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EDGE TRIMAGIC TOTAL LABELING OF BIPARTITE GRAPHS

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ABSTRACT

An Edge trimagic total labeling of a graph $G(V, E)$ with p vertices and q edges is a bijection $f: V \cup E \rightarrow \{1,2,3 \ldots p + q\}$ such that for every edge uv in$E$, $f(u) + f(uv) + f(v)$ is either λ_1 or λ_2 or λ_3. In this paper, we prove that the graphs splitting graph of star $K_{1,n}$, $K_{2,n} \odot u_2(K_1), K_{3,n}$ are edge trimagic total labeling.

Keywords: *Function, Edge trimagic, bipartite graphs.*

1. INTRODUCTION

In this paper, we consider only finite and simple undirected graphs. The vertex and edge sets of a graph G are denoted by V (G) and E(G) respectively and we let [V (G)] = p and [E(G)] = q. A labeling of a graph G is mapping that carries a set of graph elements, usually vertices and/or edges into a set of numbers, usually integers. Many kinds of labelings have been studied and an excellent survey of graph labeling can be found in [1]. In 2013 C. Jayasekaran, M. Ragees and C. Devaraj [2] introduced the edge trimagic total labeling of graphs and also C.Jayasekaran and M.Ragees proved edge trimagic and super edge trimagic total labeling [3], [5], [6]. N. Sangeetha and R. SenthilAmutha also proved that Edge trimagic and Super edge trimagic total labeling [8]. An edge trimagic total labeling of a (p, q) graph G is a bijection f : V(G) ∪ E(G) ⟶{1,2,3,…,p+q} such that for each edge uv ϵ E, f(u)+f(uv)+f(v) is equal to any of the distinct constants k_1 or k_2 or k_3. A graph G is said to be edge trimagic total if it admits an edge trimagic total labeling. An edge trimagic total labeling is called super edge trimagic total labeling if G has additional property that the vertices are labeled with the smallest positive integers. S. K. Vaidya and N.H shah proved that splitting graph of star $K_{1,n}$ is graceful and odd graceful labeling [9].A. H. Rokad and G. V. Ghodasara proved that the graph $K_{2,n} \odot u_2(K_1)$ is a Fibonacci cordial labeling [7]. K. K. Kanani and M. I. Bosmia proved that $K_{3,n}$ is a cube divisor cordial labeling [4]. In this paper, we prove that the graphs splitting graph of star $K_{1,n}$,$K_{2,n} \odot u_2(K_1), K_{3,n}$ are edge trimagic total labeling.

Definition1.1: An edge trimagic total labeling of a (p, q) graph G is a bijection f: V ∪ E⟶ {1,2,3,…,p+q} such that for each edge xy∈ E(G), the value of (f(x) + f(xy) + f(y)) is equal to any of the distinct costants $k_1 or\ k_2\ or\ k_3$.A graph G is said to be an edge trimagic total if it admits an edge trimagic total labeling.An edge trimagic total labeling of a graph is called super edge trimagic if $f(v)$={1,2,…,p}. An edge trimagic total labeling of graph is called a superior edge trimagic total labeling if $f(E)$ = {1, 2,…q}.

Definition 1.2: Splitting graph is obtained by adding to each vertex v a new vertex v'so that v' is adjacent to every vertex that is adjacent to v in G.

Definition 1.3: Let (V_1, V_2) be the bipartition of $K_{m,n}$. Where $V_1 = \{u_1, u_2, \ldots, u_m\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$. The graph $Km, n \odot ui(K1)$ is defined by attaching a pendant vertex to the vertex u_i for some i.

Definition 1.4: A graph G is called a bipartite graph if V can be partitioned into two disjoint subsets V_1 and V_2 such that every line of G joins a point of V_1 to a point of V_2. (V_1, V_2) is called a bipartition of G. If further G contains every line joining the points of V_1 to the points of V_2 then G is called a complete bipartite. If V_1 contains m points and V_2 contains n points then the complete bipartite graph G is denoted by $K_{m,n}$.

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2. EDGE TRIMAGIC TOTAL LABELING OF BIPARTITE GRAPHS

Theorem 2.1: The Splitting graph of star $K_{1,n}$ has edge trimagic total labeling.

Proof: Let $v_1, v_2, \dots v_n$ be the vertices of star $K_{1,n}$ with v be the apex vertex. The order of G is $p = 2n + 2$ and size is $q = 3n$.

Let G be the splitting graph of $K_{1,n}$ and $v', v_1', v_2', \dots, v_n'$ be the newly added vertices in $K_{1,n}$ to form G.

