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STUDY OF EFFECT OF POROSITY IN THE PRESENCE OF APPLIED MAGNETIC FIELD A COSINE FORM CONVEX CURVED PLATE MODEL

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ABSTRACT

The study of effect of porosity in the presence of applied magnetic field is studied in case of cosine form convex curved plates. The magneto-hydrodynamic (MHD) Reynolds-type equation for convex curved plates is derived using the continuity equation and the MHD motion equations. According to the results obtained, the presence of applied magnetic field signifies an increase in the MHD pressure. Compared with the classical non-magnetic case, the magnetic-field effect characterized by the Hartmann number provides an enhancement to the MHD load-carrying capacity and the response time for the porous convex curved plates. It is observed that effect of porosity is to decrease the performance characteristics of the Cosine form convex curved plate model.

Keywords: *convex curved plates,cosine function, porous medium, Magnetic field.*

1. INTRODUCTION

In recent years, the characteristic of Magneto-hydrodynamics (MHD) in flow analysis are important for many engineering and industrial applications. The MHD bearings with conducting fluids possess the high thermal-conductivity and high electrical-conductivity features over the conventional bearings [1-9].The squeeze film lubrication for different configuration of bearings under the action of transverse magnetic field has been discussed by several authors [10-15].Magneto hydrodynamic flow has many applications in aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, accelerators, fluid droplets, MHD pumps, power generators and purification of crude oil. Flow through porous medium has numerous engineering and geophysical applications.

Self-lubricating porous bearings have been studied in the last few decades because of their industrial applications and machine manufacturing. These bearings have self-contained oil reservoir and hence do not require continuous lubrication. Most porous bearings have interconnecting pores which store the lubricating fluid. When the normal load is applied, the fluid is supplied through the interconnected pores to the fluid film region to support the load, and when the load is removed from the loaded zone of the bearing, fluid is reabsorbed by capillary action. Since these can operate without additional lubricant for longer period, porous bearings have been used widely, where re-lubrication would be difficult. Thus, porous metal bearings have been used in the manufacturing of vehicles, home appliances, machines, and so forth. Because of the importance of porous bearings, Wu [16] studied the squeeze film effects between two rectangular plates in which both plates have a porous material. Numerous papers are available in the literature for the study of different types of porous bearings for example, journal bearings [17], slider bearings [18], thrust bearings [19], and many more.

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The study of porosity in the presence of applied magnetic field considering cosine form convex curved plate model has not been studied so far. Hence, in this paper an attempt has been made to study the effect of porosity in the presence of applied magnetic field in case of Cosine form convex curved plates. Modified Reynolds equation is obtained using Darcy's law for porous media and expressions for the MHD pressure, load carrying capacity and the time height relation are obtained.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

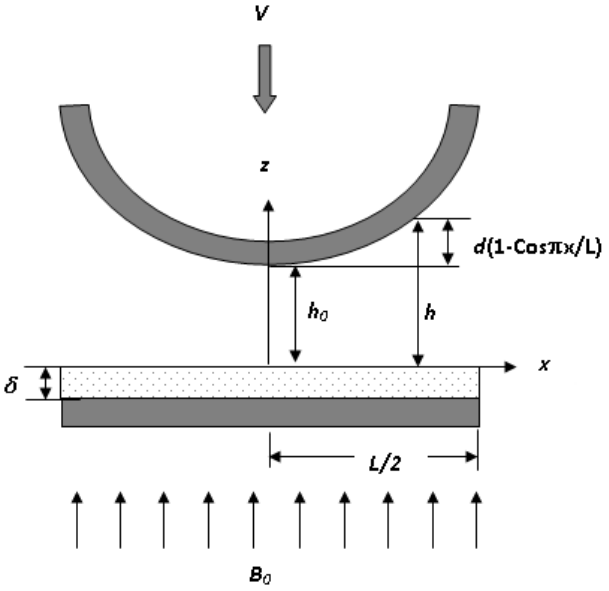

Figure-1: The physical configuration of convex curved - plates mode in the presence of transverse magnetic field.

