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# UNSTEADY MHD FLOW OF A CONDUCTING VISCO-ELASTIC [OLDROYD (1958) MODEL] LIQUID THROUGH POROUS MEDIUM BETWEEN TWO FINITE CO-AXIAL RIGHT IRCULAR CYLINDERS

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***ABSTRACT***

*The present paper is concerned with the unsteady MHD oscillatory motion of conducting visco-elastic [Oldroyd (1958) model] liquid through porous medium between two finite co-axial right circular cylinders in presence of variable transverse magnetic field applied perpendicular to the flow of liquid, when both the cylinders execute simple harmonic motion along the common axis of the cylinders. The amplitudes and frequencies have been taken different for both cylinders. The particular cases have also been discussed in detail.*

***Keywords:*** *MHD, porous medium, unsteady, harmonically.*

## INTRODUCTION

Some interesting problems in this area have been investigated by many researchers. Teipel (1981) studied the problem of the impulsive motion of a flat plate in a visco-elastic fluid. Choubey (1985) discussed the hydromagnetic flow of an electrically conducting visco-elastic Rivlin Ericksen (1955) type liquid near an infinite horizontal flat plate started impulsively from rest in its own plane with constant velocity subjected to an applied uniform transverse magnetic field. Yadav& Singh (1990) studied the impulsive motion of a porous flat plate in an elastico-viscous (Rivlin-Ericksen) liquid in the presence of a uniform transverse magnetic field. The hydromagnetic flow of two immiscible visco-elastic Walter liquids between two inclined parallel plates has been studied by Chakraborty & Sengupta (1992). Moreover some interesting problems in this area have been investigated by Ghosh and Sengupta (1993, 1996); Sengupta and Kundu (1999); Sharma and Pareek (2001); Hassanien (2002); Sengupta and Basak (2002); Pundhir and Pundhir (2003). Krishna, Rao and Sulochana (2004) have discussed the hydromagnetic oscillatory flow of a second order Rivlin Ericksen fluid in channel. Rahman and Alam Sarkar (2004) studied the unsteady MHD flow of visco-elastic Oldroyd fluid under time varying body force through a rectangular channel. Krishna and Rao (2005) investigated magneto hydrodynamic unsteady flow through a rectangular duct with a prescribed discharge. Radhakrishnamacharya and Rao (2007) studied the flow of a magnetic fluid through a non-uniform wavy tube. Kumar *et.al* (2008) studied unsteady flow of visco-elastic liquid through porous medium between two finite co-axial right circular cylinders. Nayak, Dash and panda (2013) studied unsteady MHD flow of a visco-elastic fluid along vertical porous surface with chemical reaction. Choudhury, Dhar and Dey (2014) have discussed visco-elastic MHD flow through a porous medium bounded by horizontal parallel plates moving in opposite direction in presence of heat and mass transfer.

The aim of the present paper is to study the unsteady MHD flow of a conducting visco-elastic [Oldroyd (1958) type] liquid through porous medium between two finite co-axial right circular cylinders when both cylinders oscillate harmonically with different amplitudes and frequencies. Some particular cases have also been discussed in detail.

## BASIC THEORY AND EQUATIONS OF MOTION

For motion, the Rheological equations for Oldroyd (1958) visco-elastic liquid are:

$$\mathrm{P_{ik}} = -\mathrm{p}\delta_{\mathrm{ik}} + \mathrm{P'_{ik}}$$

$$\mathrm{P'_{ik}} + \lambda_1 \frac{\mathrm{D}}{\mathrm{Dt}}\mathrm{P'_{ik}} + \mu_0 \mathrm{P'_{ij}} \mathrm{e_{ik}} - \mu_1\left(\mathrm{P'_{ij}}\mathrm{e_{ik}} + \mathrm{P'_{jk}}\mathrm{e_{ij}}\right) - \upsilon_1 \mathrm{P'_{ji}}\mathrm{e_{ji}}\delta_{\mathrm{ik}} = 2\eta_0\left[\mathrm{e_{ik}} + \lambda_2\frac{\mathrm{D}}{\mathrm{Dt}}\mathrm{e_{ik}} - 2\mu_2 \mathrm{e_{ij}}\mathrm{e_{jk}} + \upsilon_2 \mathrm{e_{ji}}\delta_{\mathrm{ik}}\right]$$

