International Journal of Mathematical Archive-7(9), 2016, 135-142
Available online through www.ijma.info ISSN 2229 – 5046

On Soft-contra-πgp-continuous in Soft Topological Spaces

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(Received On: 05-09-16; Revised & Accepted On: 28-09-16)

ABSTRACT

In this paper we investigate the notion of soft-contra-πgp-continuity, soft-almost-contra-πgp-continuous function, which is weaker than soft-contra-continuity. We also obtain some properties of soft-contra-πgp-continuous functions and discuss the relationship between other related functions. Further we apply the notion of soft-πgp-closed sets in soft topological spaces to study soft-πgp-homeomorphism.

Keyword: *soft topological spaces, soft-contra-πgp-continuity, soft-almost-contra-πgp- continuous function, soft-πgp-homeomorphism.*

INTRODUCTION

Soft system provides a general framework with the involvement of parameters. Soft set Theory has a wider application and its progress is very rapid in different fields. The soft set theory is a rapidly processing field of mathematics. Molodtsov [12] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Topological structure of soft sets was initiated by Shabir and Naz [14] and studied the concept of soft open sets, soft interior points, and soft neighbourhood of the points, soft separation axioms and subspaces of a soft topological space. N. Palaniappan [13] introduced regular generalized closed sets the concept of regular continuous functions was introduced by Arya.S.P and Gupta.R [4] in the year 1974. Athar Kharal and B.Ahmed [11] defined the notion of a mapping on soft classes and studied several properties of images and inverse images of soft sets. Hussain *et al.* [7] continued to study the properties of soft topological spaces.

In this present paper, we discuss soft-contra-πgp-continuous, soft-contra-πgp-irresolute, soft-almost-contra-πgp-continuous function and also soft-πgp-homeomorphism in soft topological space and some characterization of these mappings are obtained.

2. PRELIMINARIES

Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let P (U) denote the power set of U, and let $A \subseteq E$.

Definition 2.1[12]: A pair (F, A) is called a soft set over U, where F is a mapping given by F: $A \rightarrow$ P (U).In other words, a soft set over U is a parameterized family of subsets of the universe U. For a particular $e \in A$. F (e) may be considered the set of e-approximate elements of the soft set (F, A).

Definition 2.2[6]: Two soft set (F, A) and (G, B) over a common universe U is said to be soft equal if (F, A) is a soft subset (G, B) and (G, B) is a soft subset of (F, A).

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Definition 2.3[14]: Let τ be the collection of soft sets over X. then τ is called a soft topology on X if τ satisfies the following axioms:

(i) ϕ, $\tilde{X}$ belong to τ
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X.

Definition 2.4: A soft subset (A, E) of X is called

(i) a soft generalized closed (Soft-g-closed)[10], if Cl(A, E) $\tilde{\subseteq}$(U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft open in X.
(ii) a soft-regular open[1], if (A, E)= Int(Cl(A, E)).
(iii) a soft-pre-open [5], if (A, E) $\tilde{\subseteq}$ Int (Cl(A, E)).
(iv) a soft-clopen[5], if (A, E) is both soft open and soft closed.
(v) a soft-πgr-closed[8], if srCl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft π-open in X.

The complement of the soft regular open, soft pre-open sets are their respective, soft regular closed, soft pre-closed and set sets.

The finite union of soft regular open sets is called soft π-open set and its complement is soft-π-closed set. The soft regular open set of X is denoted by SRO(X) or SRO(X, τ, E).

Definition 2.5: [3] Let (F, E) be a soft set X. The soft set (F, E) is called a soft point , denoted by (X_e, E), if for the element e ∈ E, F(e)={x} and F(e')= ϕ for all e'∈E-{e}.