Figure.1 shows the squeeze film geometry between the cosine form convex curved plates. The lower plate with a porous facing of thickness δ is fixed and the upper plate has a squeezing velocity $\left(\frac{dh}{dt}\right)$ is approaching the lower plate under a constant load. The film thickness h for the squeeze film can be generated by the form of a cosine function $h = h_0 + d\left\{1-\cos\left(\frac{\pi}{L}x\right)\right\}$. In the equation, L is the length of the plates, d is the amplitude of the cosine function, and h_0 is the minimum film thickness. At the central position: $x = 0$, the film thickness is equal to the minimum film thickness h_0. At the edge position: $x = \frac{L}{2}$, the film thickness is equal to the sum of the minimum film thickness and the amplitude $h_0 + d$ .A uniform transverse magnetic field B_0 is applied to the bearing in the $z-$ direction as shown in the above figure1.It is assumed that the fluid film is thin, the body forces and the body couples are negligible. Under these assumptions, the hydrodynamic lubrication theory applicable to thin films, the continuity equation and the MHD governing equations of motion in rectangular coordinate are

$$\frac{\partial^2 u}{\partial z^2} - \frac{M_0^2}{h_0^2}u = \frac{1}{\mu}\frac{\partial p}{\partial x} \tag{1}$$

$$\frac{\partial p}{\partial z} = 0 \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

Where $M = \frac{M_0}{h_0}$ and $M_0 = B_0 h_0 (\frac{\sigma}{\mu})^{1/2}$ is the Hartmann number.

The relevant boundary conditions for the velocity components are

i) At the upper surface $z = h$

$$u = 0\ (no\ Slip)\ w = V = dh/dt\ (SqueezingVelocity) \tag{4}$$

ii) At the lower surface $z = 0$

$$u = 0 \ (no \ Slip) \ w = w^* \tag{5}$$

Where w^* is the modified Darcy velocity component in the $z-$direction in the porous region. Modified form of the Darcy law for porous material is given by

$$u^* = -\frac{k}{\mu\left(1+\frac{kM_0^2}{mh_0^2}\right)}\frac{\partial p^*}{\partial x} \tag{6}$$

$$w^* = -\frac{k}{\mu}\frac{\partial p^*}{\partial z} \tag{7}$$

Where k is the permeability of porous matrix, m porosity and p^* is the pressure in the porous region.

The solution of equation (1) subject to the boundary conditions eqns. (4) and (5) is given by

$$u = \frac{1}{\mu M^2}\frac{\partial p}{\partial x}\left(\frac{2\sinh\frac{Mz}{2}\sinh\frac{M(z-h)}{2}}{\cosh\frac{Mh}{2}}\right) \tag{8}$$

Substituting above value of u in the integral form of continuity equation:

$$\frac{\partial}{\partial x}\left\{\int_0^h u dz\right\} + w_h - w_0 = 0 \tag{9}$$

Substituting equations (6) and (7) in the following continuity equation for lower porous region

$$\frac{\partial u^*}{\partial x} + \frac{\partial w^*}{\partial z} = 0 \tag{10}$$

Integrating above equation once with respect to z from $-\delta$ to 0 and using the Morgan-Cameron approximation with the condition $\partial p^* / \partial z = 0$ when $z = -\delta$ gives

$$\left(\frac{\partial p^*}{\partial z}\right)_{z=0} = -\frac{\delta}{c^2}\frac{\partial^2 p}{\partial x^2} \tag{11}$$

where $c^2 = \left(1+\frac{kM_0^2}{mh_0^2}\right)$ and δ is the thickness of the porous layer.

Substituting (8) and (11) in (9) gives

$$\frac{\partial}{\partial x}\left\{\frac{1}{\mu M^3}\frac{\partial p}{\partial x}\left(Mh - 2tanh\frac{Mh}{2}\right) + \frac{k\delta}{\mu c^2}\frac{\partial p}{\partial x}\right\} = -V \tag{12}$$

Introducing the dimensionless quantities

$$x^* = \frac{x}{L}, h^* = \frac{h}{h_0}, P^* = -\frac{ph_0^3}{\mu L^2(dh/dt)}, A = \frac{d}{h_0}, \psi = \frac{k\delta}{h_0^3}, \delta^* = \frac{\delta}{h_0}$$