Let $E(G) = \{vv_i'/1 \le i \le n\} \cup \{vv_i/1 \le i \le n\} \cup \{v_i v'/1 \le i \le n\}$

Define the function $f: V(G) \cup E(G) \to \{1, 2, 3, \dots 5n + 2\}$ by

$f(v') = 2$
$f(v) = 1$
$f(v_i') = i + 2 \; for \; 1 \le i \le n$
$f(v_i) = 4n - i + 3 \; for \; 1 \le i \le n$
$f(vv_i') = 2n - i + 3 \; for \; 1 \le i \le n$
$f(vv_i) = 2n + i + 2 \; for \; 1 \le i \le n$
$f(v_i v') = 4n + i + 2 \; for \; 1 \le i \le n$

Now we prove that the splitting graph $K_{1,n}$ admits an edge trimagic total labeling.

For the edges vv_i', $1 \le i \le n$

$f(v) + f(vv_i') + f(v_i') = 1 + 2n - i + 3 + i + 2 = 2n + 6 = \lambda_1$

For the edges vv_i, $1 \le i \le n$

$f(v) + f(vv_i) + f(v_i) = 1 + 2n + i + 2 + 4n - i + 3 = 6n + 6 = \lambda_2$

For the edges $v_i v'$, $1 \le i \le n$

$f(v_i) + f(v_i v') + f(v') = 2 + 4n - i + 3 + 4n + i + 2 = 8n + 7 = \lambda_3$

Hence for each edge $uv \; E, f(u) + f(uv) + f(v)$ admits any one of the trimagic constants

$\lambda_1 = 2n + 6, \lambda_2 = 6n + 6, \lambda_3 = 8n + 7.$

Hence the splitting graph $K_{1,n}$ admits edge trimagic total labeling.

Example 2.2: The splitting graph $K_{1,n}$ given in figure is edge trimagic total labeling.

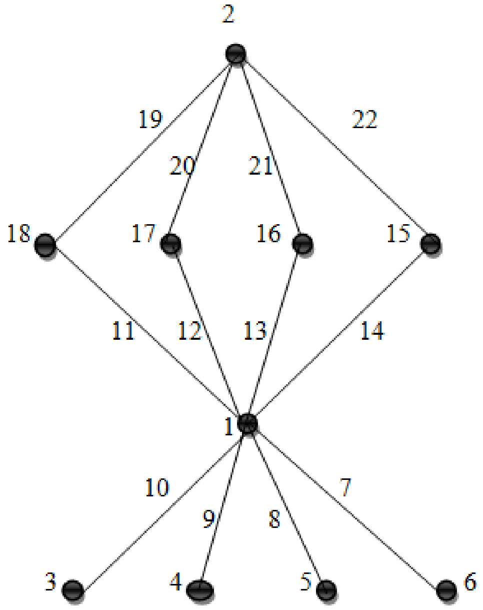

Figure-2.1: Splitting graph $K_{1,n}$ with λ_1= 14, λ_2 = 30, λ_3=39.

Theorem 2.3: The graph $K_{2,n} \odot u_2(K_1)$ has edge trimagic total labeling.

Proof: Let G $= K_{2,n} \odot u_2(K_1)$. Let $V = V_1 \cup V_2$ be the bipartition of $K_{2,n}$ such that

$V_1 = \{u_1, u_2\}$ and $V_2 = \{w_1, w_2, \ldots, w_n\}$ and pendant vertex v is adjacent to vertex u_2 in G.

$E(G) = \{u_1 w_i / 1 \le i \le n\} \cup \{u_2 w_i / 1 \le i \le n\} \cup \{u_2 v\}$. The order of G is $p = n + 3$ and size is $q = 2n + 1$.

Let us define the function $f: V \cup E \to \{1, 2, \ldots 3n + 4\}$ such that

$f(u_1) = 1;$
$f(u_2) = 2;$
$f(v) = 3n + 3;$
$f(w_i) = 3n - i + 3 \; for \; 1 \le i \le n$
$f(u_2 v) = 3n + 4;$
$f(u_1 w_i) = i + 2, for \; 1 \le i \le n$
$f(u_2 w_i) = n + i + 2 \; for \; 1 \le i \le n$

Now we prove that the graph $K_{2,n} \odot u_2(K_1)$ admits an edge trimagic total labeling.

For the edge $u_2 v$,
$f(v) + f(u_2 v) + f(u_2) = 3n + 3 + 3n + 4 + 2 = 6n + 9 = \lambda_1$

For the edges $u_1 w_i, 1 \le i \le n$
$f(u_1) + f(u_1 w_i) + f(w_i) = 1 + i + 2 + 3n - i = 3n + 6 = \lambda_2$

For the edge $u_2 w_i, 1 \le i \le n$
$f(u_2) + f(u_2 w_i) + f(w_i) = 2 + n + i + 2 + 3n - i + 3 = 4n + 7 = \lambda_3$

Hence for each edge $uv \in E, f(u) + f(uv) + f(v)$ admits any one of the trimagic constants
$\lambda_1 = 6n + 9, \lambda_2 = 3n + 6, \lambda_3 = 4n + 7.$

Hence the splitting graph $K_{2,n} \odot u_2(K_1)$ admits edge trimagic total labeling.