Definition 2.6: Let (X, τ, E) and (Y, τ*, E) be two soft topological spaces. A function f: (X, τ, E) → (Y, τ*, E) is said to be

(i) Soft-pre-continuous [15], if f^{-1}(F, E) is soft-pre-open in (X, τ, E), for every soft-open set (F, E) of (Y, τ*, E).
(ii) Soft-πgr-continuous [8], if f^{-1}(F, E) is soft-πgr-open in (X, τ, E), for every soft-open set (F, E) of (Y, τ*, E).
(iii) Soft-πg-continuous [1], if f^{-1}(F, E) is soft-πg-open in (X, τ, E), for every soft-open set (F, E) of (Y, τ*, E).
(iv) Soft-continuous [5], if f^{-1}(F, E) is soft-open in (X, τ, E), for every soft-open set (F, E) of (Y, τ*, E).
(v) Soft-g-continuous [2], if f^{-1}(F, E) is soft-g-open in (X, τ, E), for every soft-open set (F, E) of (Y, τ*, E).
(vi) Soft-contra-continuous [9] if f^{-1}(F, E) is soft-closed in (X, τ, E), for each soft-open set in (Y, τ*, E).
(vii) Soft-contra-g-continuous [9] if f^{-1}(F, E) is soft-g-closed in (X, τ, E), for each soft-open set in (Y, τ*, E)
(viii) Soft-contra-pre-continuous if f^{-1}(F, E) is soft-pre-closed in (X, τ, E), for each soft-open set in (Y, τ*, E)
(ix) Soft-contra-πgr-continuous [8] if f^{-1}(F, E) is soft-πgr-closed in (X, τ, E), for each soft-open set in (Y, τ*, E)
(x) Soft-contra-πg-continuous [2] if f^{-1}(F, E) is soft-πg-closed in (X, τ, E), for each soft-open set in (Y, τ*, E)

Definition: 2.9 [5]:

(i) A soft subset (A, E) of a soft topological space X is called soft-πgp-closed set in X if spcl(A, E) $\tilde{\subseteq}$(U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft-π-open in X.
By SπGPC(X), we mean the family of all soft-πgp-closed subsets of the space X.
(ii) Let X and Y be two topological spaces and the function f: X→Y. Then the function f is soft-πgp-irresolute if f^{-1}(F, E) is soft-πgp-open in X, for every soft-πgp-open set (F, E) of Y.

Definition 2.10 [5]: Let (X, τ, E) and (Y, τ*, E) be two soft topological spaces and f:(X, τ, E)→(Y, τ*, E) be a function. Then the function f is said to be soft-πgp-continuous function if f^{-1}(G, E) is soft-πgp-closed(open) set in (X, τ, E) for every soft-closed (open) set (G, E) of (Y, τ*, E).

Definition 2.11 [2]: Let (A, E) be a subset of a space X. The set ∩{(U, E) ∈ τ:(A, E) $\tilde{\subseteq}$(U, E) } is called the Kernal of (A, E) and it is denoted by Ker(A, E).

Definition 2.12 [5]: Let (X, τ, E) and (Y, τ*, E) be soft topological spaces and f: (X, τ, E)→ (Y, τ*, E) be a function. Then the function is called soft-open mapping if f (F, E) ∈(Y, τ*, E) for all (F, E)∈τ. Similarly, a function f:(X, τ, E)→(Y, τ*, E) is called a soft-closed mapping if for a closed set (G, E) in τ, the image f(G, E) is soft-closed in τ*.

Throughout this paper we denote (X, τ, E), (Y, τ*, E) and (Z, τ**, E) as X, Y and Z.

3. Soft-contra-πgp-continuous function

Definition 3.1.1: Let (X, τ, E) and (Y, τ^*, E) be two soft topological spaces and f: $(X, \tau, E) \rightarrow (Y, \tau^*, E)$ be a function. Then the function f is soft-contra-πgp-continuous if f^{-1} (F, E) is soft-πgp-closed in X, for every soft-open (F, E) in Y.

Definition: 3.1.2: A space (X, τ) is called πgp^s-space, if every soft-πgp-open set is soft-closed set.

Theorem 3.1.3:

(i) Every soft-contra-continuous is soft-contra-πgp-continuous.
(ii) Every soft-contra-pre-continuous is soft-contra-πgp-continuous.
(iii) Every soft-contra-g-continuous is soft-contra-πgp-continuous.
(iv) Every soft-contra-πg-continuous is soft-contra-πgp-continuous.
(v) Every soft-contra-πgr-continuous is soft-contra-πgp-continuous.

Proof: The proof follows from the definition.

None of the implications is Reversible as shown in the following example.