Equation (12) takes the form

$$\frac{\partial}{\partial x^*}\left\{\frac{\partial P^*}{\partial x^*}F(h^*, M_0, \psi)\right\} = -1 \tag{13}$$

Where $F(h^*, M_0, \psi) = \frac{1}{M_0^3}\left(M_0 h^* - 2tanh\frac{M_0 h^*}{2}\right) + \frac{\psi}{C^2}$,

$C^2 = \left(1+\frac{\psi M_0^2}{m\delta^*}\right)$ and $h^* = 1 + A\{1-\cos(\pi x^*)\}$.

The pressure conditions are: $P^* = 0$ at $x^* = \pm\frac{1}{2}$ and $\frac{dP^*}{dx^*} = 0$ at $x^* = 0$.

Integrating (13) and using the pressure conditions, one can obtain

$$P^* = -\int_{1/2}^{x^*} \frac{x^*}{F(h^*, M_0, \psi)} dx^*$$

$$P^* = \int_{x^*}^{1/2} \frac{x^*}{F(h^*, M_0, \psi)} dx^* \tag{14}$$

Integrating the film pressure, one can obtain the load-carrying capacity

$$W = b \int_{-L/2}^{L/2} p dx \tag{15}$$

Where b denotes the width of the curved plates. Introducing a non-dimensional form gives

$$W^* = \frac{h_0^3 W}{\mu L^3 b(-dh/dt)} = \int_{-1/2}^{1/2} P^* dx^* \tag{16}$$

After performing the integration, one can obtain the non-dimensional load capacity.

$$W^* = \int_{-1/2}^{1/2} \left\{ \int_{x^*}^{1/2} \frac{x^*}{F(h^*, M_0, \psi)} dx^* \right\} dx^* \tag{17}$$

Introducing non-dimensional time as:

$$T^* = \frac{W h_0^{\,2} t}{\mu L^3 b} \tag{18}$$

The elapsed time required for the upper curved plate to approach the lower plate is given by

$$T^* = \int_{h_0^*}^{1} \left[\int_{-1/2}^{1/2} \left\{ \int_{x^*}^{1/2} \frac{x^*}{F(h^*, M_0, \psi)} dx^* \right\} dx^* \right] dh_0^* \tag{19}$$

for $T^*, h^* = h_0^* + A\{1 - \cos(\pi x^*)\}$.

Using the numerical method of integration, the film pressure (14), the load capacity (17) and the elapsed time (19) can be calculated.

3. RESULTS AND DISCUSSIONS

The effect of porosity on cosine form convex curved plates in the presence of magnetic field is observed and the squeeze film characteristics are analysed with respect to the non-dimensional parameters namely the Hartmann number M_0 and permeability parameter ψ .All required characteristic features of squeeze film bearings such as the pressure distribution, load-carrying capacity, and squeezing time have been obtained as functions of dimensionless Hartmann number M_0 and permeability parameter ψ .

3.1. Squeeze film pressure

Figure 2, 3 represents the variation of non-dimensional pressure P^* with horizontal coordinate x^* for different values of ψ and M_0 . These figures show the effect of magnetic field in terms of pressure distribution when other parameters are held fixed. It is observed that the pressure distribution increases for increasing values of M_0 compared to nonmagnetic field $(M_0 = 0)$. An increase in the values of M_0 makes lubricant to acquire more magnetization which in turn interacts with the magnetic field imposed. Also applied magnetic field normal to the flow reduces the velocity of the fluid in the film region which retains substantial amount of fluid. This fluid generates the pressure distribution.

3.2. Load carrying capacity

Figure 4, 5 shows the variation of non-dimensional W^* with amplitude ratio A for different values of ψ and M_0. It is clearly observed from this figure that the load capacity increases for increasing magnetic number compared to $M_0 = 0$. For increasing values of M_0, pressure distribution increases in the fluid region, hence, load capacity also increases. The effects are more distinct in convex curved plates in presence of transverse magnetic field. The increase in W^* is more evident for larger values of M_0.

