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On αg*p-Connectedness and αg*p-Compactness in Topological Spaces

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ABSTRACT

*In this paper, we introduce the concept of αg*p-connectedness and αg*p-compactness in topological spaces. We investigate and study their basic properties. We also discuss their relationship with already existing concepts.*

***Keywords**: αg*-connected space, gp*-connected space, g*p-connected space, αg*p-connected space, αg*p-compact space.*

***Mathematics Subject Classification:** 54 D 05, 54 D 30.*

1. INTRODUCTION

The notion of connectedness and compactness are useful and fundamental notions of not only general topology but also of other advanced branches of mathematics in topological spaces. In 1974, Das defined the concept of semi-connectedness in topology and investigated its properties. Compactness is one of the most important, useful and fundamental concepts in topology. In 1981, Dorsett introduced and studied the concept of semi-compact spaces. In 2011, P.G.Patil, T.D.Rayanagoudar and Mahesh K.Bhat defined the concept of g*p-compactness and g*p-connectedness in topological spaces. Recently, S.Sekar and P.Jayakumar introduced and studied the concept of gp*-compact and gp*-connected spaces.

The authors[12] introduced αg*p-closed sets and αg*p-open sets in topological spaces and established their relationships with some generalized sets in topological spaces. The aim of this paper is to introduce the concept of αg*p-connected and αg*p-compactness in topological spaces. We investigate their basic properties. We also discuss their relationship with already existing concepts.

Throughout this paper (X,τ) and (Y,σ) represents topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The union of all αg*p-open sets of X contained in A is called αg*p-interior of A and it is denoted by αg*p-int (A).The intersection of all αg*p-closed sets of X containing A is called αg*p-closure of A and it is denoted by αg*p-cl(A).

2. PRELIMINARIES

We recall the following definitions which are useful in the sequel.

Definition 2.1: A subset A of a topological space (X, τ) is called
(i) preopen [7] if $A \subseteq int(cl(A))$ and preclosed if $cl(int(A)) \subseteq A$.
(ii) α-open [8] if $A \subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A))) \subseteq A$.

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Definition 2.2: A subset A of a topological space (X, τ) is called

(i) generalized closed (briefly, g-closed) [4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(ii) α-generalized closed(briefly, αg-closed) [5] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(iii) generalized preclosed(briefly, gp-closed) [6] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(iv) generalized star preclosed (briefly, g*p-closed set) [14] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.
(v) generalized pre star closed (briefly gp*-closed set) [3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gp-open in X.
(vi) α-generalized star closed (briefly , αg^*-closed set) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in X.
(vii) alpha generalized star preclosed(briefly, αg^*p-closed) [12] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in X.

Definition 2.3: A subset A of a topological space (X, τ) is called

(i) αg^*p-continuous[10] if $f^{-1}(V)$ is αg^*p-closed set in (X, τ) for every closed set V in (Y, σ).
(ii) αg^*p-irresolute[10] if $f^{-1}(V)$ is αg^*p-closed set in (X, τ) for every αg^*p-closed set V in (Y, σ).
(iii) contra αg^*p-continuous[11] if $f^{-1}(V)$ is αg^*p-closed set in X for every open set V in Y.

Definition 2.4 [15]: A topological space X is said to be connected if X cannot be written as the disjoint union of two non empty open sets in X.

Definition 2.5: A topological space X is said to be αg^*-connected if X cannot be written as the disjoint union of two non empty αg^*-open sets in X.

Definition 2.6 [9][13]: A topological space X is said to be g*p-connected(resp. gp*-connected) if X cannot be written as the disjoint union of two non empty g*p-open(resp. gp*-open) sets in X.

Definition 2.7: [10] A space (X,τ) is called if αg^*p-space if every αg^*p-closed set is closed.

Lemma 2.8 [12]:

(i) Every closed set is αg^*p-closed.
(ii) Every αg^*-closed set is αg^*p-closed.
(iii) Every gp*-closed set is αg^*p-closed.
(iv) Every αg^*p-closed set is g*p-closed.

Lemma 2.9 [10]: Let $f: (X,\tau) \to (Y,\sigma)$ be a function. Then the following are equivalent.