Example 3.1.4: Let X= {a, b, c, d}, E= $\{e_1, e_2\}$. Let $F_1, F_2, \ldots, F_6$ are functions from E to P(X) and are defined as follows:

$F_1(e_1)$ ={c}, $F_1(e_2)$ ={a},
$F_2(e_1)$ ={d}, $F_2(e_2)$ ={b},
$F_3(e_1)$ ={c, d}, $F_3(e_2)$ ={a, b},
$F_4(e_1)$ ={a, d}, $F_4(e_2)$ ={b, d},
$F_5(e_1)$ ={b, c, d}, $F_5(e_2)$ ={a, b, c},
$F_6(e_1)$ ={a, c, d}, $F_6(e_2)$ ={a, b, d},

Then τ_1 = {Φ, X, (F_1, E), (F_6, E)} is a soft topology and elements in τ are soft- open sets.

Let H_1, H_2, H_3, H_4 are functions from E to P(Y) and are defined as follows:

$H_1(e_1)$ ={a}, $H_1(e_2)$ ={d},
$H_2(e_1)$ ={b}, $H_2(e_2)$ ={c},
$H_3(e_1)$ ={a, b}, $H_3(e_2)$ ={c, d},
$H_4(e_1)$ ={b, c, d}, $H_4(e_2)$ ={a, b, c},

Then τ_2= {Φ, X, (H_1, E), (H_4, E)} is a soft topology on Y. Let f: X→Y be an function of f(a)=d, f(b) = c, f(c)=b, f(d)=a.

Here the inverse image of the soft-closed set (A, E) = {{c, d}, {a, b}} in Y is not soft-open, soft-g-open, soft-pre-closed, soft-πgr-closed in X. Hence f is not soft-contra-continuous, soft-contra-g-continuous, soft-contra-pre-continuous, and soft-contra-πgr-continuous. Also soft closed set (B, E) = {{b, c, d}, {a, b, c}} in Y is not soft-πg-open set in X. Hence f is not soft-contra-πg-continuous.

Theorem 3.1.5: Let X and Y be the two soft topological spaces and f: X→Y be a function. Then f is soft-πgp-continuous and the space X is πgp^s-space, then f is soft-contra-continuous.

Proof: Let (F, E) be a soft-open set in Y. Since f is soft-πgp-continuous, f^{-1}(F, E) is soft-πgp-open set in X. Since X is πgp^s-space, f^{-1}(F, E) is soft-closed in X. Hence f is soft-contra-continuous.

Theorem 3.1.6: Suppose πGPO(X) is soft-closed under arbitrary union. Then the followings are equivalent for a function f: X→Y

(i) f is soft-contra-πgp-continuous.
(ii) For every soft-closed subsets of (F, E) of Y, $f^{-1}(F, E) \in \pi GPO(X)$.
(iii) For each $x \in X$ and each $(F, E) \in SC(Y, f(x))$, there exist $(A, E) \in S\pi GPO(X, x)$ such that $f(A, E) \tilde{\subseteq} (F, E)$.

Proof:
(i)⇔ (ii), (ii) ⇒ (iii): is obvious.

(iii) ⇒ (ii): Let (F, E) be any closed set of Y and $x \in f^{-1}(F, E)$.Then $f(x) \in (F, E)$ and there exist $(A, E)_x \in S\pi GPO(X)$ such that $f(A, E)_x) \tilde{\subseteq}$ (F, E).Therefore $f^{-1}(F, E) = \cup \{(A, E)_x : x \in f^{-1}(F)\}$ and f^{-1}(F, E) is soft-πgp-open.

(i) ⇒ (iii): Let $x \in X$ and (F, E) be a closed set in Y with $f(x) \in (F, X)_x$, By (i),it follows that $f^{-1}(Y-(F, E)) = X-f^{-1}(F, E)$ is soft-πgp-closed and so f^{-1}(F, E) is soft-πgp-open. Take (A, E) = f^{-1} (F, E), we obtain that $x \in (A, E)$ and f(A, E)) $\tilde{\subseteq}$ (F, E).

Theorem 3.1.7: Suppose SπGPO(X) is soft-closed under arbitrary unions. If f: X→Y is soft-contra-πgp-continuous and Y is soft-regular, then f is soft-πgp-continuous.