3.3. Non-dimensional Squeeze film time

Figure 6, 7, 8 represents Variation of non-dimensional squeeze film time T^* with h_0^* for different values of M_0 and ψ. The response times of convex curved porous squeeze films are significantly extended in the presence of external magnetic field. As magnetic parameter increases, the squeeze film time also increases. The applied magnetic field strongly opposes the fluid flow in the film region which retains large number of fluid in the film region. Thus, there is significant increase in squeezing time.

Table.1-Illustrates the effect of transverse magnetic field on the squeeze film characteristics, and is evaluated by the relative percentage difference. The relative variation of load capacity and the approaching time of porous convex curved squeeze films is tabulated, and it is observed that the performance characteristics are improved for higher values of magnetic parameter M_0. The increase in the non-dimensional load carrying capacity R_{W^*} and non-dimensional squeeze film time R_{T^*} are defined by $R_{W^*} = \left\{\left(W^*_{magnetic} - W^*_{non-magnetic}\right)\Big/ W^*_{non-magnetic}\right\} \times 100$

And $R_{T^*} = \left\{\left(T^*_{magnetic} - T^*_{non-magnetic}\right)\Big/ T^*_{non-magnetic}\right\} \times 100$

The values of R_{W^*} and R_{T^*} are listed in Table. 1 for various values of M_0 and ψ.

4. CONCLUSION

The study of effect of porosity on the performance characteristics of cosine form convex curved plates in the presence of magnetic field is studied. The squeeze film characteristics for porous convex curved plates are improved in presence of external magnetic fields i.e, the load carrying capacity and squeeze film time increases for increasing values of magnetic parameter M_0. It is observed that the pressure distribution, load-carrying capacity, and squeeze time increases for increasing values of Hartmann number, It is observed that effect of porosity on the permeability of cosine form convex curved is found to decrease these effects.

Nomenclature:

B_0	applied magnetic field in the $z-$direction
L	length of the plates
d	amplitude of the cosine function
b	width of the curved plates
A	amplitude ratio of the cosine-form convex curved plates
h_0	minimum film thickness
h_0^*	dimensionless film thickness after time Δt
k	permeability of the porous matrix
p	hydrodynamic film pressure
p^*	pressure in the porous region
M_0	Magnetic Parameter $\left(= B_0 h_0 (\frac{\sigma}{\mu})^{1/2}\right)$
u, v	Velocity components in film region
u^*, w^*	Velocity components in porous region

x, z local Cartesian co-ordinates

m porosity

P^* dimensionless pressure $\left(= \frac{ph_0^3}{\mu L^2(-dh/dt)} \right)$

t mean time of approach

T^* Dimensionless time of approach $\left(= \frac{h_0^2 Wt}{\mu L^3 b} \right)$

V Squeezing Velocity $\left(-\frac{dh}{dt} \right)$

W^* Dimensionless load carrying capacity $\left(= \frac{h_0^3 W}{\mu L^3 b(-dh/dt)} \right)$