(i) f is αg^*p-continuous.
(ii) The inverse image of each closed set in Y is αg^*p-closed in X.
(iii) The inverse image of each open set in Y is αg^*p-open in X.

Lemma 2.10 [10]: A function $f: X \to Y$ is αg^*p-irresolute if and only if the inverse image of every αg^*p-open set in Y is αg^*p-open in X

3. αg^*p -CONNECTED SPACES

In this section we introduce αg^*p-connected spaces and investigate their basic properties.

Definition 3.1: A topological space X is said to be αg^*p-connected if X cannot be written as the disjoint union of two non empty αg^*p-open sets in X.

Example 3.2: Let X = {a, b, c, d} be given the topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$.
αg^*p-O(X) = $\{\phi, \{a\},\{b\},\{a, b\},\{a, b, c\},\{a, b, d\}, X\}$. Then the space X is αg^*p-connected.

Definition 3.3: A subset S of a topological space X is said to be αg^*p-connected relative to X if S cannot be written as the disjoint union of two non empty αg^*p-open sets in X.

Theorem 3.4: For a topological space X, the following are equivalent.

(i) X is αg^*p-connected
(ii) The only subsets of X which are both αg^*p-open and αg^*p-closed are the empty set and X.
(iii) Each αg^*p-continuous function of X into a discrete space Y with at least two points is a constant map.

Proof: Suppose X is αg*p-connected. Let S be a proper subset which is both αg*p-open and αg*p-closed in X. Then its complement X \ S is also αg*p-open and αg*p-closed. Then X = S ∪ (X \ S), a disjoint union of two non empty αg*p-open sets which contradicts (i). Therefore S = ϕ or X. This proves (i) ⇒ (ii).

Suppose (ii) holds. Let X = A ∪ B where A and B are disjoint non empty αg*p-open subsets of X. Since A = X \ B and B = X \ A, A and B are both αg*p-open and αg*p-closed. By assumption, A = ϕ or X which is a contradiction. Therefore X is αg*p-connected. This proves (ii) ⇒ (i).

Now to prove (ii) ⇒ (iii). Suppose (ii) holds. Let f : X → Y be an αg*p-continuous function where Y is a discrete space with atleast two points. Then $f^{-1}(\{y\})$ is αg*p-closed and αg*p-open for each y∈Y. Since (ii) holds, $f^{-1}(\{y\}) = \phi$ or X. If $f^{-1}(\{y\}) = \phi$ for all y∈Y, f will not be a function. That implies $f^{-1}(\{y\}) = X$ for some y∈Y. Therefore for the fixed y, f(x) = y for all x ∈X. This proves that f is a constant map. This proves (ii) ⇒ (iii).

Now suppose (iii) holds. Let S be both αg*p-open and αg*p-closed in X. Suppose S ≠ ϕ. Let f: X → Y be an αg*p-continuous function defined by f(S) ={y} and f(X \ S) = {w} for some distinct points y and w in Y. By (iii) f is a constant function. Therefore S = X. Hence (ii) holds. This proves (iii) ⇒(ii).

Theorem 3.5: Let f: X → Y be a function.

(i) If X is αg*p-connected and if f is αg*p-continuous, surjective then Y is connected.
(ii) If X is αg*p-connected and if f is αg*p-irresolute, surjective then Y is αg*p-connected.

Proof:

(i) Let X be αg*p-connected and f be αg*p-continuous, surjective. Suppose Y is disconnected. Then Y = A∪ B, where A and B are disjoint non empty open subsets of Y. Since f is αg*p-continuous surjective, by using lemma 2.9, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$, $f^{-1}(B)$ are disjoint non empty αg*p-open subsets of X. This contradicts the fact that X is αg*p-connected. Therefore Y is connected. This proves (i).
(ii) Let X be αg*p-connected and f be αg*p-irresolute, surjective. Suppose Y is not αg*p-connected. Then Y = A∪B where A and B are disjoint non empty αg*p-open subsets of Y. Since f is αg*p-irresolute surjective, by using lemma 2.10, $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$, $f^{-1}(B)$ are disjoint non empty αg*p-open subsets of X. This implies X is not αg*p-connected, a contradiction. Therefore Y is αg*p-connected.

Theorem 3.6: Every αg*p-connected space is connected.