Proof: Let x be an arbitrary point of X and (V, E) be an soft-open set of Y containing f(x).The regularity of Y implies that there exist an soft-open set in Y containing f(x) such that s-cl(W, E) $\tilde{\subseteq}$ (V, E). Since f is soft-contra-πgp-continuous, then there exist (U, E) ∈ s-πGPO(X) contains, such that f(U, E) $\tilde{\subseteq}$ s-cl(W, E). Then f(U, E) $\tilde{\subseteq}$ s-cl(W, E) $\tilde{\subseteq}$ (V,E). Hence f is soft-πgp-continuous.

Theorem 3.1.8: If a function f: X→Y is soft-contra-πgp-continuous and U is soft-open in X; then f/U: (X, τ, E) → (Y, τ*, E) is soft-contra-πgp-continuous.

Proof: Let (B, E) be soft-closed in Y. Since f: X→Y is soft-contra-πgp-continuous; f^{-1}(B, E) is soft-πgp-open in X. $(f/U)^{-1}$(B, E) = f^{-1}(B, E) ∩ U is soft-πgp-open in X. Hence $(f/U)^{-1}$(B, E) is soft-πgp-open in U.

Definition 3.1.9:
- (i) The Soft πgp-Closure of a soft set (G, E) is defined to be the intersection of all soft πgp-closed sets containing the soft set (G, E) and is denoted by s-πgp-cl(G, E).
- (ii) The Soft πgp-Interior of a soft set (G, E) is defined to be the union of all soft πgp-open sets contained the soft set (G, E) and is denoted by s-πgp-int(G, E).

Theorem 3.1.10: Suppose that πGPC(X) is soft-closed under arbitrary intersections. Then the following are equivalent
- (i) f is soft-contra-πgp-continuous.
- (ii) The inverse images of every closed set of Y are soft-πgp-open.
- (iii) For each x∈X and each closed set (B, E) in Y with f(x)∈(B, E),there exist a soft-πgp-open set (A, E) in X such that x∈(A, E) and f(A, E) $\tilde{\subseteq}$ (B, E).
- (iv) f (sπgp-cl(A, E)) $\tilde{\subseteq}$ ker f(A, E) for every subset (A, E) of X.
- (v) sπgp-cl(f^{-1}(B, E))) $\tilde{\subseteq}$ f^{-1}(ker(B, E)) for every subset (B, E) of Y.

Proof:
(i) ⇒(ii): and **(ii)⇒(i)** is obviously true.

(i)⇒(iii): Let x ∈ X and (B, E) be soft-closed set in Y with f(x)∈(B, E).By (i), it follows that f^{-1}(Y-(B, E)) is soft-πgp-closed set and so f^{-1}(B, E) is soft-πgp-open. Take (A, E)=f^{-1}(B, E).we obtain that x∈(A, E) and f(A, E) $\tilde{\subseteq}$ (B, E).

(iii)⇒(ii): Let (B, E) be a soft-closed set in Y with x∈f^{-1}(B. E). Since f(x)∈(B, E),by (iii),there exist a soft-πgp-open set (A, E) in X containing x such that f(A, E) $\tilde{\subseteq}$ (B, E).It follows that x∈(A,E) $\tilde{\subseteq}$ f(B,E).Hence f^{-1}(B,E) is soft-πgp-open.

(ii)⇒(iv): Let (A,E) be any set of X. Let y∉ ker f(A, E).Then there exist a soft-closed set (F, E) containing y such that f(A, E)∩(F, E) = ϕ. Hence, we have (A, E) ∩f^{-1}(F, E)= ϕ. s-πgp-cl(A,E)∩ f^{-1} (F,E)= ϕ. Thus f(s-πgp-cl(A,E))⊂(F,E) = ϕ and y ∉ f(s-πgp-cl(A, E)) and hence f(s-πgp-cl(A, E)) ⊂ ker f(A, E).

(iv) ⇒ (v): Let (B, E) be any subset of Y. By (iv), f(s-πgp-cl(f^{-1}(B, E)))⊂ ker (B, E) and sπgp-cl (f^{-1}(ker(B, E))).