Greek symbols

σ Conductivity of fluid

ψ nondimensional permeability parameter

δ thickness of the porous layer

δ^* dimensionless thickness of the porous layer

μ lubricant viscosity

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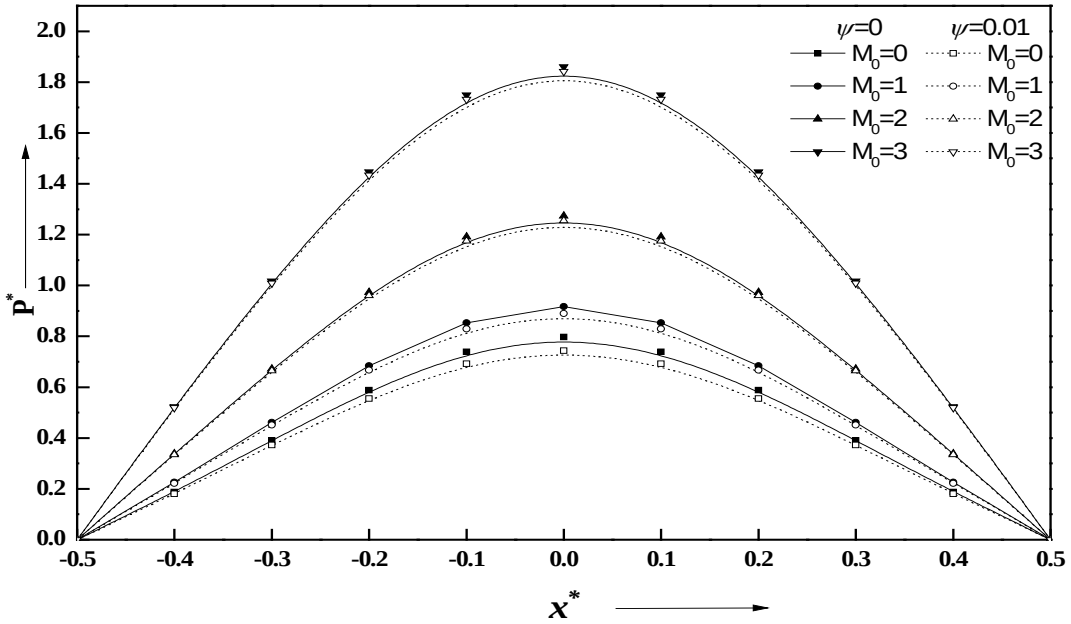

Fig.2 Variation of non-dimensional P^* with x^* for different values of ψ and M_0 with δ=0.01

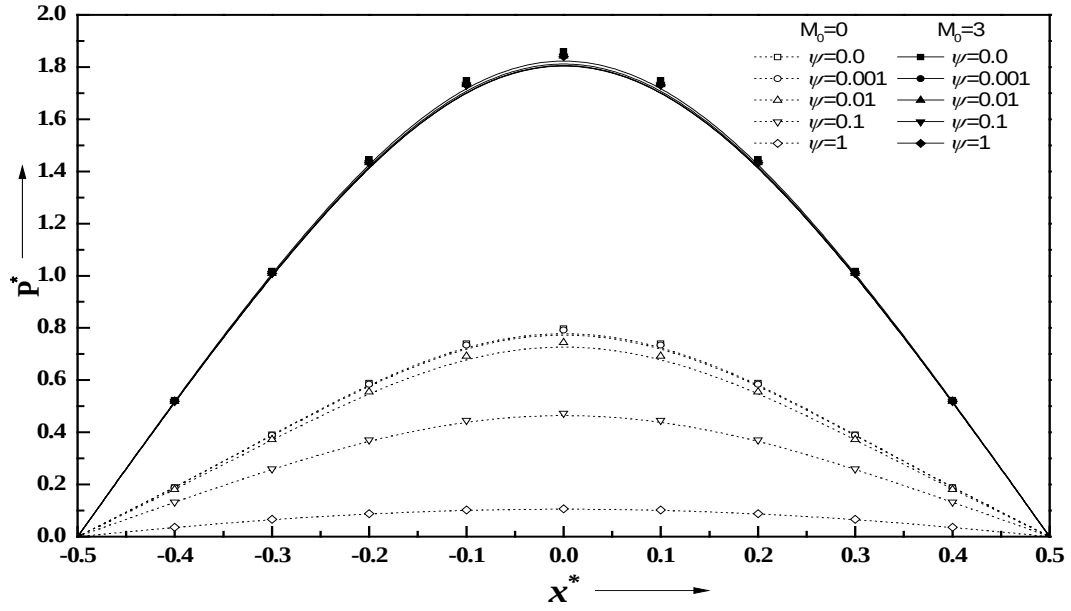

Fig.3 Variation of non-dimensional P^* with x^* for different values of ψ and M_0 with δ=0.01