Proof: Let X be an αg*p-connected space. Suppose X is not connected. Then there exists a proper non empty subset B of X which is both open and closed in X. Since every closed set is αg*p-closed, B is a proper non empty subset of X which is both αg*p-open and αg*p-closed in X. Then by using Theorem 3.4, X is not αg*p-connected. This proves the theorem.

The converse of Theorem 3.6 is not true as shown in the following example.

Example 3.7: Let X = {a, b, c, d} be given the topology τ = {ϕ , {a}, {b, c}, {a, b, c}, X}.
αg*p-O(X) = {ϕ ,{a},{b},{c},{a, b},{a, c},{b, c},{a, b, c},{a, b, d},{a, c, d}, X}.
The space X is connected but not αg*p-connected.

Lemma 3.8: For a topological space X, the following are equivalent.

(i) X is αg*(resp. gp*, g*p)-connected
(ii) The only subsets of X which are both αg*(resp. gp*, g*p)-open and αg*(resp. gp*, g*p)-closed are the empty set and X.

Theorem 3.9: Every αg*p-connected space is αg*-connected.

Proof: Let X be an αg*p -connected space. Suppose X is not αg*-connected. Then by using Lemma 3.8, there exists a proper non empty subset B of X which is both αg*-open and αg*-closed in X. Since every αg*-closed (open) is αg*p-closed (open) then X is not αg*p-connected. This proves the theorem.

The converse of Theorem 3.9 is not true as shown in the following example.

Example 3.10: Let X= {a, b, c, d} be given the topology τ = {ϕ , {c, d}, X}.
αg*-O(X) = {ϕ ,{c},{d},{c, d},{ a, c, d},{b, c, d}, X}.
αg*p -O(X) = {ϕ ,{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}, X}.

The space X is αg^*-connected but not αg^*p-connected.

Theorem 3.11: Every αg^*p-connected space is gp^*-connected.

Proof: Let X be an αg^*p-connected space. Suppose X is not gp^*-connected. Then by using Lemma 3.8, there exists a proper non empty subset B of X which is both gp^*-open and gp^*-closed in X. Since every gp^*-closed (open) is αg^*p -closed (open) then X is not αg^*p-connected. This proves the theorem.

The converse of Theorem 3.11 is not true as shown in the following example.

Example 3.12: Let X = {a, b, c, d} be given the topology $\tau = \{\phi, \{a, b\}, X\}$.
$gp^*\text{-}O(X) = \{\phi, \{a, b\}, X\}$.
$\alpha g^*p\text{-}O(X) = \{\phi, \{a\},\{b\},\{a, b\},\{a, c\},\{a, d\},\{b, c\},\{b, d\},\{a, b, c\},\{a, b, d\},\{a, c, d\},\{b, c, d\}, X\}$.

The space X is gp^*-connected but not αg^*p-connected.

Theorem 3.13: Every g^*p-connected space is αg^*p-connected.

Proof: Let X be an g^*p-connected space. Suppose X is not αg^*p-connected. Then by using Lemma 3.8, there exists a proper non empty subset B of X which is both αg^*p-open and αg^*p-closed in X. Since every αg^*p-closed (open) is g^*p-closed (open) then X is not g^*p-connected. This proves the theorem.

The converse of Theorem 3.13 is not true as shown in the following example.

Example 3.14: Let X = {a, b, c, d} be given the topology $\tau = \{\phi, \{a\},\{b\},\{a, b\},\{a, b, c\}, X\}$.
$g^*p\text{-}O(X) = \{\phi, \{a\},\{b\},\{c\},\{a, b\},\{a, c\},\{b, c\},\{a, b, c\},\{a, b, d\}, X\}$.
$\alpha g^*p\text{-}O(X) = \{\phi, \{a\},\{b\},\{a, b\},\{a, b, c\},\{a, b, d\}, X\}$. The space X is αg^*p-connected but not g^*p-connected but not αg^*p-connected.

Remark 3.15: If A is αg^*p-closed in (X, τ), then A is closed in $(X, \tau_{\alpha g^*p})$ provided $\tau_{\alpha g^*p}$ is a topology.

Theorem 3.16: Suppose X is a topological space with $\tau_{\alpha g^*p} = \tau$. Then X is connected if and only if X is αg^*p-connected.