(v) ⇒ (i): Let (B, E) be any soft open set in Y. By (v), s-πgp-cl(f^{-1}(B,E))⊂f^{-1}(ker(B,E))=f^{-1}(B,E) s-πgp-cl(f^{-1}(B,E)) = f^{-1}(B, E), We obtain f^{-1}(B, E) is s-πgp-closed in X. Hence f is soft-contra-πgp-continuous.

3.2. soft-contra-πgp-irresolute

Definition: 3.2.1: A Map f: X→Y is said to be soft-contra-πgp-irresolute if f^{-1}(F, E) is soft-πgp-closed in X, for each (F, E) is soft-πgp-open in Y.

Theorem 3.2.2: Let f: X→Y and g: Y→Z be two maps in soft topological space such that gof: X→Z. Then
- (i) If g is soft-πgp-continuous and f is soft-contra-πgp-irresolute, then gof is soft-contra-πgp-continuous.
- (ii) If g is a soft-πgp-irresolute and f is soft-contra-πgp-irresolute, then gof is soft-contra-πgp-irresolute.

Proof:
- (i) Let (F, E) be soft-closed set in Z. Then g^{-1}(F, E) is soft-πgp-closed set in Y. Since f is contra-soft-πgp-irresolute, f^{-1}(g^{-1}(F, E)) is soft-πgp-open set in X. Hence gof is soft-contra-πgp-continuous.
- (ii) Let (F, E) be soft-πgp-closed set in Z. Then g^{-1}(F, E) is soft-πgp-closed set in Y. Since f is soft-contra-πgp-irresolute, f^{-1}(g^{-1}(F, E)) is soft-πgp-open set in X. Hence gof is soft-contra-πgp-irresolute.

Theorem 3.2.3: Suppose that s-πGPC(Y) is soft-closed under arbitrary intersections. If f: X→Y is surjective soft-πgp-open function and g: Y→Z is a function such that gof: X→Z is soft-contra-πgp-continuous, then g is soft-contra-πgp-continuous

Proof: Suppose that x and y are two soft-points in X and Y such that f(x) = y. Let (B, E) ∈ SC(Z,(gof)(x)).Then there exist a s-πgp-open set (A, E) in X containing x such that g(f(A,E)) $\tilde{\subseteq}$ (B,E). Since f is s-πgp-open, f (A, E) is s-πgp-open set in Y containing y such that g(f(A, E)) $\tilde{\subseteq}$ (B, E).This implies g is soft-contra-πgp-continuous.

Theorem 3.2.4: Every soft-contra-πgp-irresolute is soft-contra-πgp-continuous.

Proof: The proof is obvious.

Remark 3.2.5: Converse of the above need not be true as seen in the following example.

Example 3.2.6: Let X= {a, b, c, d}, E={e_1, e_2}. Let F_1, F_2,…,F_6 are functions from E to P(X) and are defined as follows:

$F_1(e_1)$ ={c}, $F_1(e_2)$ ={a},
$F_2(e_1)$ ={d}, $F_2(e_2)$ ={b},
$F_3(e_1)$ ={c, d}, $F_3(e_2)$ ={a, b},
$F_4(e_1)$ ={a, d}, $F_4(e_2)$ ={b, d},
$F_5(e_1)$ ={b, c, d}, $F_5(e_2)$ ={a, b, c},
$F_6(e_1)$ ={a, c, d}, $F_6(e_2)$ ={a, b, d},

Then τ_1 = {Φ, X, (F_1, E), (F_6, E)} is a soft topology and elements in τ are soft- open sets.

Let G_1, G_2, G_3, G_4 are functions from E to P(Y) and are defined as follows:

$G_1(e_1)$ ={a}, $G_1(e_2)$ ={d},
$G_2(e_1)$ ={b, c, d}, $G_2(e_2)$ ={a, b, c},

Then τ_2= {Φ, X, (G_1, E), (G_4, E)} is a soft topology on Y. Let f: X→Y be an identity function. Hence it is soft-contra-πgp-continuous. But the inverse image of (A, E) = {{a, b}, {c, d}} in Y is not soft-πgp-closed set in X. Hence not soft-contra-πgp-irresolute.

3.3 soft-Almost-contra-πgp-continuous functions

Definition 3.3.1: A function f: X→Y is said to be soft -almost-contra-continuous if f^{-1}(F, E) is closed in X, for each soft-regular-open set (F, E) of Y.