with the equation of incompressibility

$$e_{ii} = 0$$

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where

$$\frac{D}{Dt} b_{ik} = \frac{\partial}{\partial t} b_{ik} + \upsilon_{ij}\, b_{ik,j} + w_{ij}\, b_{jk} + w_{kj}\, b_{ij}$$

$$e_{ij} = \frac{1}{2}\left(\nu_{k,i} + \nu_{i,k}\right)$$

$$w_{ik} = \frac{1}{2}\left(\nu_{k,i} - \nu_{i,k}\right)$$

where

$P_{ik}$ = stress tensor
$e_{ik}$ = rate of strain tensor
$\lambda_1$ = relaxation time
$\lambda_2$ = retardation time
$\delta_{ik}$ =The metric tensor (Kronecker delta)
$\eta_0$ = Cofficient of viscosity

and $\mu_0, \mu_1, \mu_2, \upsilon_1, \upsilon_2$ are material constants.

**FORMULATION OF THE PROBLEM**

Let (r, $\theta$, z) be the cylindrical polar coordinates and $v_r, v_\theta, v_z$ are the components of velocity of liquid in the increasing directions of r, $\theta$ , and z – axis respectively.

For the present geometry, appropriate equation of motion for Oldroyd (1958) visco-elastic liquid through porous medium in presence of variable magnetic field when induced magnetic field is neglected is given by:

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} = -\frac{1}{\rho}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \upsilon\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \left(\frac{\sigma B_0^2}{\rho} + \frac{\upsilon}{K}\right)\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) w \tag{1}$$

where r represents radius of cylinder, t the time and K is the permeability of porous medium, $\mu$ the coefficient of viscosity in the direction of oscillation, p the fluid pressure, w the velocity of liquid, $\upsilon = \frac{\mu}{\rho}$ = the kinetic viscosity, $\rho$ the density of liquid, $\sigma$ the electrical conductivity of the liquid and $B_0$ is the magnetic inductivity of the field.

Here consider the motion of conducting visco-elastic liquid through porous medium bounded between two co-axial right circular cylinders of radii a and b $(a > b)$ executing longitudinal harmonic oscillations along their common axis with different amplitudes and frequencies.

Assuming the pressure gradient to be zero, the equation (1) becomes

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} = \upsilon\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \left(\frac{\sigma B_0^2}{\rho} + \frac{\upsilon}{K}\right)\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) w \tag{2}$$

and boundary conditions are:

$$\left.\begin{aligned} w &= \nu_1 e^{-i\omega_1 t} \text{ where } r = a \\ w &= \nu_2 e^{-i\omega_2 t} \text{ where } r = b \end{aligned}\right\} \tag{3}$$

where $\nu_1, \omega_1$ and $\nu_2, \omega_2$ are the respective amplitudes and frequencies of the outer and inner cylinders.

Introducing the following non-dimensional quantities:

$$r^* = \frac{r}{a}, t^* = \frac{\upsilon}{a^2} t, w^* = \frac{\upsilon}{a^2} w,\ \lambda_1^* = \frac{\upsilon}{a^2}\lambda_1, \lambda_2^* = \frac{\upsilon}{a^2}\lambda_2,$$

$$\nu_1^* = \frac{a}{\upsilon}\nu_1, \nu_2^* = \frac{a}{\upsilon}\nu_2, \omega_1^* = \frac{a^2}{\upsilon}\omega_1, \omega_2^* = \frac{a^2}{\upsilon}\omega_2, K^* = \frac{1}{a^2} K$$