Proof: Suppose X is not αg^*p-connected. Then there exists a proper non empty subset B of X which is both αg^*p-open and αg^*p-closed in X. Since $\tau_{\alpha g^*p} = \tau$, every αg^*p-closed set is closed. Therefore B is both open and closed in X that implies X is not connected. This proves that connectedness implies αg^*p-connectedness.

Theorem 3.17: Suppose X is an αg^*p-space. Then X is αg^*p-connected if and only if X is αg^*-connected.

Proof: Suppose X is αg^*p-connected. Then by using Theorem 3.9, X is αg^*-connected.

Conversely we assume that X is αg^*-connected. Suppose X is not αg^*p-connected. Then there exists a proper non empty subset B of X which is both αg^*p-open and αg^*p-closed in X. Since X is αg^*p-space, B is both open and closed in X. Again since every closed set is αg^*-closed, B is both αg^*-open and αg^*-closed in X which shows that X is not αg^*-connected, a contradiction. Therefore X is αg^*p-connected.

Theorem 3.18: A contra αg^*p-continuous image of an αg^*p-connected space is connected.

Proof: Let $f: (X, \tau) \to (Y, \sigma)$ be a contra αg^*p-continuous function from an αg^*p-connected space X on to a space Y. Assume that Y is disconnected. Then $Y = A \cup B$ where A and B are non empty clopen sets in Y with $A \cap B = \phi$. Since f is contra αg^*p-continuous, we have that $f^{-1}(A)$ and $f^{-1}(B)$ are non empty αg^*p-open sets in X with $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A \cup B) = f^{-1}(Y) = X$ and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\phi) = \phi$. This means that X is not αg^*p-connected, which is a contradiction. This proves the theorem.

Definition 3.19: Let X be a topological space. Two non-empty subsets A and B of X are called αg^*p-separated iff $\alpha g^*p\text{-}cl(A) \cap B = A \cap \alpha g^*p\text{-}cl(B) = \phi$.

Theorem 3.20: Let X be a topological space, then the following statements are equivalent :

(i) X is a αg^*p-connected space.
(ii) X is not the union of any two αg^*p-separated sets.

Proof: ($\Rightarrow$) Let A and B be a two αg*p-separated sets such that $X = A \cup B$. Then $\alpha g^*p\text{-cl}(A) \cap B = A \cap \alpha g^*p\text{-cl}(B) = \phi$. Since $A \subseteq \alpha g^*p\text{-cl}(A)$ and $B \subseteq \alpha g^*p\text{-cl}(B)$, then $A \cap B = \phi$. Now $\alpha g^*p\text{-cl}(A) \subseteq X \setminus B = A$. Hence $A = \alpha g^*p\text{-cl}(A)$. Then A is αg*p-closed set. By the same way we can show that B is αg*p-closed set which is a contradiction. Hence X cannot be written as the union of two αg*p-separated sets.

($\Leftarrow$) Let A and B be a two disjoint non-empty and αg*p-closed sets such that $X = A \cup B$. Then $\alpha g^*p\text{-cl}(A) \cap B = A \cap \alpha g^*p\text{-cl}(B) = A \cap B = \phi$ which is a contradicts with the hypothesis. Therefore X is a αg*p-connected space.

Theorem 3.21: Let A be a αg*p-connected set and H , K be an αg*p-separated sets. If $A \subseteq H \cup K$ then either $A \subseteq H$ or $A \subseteq K$.

Proof: Suppose A is αg*p-connected set and H, K are αg*p-separated sets such that $A \subseteq H \cup K$. Let $A \not\subset H$ and $A \not\subset K$. Suppose $A_1 = H \cap A \neq \phi$ and $A_2 = K \cap A \neq \phi$. Then $A = A_1 \cup A_2$. Since $A_1 \subseteq H$, $\alpha g^*p\text{-cl}(A_1) \subseteq \alpha g^*p\text{-cl}(H)$. Since $\alpha g^*p\text{-cl}(H) \cap K = \phi$, $\alpha g^*p\text{-cl}(A_1) \cap A_2 = \phi$. Since $A_2 \subseteq K$, $\alpha g^*p\text{-cl}(A_2) \subseteq \alpha g^*p\text{-cl}(K)$. Since $\alpha g^*p\text{-cl}(K) \cap H = \phi$, $\alpha g^*p\text{-cl}(A_2) \cap A_1 = \phi$. But $A = A_1 \cup A_2$, therefore A is not αg*p-connected space which is a contradiction. Then either $A \subseteq H$ or $A \subseteq K$.