Definition 3.3.2: A function f: X→Y is said to be soft -almost-contra-πgp-continuous if f^{-1}(F, E) ∈ SπGPC(X), for each (F, E) ∈ SRO(Y).

Theorem 3.3.3: Suppose soft-πgp-open set of X is soft-closed under arbitrary unions. The following statement is equivalent for a function f: X→Y,

(i) f is soft-almost-contra-πgp-continuous.
(ii) f^{-1}(F, E) ∈ soft-πgp-open in X, for every (F, E) ∈ SRC(Y).
(iii) For each x∈X and each soft-regular closed set (F, E) in Y containing f(x), there exist a soft-πgp-open set (A, E) in X containing x such that f(A, E) $\tilde{\subseteq}$ (F, E).
(iv) For each x∈X and each soft-regular open set (B, E) in Y not containing f(x), there exists a soft-πgp-closed set (G, E) in X not containing x such that f^{-1}(B, E) $\tilde{\subseteq}$ (G, E).
(v) f^{-1}(s-int (cl (G, E))) ∈s-πGPC(X) for every soft-open subset (G, E) of Y.
(vi) f^{-1}(s-int (cl (F, E)))∈SπGPO(X) for every soft-closed subset (F, E) of Y.

Proof:
(i)⇒ (ii): Let (F, E) ∈SRC(Y).Then Y-(F, E)∈SRO(Y). Since f is soft-almost-contra-πgp-continuous. Hence f^{-1}(Y-(F, E)) =X-f^{-1} (F, E) ∈SπGPC(X).This implies f^{-1}(F, E)∈SπGPO(X).

(ii)⇒ (i): Let (F, E) ∈ SRO(Y).Then by assumption (F-(F, E)) ∈ SRC(Y). Since for each (F, E) ∈ SRC(Y).Hence f^{-1}(Y-(F, E))=X-f^{-1}(F,E) ∈ SπGPO(X) This implies f^{-1}(F, E) ∈ SπGPC(X).

(ii)⇒(iii): Let (F, E) be any soft-regular closed set in Y containing f(x).f^{-1}(F, E) ∈ SπGPO(X) and x ∈ f^{-1}(F,E). Take (A, E)= f^{-1}(F, E).then f(A, E) $\tilde{\subseteq}$ (F, E).

(iii)⇒(ii): Let (F, E) ∈ SRC(Y) and x ∈f^{-1}(F, E).From (iii), there exist a soft-πgp-open set (A, E) in X containing x such that (A, E) $\tilde{\subseteq}$ f^{-1}(F, E).we have f^{-1}(F, E) = ∪ {(A, E):x∈ f^{-1}(F, E)}.Then f^{-1}(F, E) is soft-πgp-open.

(iii) ⇒ (iv): Let (B, E) be any soft-regular open set in Y containing f(x). Then Y-(B, E) is a soft regular closed set containing f(x). By (iii), there exists a soft-πgp-open set (A, E) in X containing x such that f(A,E) $\tilde{\subseteq}$ Y-(B, E). Hence (A, E) $\tilde{\subseteq}$ f^{-1}(Y-(B, E)). Then f^{-1}(B, E) $\tilde{\subseteq}$ X-(A, E). Let us set (G, E)= X-(A, E).We obtain a soft-πgp-closed set in X not containing x such that f^{-1}(B, E) $\tilde{\subseteq}$ (G, E).

(iv) ⇒ (iii): Let (F, E) be soft-regular closed set in Y containing f(x). Then Y-(F, E) is soft-regular open set in Y containing f(x). By (iv) there exists a soft-πgp-closed set (G, E) in X not containing x such that f^{-1}(Y-(F, E)) $\tilde{\subseteq}$ (G, E). Then X-f^{-1}(F, E) $\tilde{\subseteq}$ (G, E) implies X-(G, E) $\tilde{\subseteq}$ f^{-1}(F, E). Hence f(X-(G, E)) $\tilde{\subseteq}$ (F, E).Take (A, E)= X-(G, E). Then (A, E) is soft-πgp-open set in X containing x such that f(A, E) $\tilde{\subseteq}$ (F, E).