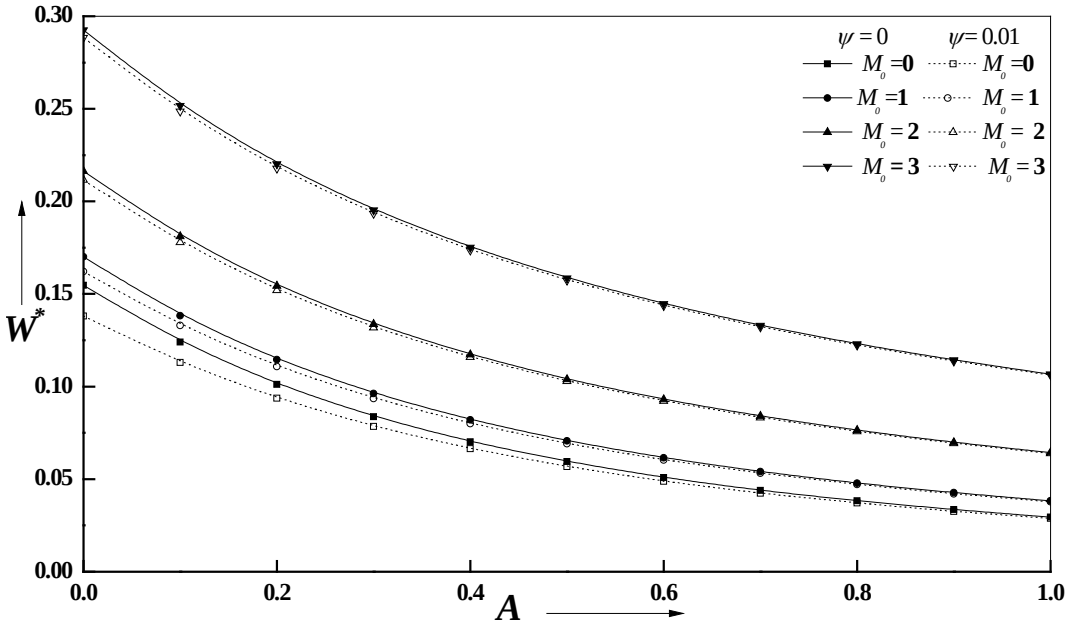

Fig. 4 Variation of non-dimensional W^* with amplitude ratio A for different values of ψ and M_0.

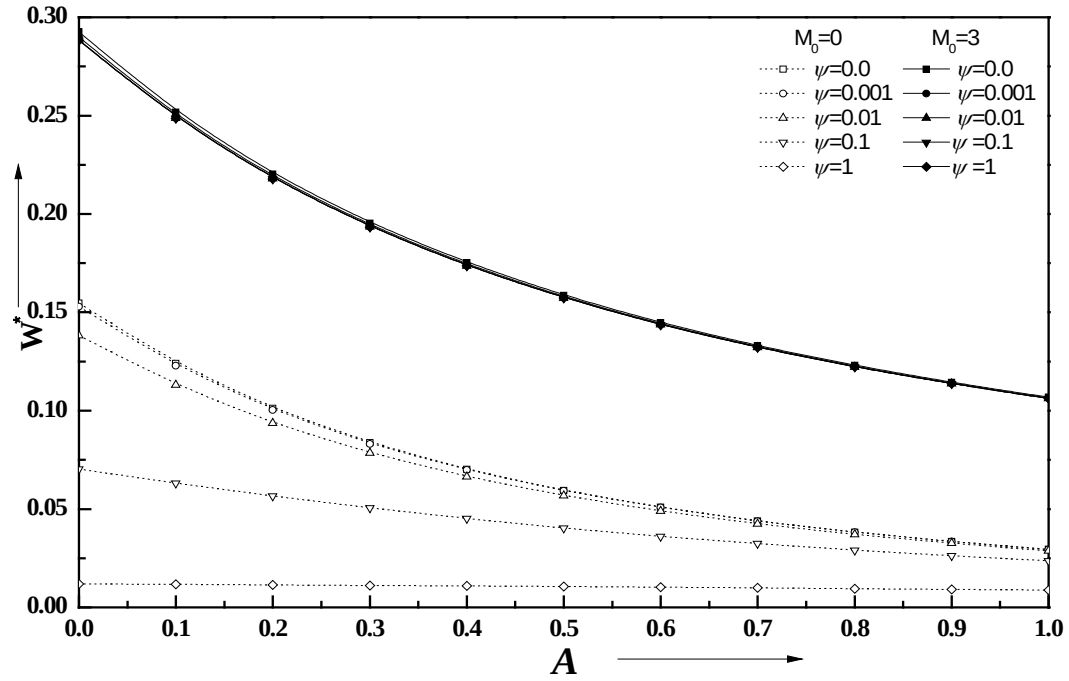

Fig.5 Variation of non-dimensional W^* with A for different values of ψ and M_0 with δ=0.01