Theorem 3.22: If H is αg*p-connected set and $H \subseteq E \subseteq \alpha g^*p\text{-cl}(H)$ then E is αg*p-connected.

Proof: If E is not αg*p-connected, then there exists two sets A, B such that $\alpha g^*p\text{-cl}(A) \cap B = A \cap \alpha g^*p\text{-cl}(B) = \phi$ and $E = A \cup B$. Since $H \subseteq E$, either $H \subseteq A$ or $H \subseteq B$. Suppose $H \subseteq A$ then $\alpha g^*p\text{-cl}(H) \subseteq \alpha g^*p\text{-cl}(A)$ .Thus $\alpha g^*p\text{-cl}(H) \cap B = \alpha g^*p\text{-cl}(A) \cap B = \phi$. But $B \subseteq E \subseteq \alpha g^*p\text{-cl}(H)$ then $\alpha g^*p\text{-cl}(H) \cap B = B$. Therefore $B = \phi$ which is a contradiction.

Thus E is αg*p-connected set. If $H \subseteq E$, then by the same way we can prove that $A = \phi$ which is a contradiction. Then E is αg*p-connected.

Corollary 3.23: If a space X contains a αg*p-connected subspace A such that $\alpha g^*p\text{-cl}(A) = X$ then X is αg*p-connected.

Proof: Suppose A is a αg*p-connected subspace of X such that $\alpha g^*p\text{-cl}(A) = X$. Since $A \subseteq X = \alpha g^*p\text{-cl}(A)$ then by Theorem 3.22 , X is αg*p-connected.

Theorem 3.24: If A is αg*p-connected set then $\alpha g^*p\text{-cl}(A)$ is αg*p-connected.

Proof: Suppose A is αg*p-connected set and $\alpha g^*p\text{-cl}(A)$ is not. Then there exist two αg*p-separated sets H ,K such that $\alpha g^*p\text{-cl}(A) = H \cup K$ .But $A \subseteq \alpha g^*p\text{-cl}(A)$, then $A \subseteq H \cup K$ and since A is αg*p-connected set then either $A \subseteq H$ or $A \subseteq K$ (by theorem 3.21).

Case-(i): If $A \subseteq H$, then $\alpha g^*p\text{-cl}(A) \subseteq H$. But $\alpha g^*p\text{-cl}(H) \cap K = \phi$, hence $\alpha g^*p\text{-cl}(A) \cap K = \phi$.

Since $K \subseteq \alpha g^*p\text{-cl}(A)$ then $K = \phi$ which is a contradiction.

Case-(ii): If $A \subseteq K$, then the same way we can prove that $H = \phi$ which is a contradiction.

Therefore $\alpha g^*p\text{-cl}(A)$ is αg*p-connected set.

Theorem 3.25: Let X be a topological space such that any two elements a and b of X are contained in some αg*p-connected subspace of X . Then X is αg*p-connected.

Proof: Suppose X is not αg*p-connected space. Then X is the union of two αg*p-separated sets A, B. Since A, B are non-empty sets, thus there exist a,b such that $a \in A$, $b \in B$. Let H be a αg*p-connected subspace of X which contains a and b. Therefore by theorem 3.21 either $H \subseteq A$ or $H \subseteq B$ which is a contradiction since $A \cap B = \phi$ .Then X is αg*p-connected space.

Theorem 3.26: If A and B are αg*p-connected subspace of a space X such that $A \cap B \neq \phi$, then $A \cup B$ is αg*p-connected subspace.

Proof: Suppose that $A \cup B$ is not αg*p-connected. Then there exist two αg*p-separated sets H and K such that $A \cup B = H \cup K$. Since $A \subseteq A \cup B = H \cup K$ and A is αg*p–connected then either $A \subseteq H$ or $A \subseteq K$.

Since $B \subseteq A \cup B = H \cup K$ and B is αg*p-connected, either $B \subseteq H$ or $B \subseteq K$.