(i) ⇒ (v): Let (G, E) be the soft-open subset of Y. Since s-int(cl(G, E)) is soft-regular open, then by (i), f^{-1}(s-int(cl(G,E))) ∈ SπGPC(X).

(v) ⇒ (i): Let (G, E) ∈ SRO(Y). Then (G, E) is soft-open set in Y. By (v), f^{-1}(s-(int(cl(G,E)))) ∈ SπGPC(X). This implies f^{-1}(G,E) ∈ SπGPC(X).

(ii) ⇔ (vi): is similar as (i)⇔(v).

Theorem 3.3.4: Every soft-contra-πgp-continuous function is soft-almost-contra-πgp-continuous.

Proof: The proof is straight forward.

Remark 3.3.5: Converse of the above need not be true as seen in the following example.

Example 3.3.6: Let X= {a, b, c, d}, E={e_1, e_2}. Let F_1, F_2,….,F_6 are functions from E to P(X) and are defined as follows:

$F_1(e_1)$ ={c}, $F_1(e_2)$ ={a},
$F_2(e_1)$ ={d}, $F_2(e_2)$ ={b},
$F_3(e_1)$ ={c, d}, $F_3(e_2)$ ={a, b},
$F_4(e_1)$ ={a, d}, $F_4(e_2)$ ={b, d},
$F_5(e_1)$ ={b, c, d}, $F_5(e_2)$ ={a, b, c},
$F_6(e_1)$ ={a, c, d}, $F_6(e_2)$ ={a, b, d},

Then τ_1 = {Φ, X, (F_1, E), (F_6, E)} is a soft topology and elements in τ are soft- open sets.

Let G_1, G_2, G_3, G_4 are functions from E to P(Y) and are defined as follows:

$G_1(e_1)$ ={a}, $G_1(e_2)$ ={d},
$G_2(e_1)$ ={b, c, d}, $G_2(e_2)$ ={a, b, c},

Then τ_2= {Φ, X, (G_1, E), (G_4, E)} is a soft topology on Y. Let f: X→Y be an identity function. Hence it is soft-almost-πgp-continuous. But the inverse image of (A, E) = {{Φ}, {d}} = {{Φ}, {a}} in Y is not soft-πgp-closed set in X. Hence not soft-πgp-continuous.

Theorem 3.3.7: If f: X→Y is an soft-almost-contra-πgp-continuous function and (A, E) is soft-open subset of X, then the restriction f/(A, E): (A, E) →Y is soft-almost-contra-πgp-continuous.

Proof: Let (F, E) ∈ SRC(Y). Since f is soft-almost-contra-πgp-continuous, f^{-1}(F, E) ∈ SπGPO(X). Since (A, E) is soft-open set, it follows that $(f/(A, E))^{-1}$(F, E) = (A, E) ∩ f^{-1}(F, E) ∈ SπGPO(A, E). Therefore f/(A, E) is an soft-almost-contra-πgp-continuous.

4. soft-πgp-homeomorphism

Definition 4.1: A bijection f: X → Y is called soft πgp-homeomorphism if f is both soft-πgp-continuous and soft-πgp-open map.

Definition 4.2: A bijection f: X → Y is called soft-πgpC-homeomorphism if f is both soft-πgp-irresolute and f^{-1} is soft-πgp-irresolute.

Definition 4.3: A soft topological space X is called a soft-πgp-space if every soft-πgp-closed set is soft-closed set in X.

Theorem 4.4: For any bijection f: X → Y, the following statements are equivalent.

(i) f^{-1}:Y→X is soft-πgp-continuous.
(ii) f is a soft-πgp-open map.
(iii) f is a soft-πgp-closed map.

Proof:

(i) ⇒ (ii): Let (A, E) is a soft open set in X. Then X - (A, E) is soft closed in X. Since f^{-1} is soft- πgp-continuous, $(f^{-1})^{-1}$(X- (A, E)) = f(X- (A, E)) =Y-f ((A, E)) is soft-πgp-closed in Y. Then f((A, E)) is soft-πgp-open in Y. Hence f is a soft-πgp-open map.

(ii) ⇒ (iii): Let f be a soft-πgp-open map. Let (A, E) be a soft-closed set in X. Then X- (A, E) is soft-open in X. Since f is soft-πgp-open, f(X- (A, E)) = Y-f((A, E)) is soft-πgp-open in Y. Then f((A, E)) is soft-πgp-closed in Y. Hence f is soft-πgp-closed.