In (2) and (3) and then dropping the stars, it is found

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} = \left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \left(\frac{1}{K} + H\right)\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) w \tag{4}$$

and boundary conditions

$$\left.\begin{aligned} w &= \nu_1 e^{-i\omega_1 t} \text{ where } r = 1 \\ w &= \nu_2 e^{-i\omega_2 t} \text{ where } r = {}^{b}\!/\!{}_{a} \end{aligned}\right\} \tag{5}$$

where $H = \dfrac{\sigma B_0^2}{\rho}$

 

## SOLUTION OF THE PROBLEM

To found solution of equation (4) in the form

$$w = \nu_1 f(r) e^{-i\omega_1 t} + \nu_2 g(r) e^{-i\omega_2 t} \tag{6}$$

which is evidently periodic in t.

Substituting (6) in (4), it is found

$$\left[(1 - i\omega_1\lambda_2)\left\{\frac{d^2f}{dr^2} + \frac{1}{r}\frac{df}{dr}\right\} + \left\{(1 - i\omega_1\lambda_1)\left(i\omega_1 - \frac{1}{K} - H\right)\right\} f(r)\right] \times \nu_1 e^{-i\omega_1 t}$$
$$+ \left[(1 - i\omega_2\lambda_2)\left\{\frac{d^2g}{dr^2} + \frac{1}{r}\frac{dg}{dr}\right\} + \left\{(1 - i\omega_2\lambda_1)\left(i\omega_2 - \frac{1}{K} - H\right)\right\} g(r)\right] \times \nu_2 e^{-i\omega_2 t} = 0 \tag{7}$$

By assumption that $\nu_1$ and $\nu_2$ are not zero, it is found

$$\frac{d^2f}{dr^2} + \frac{1}{r}\frac{df}{dr} + m^2 f = 0 \tag{8}$$

and

$$\frac{d^2g}{dr^2} + \frac{1}{r}\frac{dg}{dr} + n^2 g = 0 \tag{9}$$

where

$$m^2 = \frac{(1 - i\omega_1\lambda_1)\left(i\omega_1 - \frac{1}{K} - H\right)}{(1 - i\omega_1\lambda_2)}$$

$$n^2 = \frac{(1 - i\omega_2\lambda_1)\left(i\omega_2 - \frac{1}{K} - H\right)}{(1 - i\omega_2\lambda_2)}$$

Now boundary conditions given by equation (5) becomes

$$\left.\begin{array}{l} f(r) = 1, g(r) = 0 \text{ when } r = 1 \\ f(r) = 0, g(r) = 1 \text{ when } r = b/a \end{array}\right\} \tag{10}$$

Now the solutions of (8) and (9) subject to the boundary conditions (10) are:

$$f(r) = \frac{J_0(mr)Y_0\left(m\frac{b}{a}\right) - Y_0(mr)J_0\left(m\frac{b}{a}\right)}{J_0(m)Y_0\left(m\frac{b}{a}\right) - J_0\left(m\frac{b}{a}\right)Y_0(m)}$$

and

$$g(r) = \frac{J_0(nr)Y_0\left(n\frac{b}{a}\right) - Y_0(nr)J_0\left(n\frac{b}{a}\right)}{J_0(n)Y_0\left(n\frac{b}{a}\right) - Y_0(n)J_0\left(n\frac{b}{a}\right)}$$

Putting the values of f (r) and g (r) in (6) there is found the velocity of conductingvisco-elastic [Oldroyd (1958) type] liquid through porous medium between two oscillating co-axial right circular cylinders under the influence of variable magnetic field.