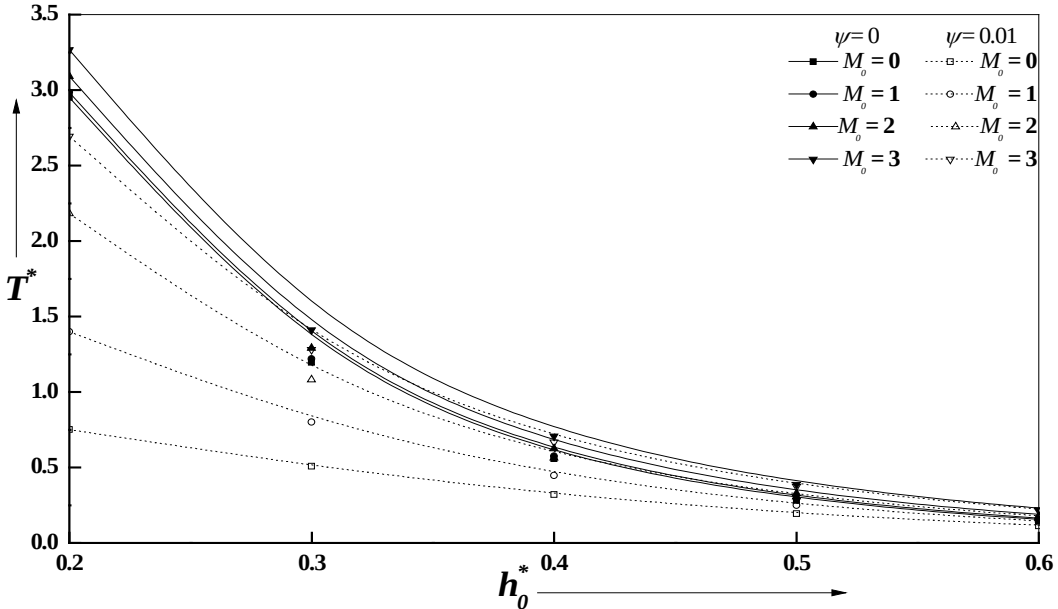

Fig. 6 Variation of non-dimensional T with h_0^* for different values of ψ and M_0 at A=0.2.