Example 2.4: The graph $K_{2,3} \odot u_2(K_1)$ given in figure is edge trimagic total labeling.

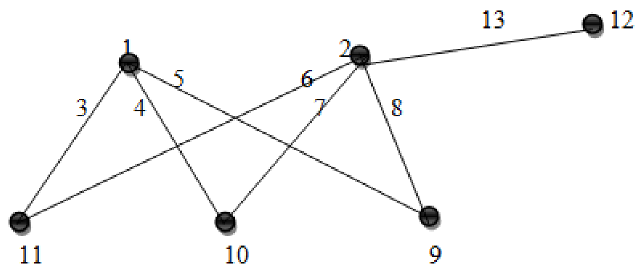

Figure-2.2: $K_{2,3} \odot u_2(K_1)$ with λ_1= 27, λ_2 = 15, λ_3=19.

Theorem 2.5: The complete bipartite graph $K_{3,n}$ has edge trimagic total labeling.

Proof: Let $K_{3,n}$be the complete bipartite graph. Let $W = U \cup V$ be the bipartition of $K_{3,n}$ such that $U = \{u_1, u_2, \ldots, u_n\}$ and $V = \{v_1 v_{2,} \ldots, v_n\}$.

Let $V(G) = \{u_1, u_2, u_3, v_i / 1 \le i \le n\}$ and $E(G) = \{u_1 v_i, u_2 v_i, u_3 v_i / 1 \le i \le n\}$. The order of $K_{3,n}$ is $p = n + 3$ and size is $q = 3n$.

Let us define the function $f: V \cup E \to \{1, 2, 3, \ldots, 4n + 3\}$ by

$f(u_1) = 1;$
$f(u_2) = 2;$
$f(u_3) = 3;$
$f(v_i) = 4n + 4 - i \;\; for \; all \; 1 \le i \le n$
$f(u_1 v_i) = i + 3 \;\; for \; all \;\; 1 \le i \le n$
$f(u_2 v_i) = n + 3 + i \; for \; all \; 1 \le i \le n$
$f(u_3 v_i) = 2n + 3 + i \;\; for \; all \; 1 \le i \le n$

Now we prove that the complete bipartite graph $K_{3,n}$ admits an edge trimagic toatal labeling.

For the edges $u_1 v_i$, $1 \le i \le n$

$$f(u_1) + f(u_1 v_i) + f(v_i) = 1 + i + 3 + 4n + 4 - i = 4n + 8 = \lambda_1$$

For the edges $u_2 v_i$, $1 \le j \le n$

$$f(u_2) + f(u_2 v_i) + f(v_i) = 2 + n + 3 + i + 4n + 4 - i = 5n + 9 = \lambda_2$$

For the edges $u_3 v_i$, $1 \le j \le n$

$$f(u_3) + f(u_3 v_i) + f(v_i) = 3 + 2n + 3 + i + 4n + 4 - i = 6n + 10 = \lambda_3$$

Hence for each edge $uv \in E, f(u) + f(uv) + f(v)$ admits any one of the trimagic constants

$$\lambda_1 = 4n + 8, \lambda_2 = 5n + 9, \lambda_3 = 6n + 10.$$

Hence the complete bipartite graph $K_{3,n}$ admits edge trimagic total labeling.

Example 2.6: The complete bipartite graph $K_{3,4}$ given in figure is edge trimagic total labeling.

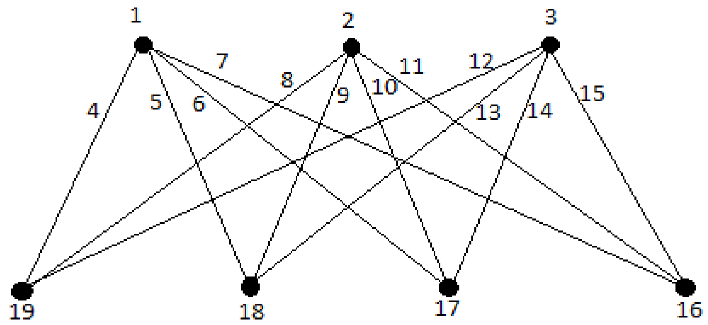

Figure-2.3: $K_{3,4}$ with λ_1= 24, λ_2 = 29, λ_3=34.

CONCLUSION

In this paper we proved that the graphs splitting graph of star $K_{1,n}$,$K_{2,n} \odot u_2(K_1), K_{3,n}$ are edge trimagic total labeling.

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