$$w = \nu_1\left\{\frac{J_0(mr)Y_0\left(m\frac{b}{a}\right) - Y_0(mr)J_0\left(m\frac{b}{a}\right)}{J_0(m)Y_0\left(m\frac{b}{a}\right) - Y_0(m)J_0\left(m\frac{b}{a}\right)}\right\} e^{-i\omega_1 t} + \nu_2\left\{\frac{J_0(nr)Y_0(n) - Y_0(nr)J_0(n)}{J_0(n)Y_0\left(n\frac{b}{a}\right) - Y_0(n)J_0\left(n\frac{b}{a}\right)}\right\} e^{-i\omega_2 t} \tag{11}$$

## PARTICULAR CASES

**Case-I:** If both cylinders oscillate with same amplitudes but different frequencies

i.e. $\nu_1 = \nu_2 = \nu$(say) then from (11), it is found

$$w = \nu\left[\left\{\frac{J_0(mr)Y_0\left(m\frac{b}{a}\right) - Y_0(mr)J_0\left(m\frac{b}{a}\right)}{J_0(m)Y_0\left(m\frac{b}{a}\right) - Y_0(m)J_0\left(m\frac{b}{a}\right)}\right\} e^{-i\omega_1 t} + \left\{\frac{J_0(nr)Y_0(n) - Y_0(nr)J_0(n)}{J_0(n)Y_0\left(n\frac{b}{a}\right) - Y_0(n)J_0\left(n\frac{b}{a}\right)}\right\} e^{-i\omega_2 t}\right] \tag{12}$$

**Case-II:** If both cylinders oscillate with same frequencies but different amplitudes

i.e. $\omega_1 = \omega_2 = \omega$ (say) then from (11), it is found

$$w = \left[\nu_1\left\{\frac{J_0(mr)Y_0\left(m\frac{b}{a}\right) - Y_0(mr)J_0\left(m\frac{b}{a}\right)}{J_0(m)Y_0\left(m\frac{b}{a}\right) - Y_0(m)J_0\left(m\frac{b}{a}\right)}\right\} + \nu_2\left\{\frac{J_0(nr)Y_0(n) - Y_0(nr)J_0(n)}{J_0(n)Y_0\left(n\frac{b}{a}\right) - Y_0(n)J_0\left(n\frac{b}{a}\right)}\right\}\right] e^{-i\omega t} \tag{13}$$

 

**Case-III**: If both cylinders oscillate with same amplitudes and same frequencies

i.e. $\nu_1 = \nu_2 = \nu$ (say) and $\omega_1 = \omega_2 = \omega$ (say) then from (11), it is found

$$w = \left\{ \frac{J_0(mr)\left\{Y_0\left(m\frac{b}{a}\right) - Y_0(m)\right\} - Y_0(mr)\left\{J_0\left(m\frac{b}{a}\right) - J_0(m)\right\}}{J_0(m)Y_0\left(m\frac{b}{a}\right) - Y_0(m)J_0\left(m\frac{b}{a}\right)} \right\} \nu e^{-i\omega t} \tag{14}$$

**Case-IV:** If magnetic field and porous medium both are withdrawn

i.e. H = 0 and K= ∞ then from (11), it is found

$$w = \nu_1 \left\{ \frac{J_0(mr)Y_0\left(m\frac{b}{a}\right) - Y_0(mr)J_0\left(m\frac{b}{a}\right)}{J_0(m)Y_0\left(m\frac{b}{a}\right) - Y_0(m)J_0\left(m\frac{b}{a}\right)} \right\} e^{-i\omega_1 t} + \nu_2 \left\{ \frac{J_0(nr)Y_0(n) - Y_0(nr)J_0(n)}{J_0(n)Y_0\left(n\frac{b}{a}\right) - Y_0(n)J_0\left(n\frac{b}{a}\right)} \right\} e^{-i\omega_2 t} \tag{15}$$

**Case-V:** If it is taken $\lambda_2 = 0$ in above results.

Then all results for velocity of Maxwell liquid are found.

**Case-VI:** If it is taken $\lambda_1 = 0$ and $\lambda_2 = 0$ in above results.

Then all results for velocity of purely viscous liquid are found.

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