(1) If $A \subseteq H$ and $B \subseteq H$, then $A \cup B \subseteq H$. Hence $K = \phi$ which is a contradiction.

(2) If $A \subseteq H$ and $B \subseteq K$, then $A \cap B \subseteq H \cap K = \phi$. Therefore $A \cap B = \phi$ which is a contradiction.

By the same way we can get a contradiction if $A \subseteq K$ and $B \subseteq H$ or if $A \subseteq K$ and $B \subseteq K$. Therefore $A \cup B$ is αg*p-connected subspace of a space X.

Theorem 3.27: If X and Y are αg*p-connected spaces, then $X \times Y$ is αg*p-connected space.

Proof: For any points (x_1, y_1) and (x_2, y_2) of the space $X \times Y$, the subspace $X \times \{y_1\} \cup \{x_2\} \times Y$ contains the two points and this subspace is αg*p-connected since it is the union of two αg*p-connected subspaces with a point in common. Thus $X \times Y$ is αg*p-connected.

4. αg*p-COMPACTNESS

In this section we introduce the concept of αg*p-compactness and studied some of their properties.

Definition 4.1: A collection $\{A_i: i \in \Lambda\}$ of αg*p-open sets in a topological space X is called a αg*p-open cover of a subset S if $S \subset \bigcup \{A_i: i \in \Lambda\}$

Definition 4.2: A topological space (X, τ) is called αg*p-compact if every αg*p-open cover of X has a finite subcover.

Definition 4.3: A subset S of a topological space X is said to be αg*p-compact relative to X if for every collection $\{A_i: i \in \Lambda\}$ of αg*p-open subsets of X such that $S \subset \bigcup \{A_i: i \in \Lambda\}$ there exists a finite subset Λ_0 of Λ such that $S \subset \bigcup \{A_i: i \in \Lambda_0\}$.

Definition 4.4: A subset S of a topological space X is said to be αg*p-compact if S is αg*p-compact as a subspace of X.

Definition 4.5: A space X is said to be αg*p-lindelof if cover of X by αg*p-open sets contains a countable subcover.

Theorem 4.6:

i) Every αg*p-compact space is compact.
ii) Every gp*-compact space is αg*p-compact.
iii) Every αg*p-compact space is αg*p-lindelof.

Proof: (i) Let (X, τ) be a αg*p-compact space. Let $\{A_i: i \in \Lambda\}$ be an open cover of (X, τ). By lemma 2.8, $\{A_i: i \in \Lambda\}$ is a αg*p-open cover of (X, τ).Since (X, τ) is αg*p-compact , αg*p-open cover $\{A_i: i \in \Lambda\}$ of (X, τ) has a finite subcover say $\{A_i: i = 1,2\ldots n\}$ for X. Hence (X, τ) is compact.
(ii) and (iii) follows from definitions (4.2, 4.5) and lemma (2.8) .

Theorem 4.7: A αg*p-closed subset of αg*p-compact space is αg*p-compact relative to X.

Proof: Let A be a αg*p-closed subset of a αg*p-compact space X. Then $X \setminus A$ is αg*p-open. Let $S = \{A_i \ i \in \Lambda\}$ be a αg*p-open cover for A by αg*p-open subsets in X .Then $S^* = S \cup A^c$ is a αg*p-open cover for X. By hypothesis, X is αg*p-compact and hence S* is reducible to a finite subcover of X , say $X = A_{i_1} \cup A_{i_2} \cup \ldots \cup A_{i_n} \cup A^c$, $A_{i_k} \in S$. Hence $A \subset A_{i_1} \cup A_{i_2} \cup \ldots \cup A_{i_n}$, $A_{i_k} \in S$. Thus a αg*p-open cover S of A contains a finite subcover. Hence A is αg*p-compact relative to X.

Theorem 4.8: A space X is αg*p-compact if and only if every family of αg*p-closed sets in X with empty intersection has a finite subfamily with empty intersection.