(iii) ⇒ (i): Let (A, E) be soft-closed set in X. Then f((A, E)) is soft-πgp-closed in Y. That is $(f^{-1})^{-1}$ ((A, E)) is soft-πgp-closed in Y. Hence f^{-1} is soft-πgp-continuous.

Theorem 4.5: Let f: X→ Y be a bijective and soft-πgp-continuous map. Then the following Statements are equivalent.
- (i) f is a soft-πgp-open map.
- (ii) f is a soft-πgp-homeomorphism.
- (iii) f is a soft-πgp-closed map.

Proof:

(i) ⇒ (ii): Follows from the definition.

(ii) ⇒ (iii): Let (A, E) be a soft-closed set in X. Then X- (A, E) is soft-open in X. Since f is a soft-πgp-homeomorphism, f(X- (A, E))= Y-f((A, E)) is soft-πgp-open in Y. Then f((A, E)) is soft-πgp-closed in Y. Hence f is a soft-πgp-closed map.

(iii) ⇒ (i): Let (A, E) be a soft-open set in X. Then X- (A, E) is soft-closed in X. Since f is a soft-πgp-closed map, f (X-(A, E)) =Y-f((A, E)) is soft-πgp-closed in Y. Then f ((A,E)) is soft-πgp-open in Y. Hence f is a soft-πgp-open map.

Theorem 4.6: If f: X→Y and g:Y→Z are soft-πgpC-homeomorphisms, then gof: X→Z is also a soft-πgpC-homeomorphism.

Proof: Let (A, E) be a soft-πgp-open set in Z. Now $(g\circ f)^{-1}$((A, E))=f^{-1}(g^{-1}((A, E)))=f^{-1}((A, E)), where (A, E) = g^{-1}((A, E)). By hypothesis, (A, E) is soft-πgp-open in Y and again by hypothesis, f^{-1}((A, E)) is soft-πgp-open in X. Therefore (gof) is soft-πgp-irresolute. Also for a soft-πgp-open set (G, E) in X, we have (g∘f)((G,E)) = g(f((G,E))) = g((W,E)), where (W, E) = f(G, E). By hypothesis, f((G, E)) is soft-πgp-open in Y and again by hypothesis, g(W, E) is soft-πgp-open in Z. Therefore $(g\circ f)^{-1}$is soft-πgp-irresolute. Hence g∘f is soft-πgpC-homeomorphism.

Theorem 4.7: Every soft-πgp-homeomorphism from a soft-πgp-space into another soft-πgp-space is a soft-homeomorphism.

Proof: Let f: X → Y, be a soft-πgp-homeomorphism. Then f is bijective, soft-πgp-continuous and soft-πgp-open. Let (A, E) be an soft-open set in X. Since f is soft-πgp-open and since Y is soft-πgp-space, f((A, E)) is soft-open in Y. This implies f is soft-open map. Let (A, E) be soft-closed in Y. Since f is soft-πgp-continuous and since X is soft-πgp-space, f^{-1}((A, E)) is soft-closed in X. Therefore f is soft- continuous. Hence f is a soft-homeomorphism.

Theorem 4.8: Every soft-πgp-homeomorphism from a soft-πgp-space into another soft-πgp-space is a soft-πgpC-homeomorphism.

Proof: Let f: X → Y be a soft-πgp-homeomorphism. Then f is bijective, soft-πgp-continuous and soft-πgp-open. Let (A, E) be an soft-πgp-closed set in Y. Then (A, E) is soft-closed in Y, since f is soft-πgp-continuous f^{-1}((A, E)) is soft-πgp-closed in X. Hence f is a soft-πgp-irresolute map. Let (V, E) be soft-πgp-open in X. Then (V, E) is soft-open in X. Since f is soft-πgp-open, f((V, E)) is soft-πgp-open set in Y. That is $(f^{-1})^{-1}$((V, E)) is soft-πgp-open in Y and hence f^{-1} is soft- πgp-irresolute. Thus f is soft-πgpC-homeomorphism.

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Source of support: Nil, Conflict of interest: None Declared