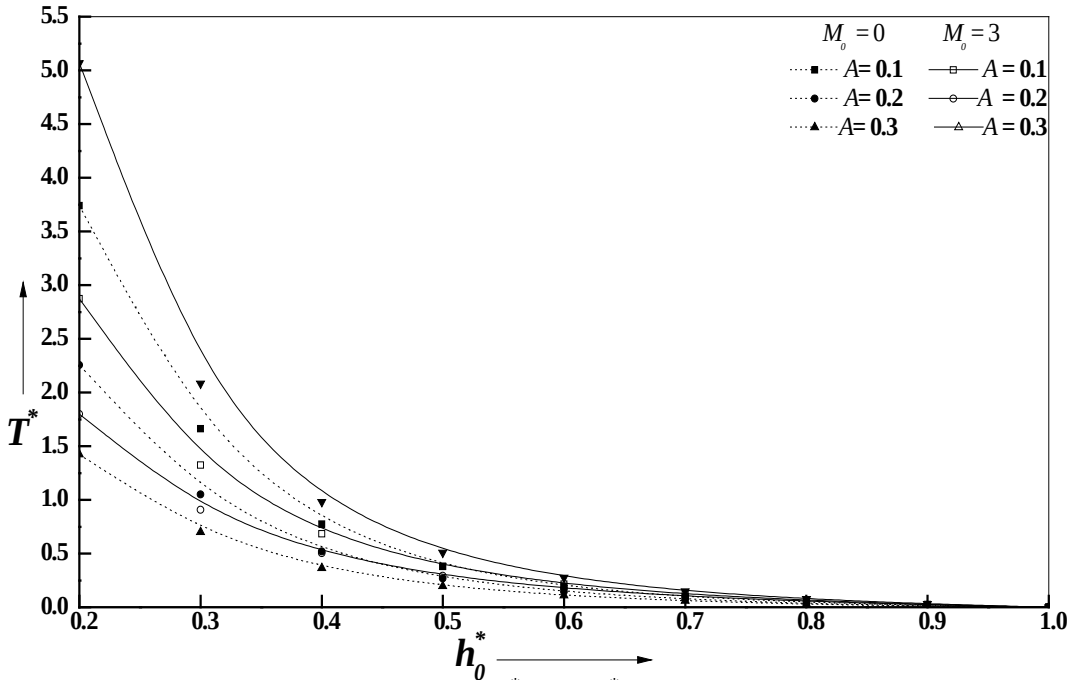

Fig. 7 Variation of non-dimensional T^* with h_0^* for different values of A and M_0 at ψ=0.001.

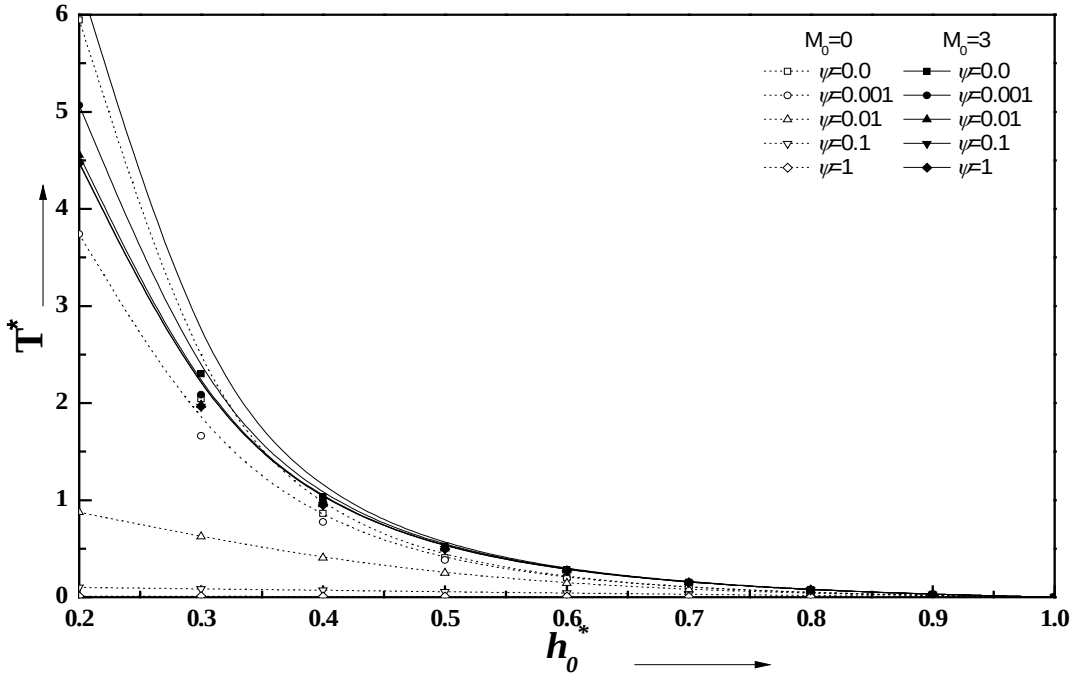

Fig.8 Variation of non-dimensional T^* with h_0^* for different values of ψ and M_0 with $A=0.1$ *and* $\delta=0.01$

Table-1: Variation of R_{W^*} and R_{T^*} for different values of $M_0, \psi, \delta = 0.01$ and $m = 0.6$

		R_{W^*}	R_{T^*}
ψ =0.0	M_0 =1	18.7857	3.3169
	M_0 =2	74.7225	13.248
	M_0 =3	166.6498	29.753
ψ =0.01	M_0 =1	21.896	35.344
	M_0 =2	81.144	68.878
	M_0 =3	177.246	101.258
ψ =1	M_0 =1	548.387	4644.48
	M_0 =2	873.719	6423.34
	M_0 =3	1391.84	7747

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