Proof: Suppose X is αg*p-compact and $\{U_\alpha : \alpha \in \Lambda\}$ is a family of αg*p-closed sets in X such that $\bigcap \{U_\alpha : \alpha \in \Lambda\} = \phi$.Then $\bigcup \{X \setminus U_\alpha : \alpha \in \Lambda\}$ is αg*p-open cover for X. Since X is αg*p-compact, this cover has a finite subcover(say) $\{X \setminus U_{\alpha_1}, X \setminus U_{\alpha_2}, \ldots, X \setminus U_{\alpha_n}\}$ for X. That is $X = \bigcup \{X \setminus U_{\alpha_i} : i = 1, 2, \ldots n\}$. This implies that $\bigcap_{i=1}^{n} U_{\alpha_i} = \phi$.

Conversely, Suppose that every family of αg^*p-closed sets in X which has empty intersection has a finite subfamily with empty intersection. Let $\{V_\alpha : \alpha\in\Lambda\}$ be a αg^*p-open cover for X. Then $\bigcup \{V_\alpha : \alpha\in\Lambda\} = X$. This implies that $\bigcap \{X \setminus V_\alpha : \alpha\in\Lambda\}= \phi$. Since $X \setminus V_\alpha$ is αg^*p-closed for each $\alpha\in\Lambda$, there is a finite subfamily $\{X \setminus V_{\alpha_1}, X \setminus V_{\alpha_2},\ldots, X \setminus V_{\alpha_n}\}$ with empty intersection. This implies $\bigcup_{i=1}^{n} V_{\alpha_i} = X$. Hence X is αg^*p-compact.

Theorem 4.9: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a surjective, αg^*p-continuous map .If X is αg^*p-compact then Y is compact.

Proof: Let $\{A_i: i\in\Lambda\}$ be an open cover of Y. Then $\{f^{-1}(A_i): i\in\Lambda\}$ is an αg^*p-open cover of X. Since X is αg^*p-compact, it has a finite subcover, say $\{f^{-1}(A_{i_1}), f^{-1}(A_{i_2}),\ldots,f^{-1}(A_{i_n})\}$. Surjectiveness of f implies $\{A_{i_1}, A_{i_2},\ldots, A_{i_n}\}$ is an open cover of Y and hence Y is compact.

Theorem 4.10: If a map f: $(X, \tau) \rightarrow (Y, \sigma)$ is αg^*p-irresolute and a subset S of X is αg^*p-compact relative to X then the image f(S) is αg^*p-compact relative to Y.

Proof: Let $\{A_i: i\in\Lambda\}$ be a collection of αg^*p-open sets in Y such that $f(S) \subset\bigcup \{A_i: i\in\Lambda\}$. Then $S \subset\bigcup \{f^{-1}(A_i): i\in\Lambda\}$, where $f^{-1}(A_i)$ is αg^*p-open in X for each i. Since S is αg^*p-compact relative to X, there exists a finite sub collection $\{A_1, A_2,\ldots, A_n\}$ such that $S \subset\bigcup \{f^{-1}(A_i) / i=1, 2,\ldots, n\}$.That is $f(S) \subset\bigcup \{A_i : i= 1,2,\ldots,n\}$.Hence f(S) is αg^*p-compact relative to Y.

Theorem 4.11: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a surjective, αg^*p-irresolute map. If X is αg^*p-compact then Y is αg^*p -compact.

Proof: Let $\{A_i: i\in\Lambda\}$ be an αg^*p -open cover of Y. Then $\{f^{-1}(A_i): i\in\Lambda\}$ is an αg^*p-open cover of X. Since X is αg^*p-compact, it has a finite subcover, say $\{f^{-1}(A_{i_1}), f^{-1}(A_{i_2}),\ldots, f^{-1}(A_{i_n})\}$. Surjectiveness of f implies $\{A_{i_1}, A_{i_2},\ldots, A_{i_n}\}$ is a subcover for Y and hence Y is αg^*p -compact.

Theorem 4.12: If f: $(X, \tau) \rightarrow (Y, \sigma)$ is a αg^*p-open function and Y is αg^*p-compact then X is compact.

Proof: Let $\{A_i: i\in\Lambda\}$ be an open cover of X. Then $\{f(A_i): i\in\Lambda\}$ is an αg^*p -open cover of Y. Since Y is αg^*p-compact, it has a finite subcover, say $\{f(A_{i_1}), f(A_{i_2}),\ldots, f(A_{i_n})\}$. Then $\{A_{i_1}, A_{i_2},\ldots, A_{i_n}\}$ is a finite subcover for X and hence X is compact.

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