, b, c and d are proportional, but,
i) a, c, b and d are not in proportion.
ii) c, a, d and b are not in proportion.
iii) b, a, d and c are not in proportion.

Proof: Since, a, b, c and d are in proportion.
Then, $\quad a: b: c: d$

$$
\therefore \frac{\mathrm{a}}{\mathrm{~b}}=\frac{\mathrm{c}}{\mathrm{~d}}
$$

i) Now, let us consider $\mathrm{a}=0$ and $\mathrm{c}=0$.

Since, $\quad \frac{a}{b}=\frac{c}{d}$
Or, $\quad \frac{a}{c}=\frac{b}{d}$
Or, $\quad \frac{0}{0}=\frac{b}{d}$

$$
\begin{aligned}
& \therefore \text { meaningless }=\frac{\mathrm{b}}{\mathrm{~d}} \\
& \therefore \frac{\mathrm{a}}{\mathrm{c}}=\frac{\mathrm{b}}{\mathrm{~d}}=\text { meaningless }
\end{aligned}
$$

Therefore, $a, ~ c, b$ and $d$ are not in proportion.
ii) Now, let us consider $\mathrm{a}=0$ and $\mathrm{c}=0$.

Since, $\quad \frac{a}{b}=\frac{c}{d}$
Or, $\quad \frac{d}{b}=\frac{c}{a}$
Or, $\quad \frac{c}{a}=\frac{d}{b}$
Or, $\quad \frac{0}{0}=\frac{d}{b}$
$\therefore$ meaningless $=\frac{\mathrm{d}}{\mathrm{b}}$
$\therefore \frac{\mathrm{c}}{\mathrm{a}}=\frac{\mathrm{d}}{\mathrm{b}}=$ meaningless
Therefore, c, a, d and b are not in proportion.
iii) Now, let us consider $\mathrm{a}=0$ and $\mathrm{b}=0$.

Since, $\quad \frac{a}{b}=\frac{c}{d}$
Or, $\quad \frac{b}{a}=\frac{d}{c}$
Or, $\quad \frac{0}{0}=\frac{d}{c}$
$\therefore$ meaningless $=\frac{\mathrm{d}}{\mathrm{c}}$
$\therefore \frac{\mathrm{b}}{\mathrm{a}}=\frac{\mathrm{d}}{\mathrm{c}}=$ meaningless
Therefore, b, a, d and c are not in proportion.

## Explanation of the proof of the mistake-3:

Explanation: Four real numbers a, b, c and d are in proportion provided $b \neq 0, \mathrm{~d} \neq 0$, then we can write a : b: : c : d
So, $\mathrm{a}, \mathrm{c}, \mathrm{b}$ and d are in proportion provided $\mathrm{c} \neq 0, \mathrm{~d} \neq 0$. then we can write a : c : : b : d.
$\mathrm{c}, \mathrm{a}, \mathrm{d}$ and b are in proportion provided $\mathrm{a} \neq 0, \mathrm{~b} \neq 0$. then we can write $\mathrm{c}: \mathrm{a}:: \mathrm{d}: \mathrm{b}$.
$b, a, d$ and $c$ are in proportion provided $a \neq 0, c \neq 0$. then we can write $b: a: d: c$.
Mistake-4: Show that the mean proportion of -2 and -8 is 4 .
Proof: Let us consider the mean proportion of -2 and -8 be $m$.
Therefore, $-2, m,-8$ are in continued proportion.

$$
\therefore-2: \mathrm{m}:: \mathrm{m}:-8
$$

Or, $\quad-2: m=m:-8$
Or, $\quad \frac{-2}{m}=\frac{m}{-8}$
Or, $\quad m^{2}=16$
Or, $\quad \mathrm{m}=\sqrt{16}$
Or, $\quad m=4$
$\therefore$ The mean proportion of -2 and -8 is 4 .

## Explanation of the proof of the mistake-4:

Explanation: It has been seen that $\mathrm{m}^{2}=16$ implies that $\mathrm{m}=\sqrt{16}$ i.e $\mathrm{m}=4$. Actually, $\mathrm{m}^{2}=16$ implies that $m= \pm \sqrt{16}$ i.e $m= \pm 4$.

Since, -2 and -8 are negative numbers then value of $m$ will be negative i.e. -4 .

## 5. IMPLEMENTATION

These mistakes may be implemented in the teaching-learning process as well as in the content of Ratio and Proportion (Algebraic Treatment) of text books of mathematics.

## 6. CONCLUSION

1. There is a special strategy in this study for learning mathematics which helps the learner to understand important mathematical concept.
2. Four mistakes and their proofs/solutions and the explanation of the proofs/solutions of the mistake on 'Ratio and Proportion (Algebraic Treatment)' have been presented here.
3. This study will help the learners to understand the mistakes clearly which is generally done by them on 'Ratio and Proportion (Algebraic Treatment)'.
4. It will also help the learners having different ability levels i.e. the above average, average and below average levels for their improvement of mathematics learning.

## 7. FURTHER RESEARCH

This study may be extended as an application of 'Ratio and Proportion (Algebraic Treatment) on large number of samples at an appropriate grade level.

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